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The Control Design on Non-Minimum Phase Nonlinear Systems with Relative Degree Two

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Abstract. The design of control for nonlinear systems is a very important issue in control theory. In this paper we will design control on a non-minimum phase nonlinear systems with relative degree two by using the input-output linearization method and redefined output. The design is begun with selecting the new output as a linear combination of the state of the system. From the design result, it is found the normal form has a stable zero dynamics so that non-minimum phase nonlinear systems become minimum phase with new output.

INTRODUCTION

The design of control in the non-minimum phase nonlinear systems is a very challenging and important issue that can be found in practical enginering applications. For example the problem of aircraft height control, the problem of determining the path of the rocket and others [1]. In [2] has been explained that if the normal form of a nonlinear system has a stable zero dynamic, the nonlinear systems is called minimum phase. Conversely, if the normal form of the nonlinear system has an unstable zero dynamic, the nonlinear systems is called non-minimum phase.

The design of control on the non-minimum phase nonlinear system has been done by many researchers. Hauser et al. in [3] has designed of control non-minimum phase nonlinear system based on a minimum phase nonlinear system approach. Next, Marino and Tomei in [4] have discussed the stability of non-minimum phase lower triangular nonlinear system by using dynamic output feedback control order $n+2(r-1)$, where n is the dimension of the system and r is the relative degree of the system. To design the control begins with the redefinition of the output so that the system has a minimum phase with a new output. furthermore [5] has discussed tracking the output of a non-minimum phase nonlinear system in which the input control is a static control obtained through input-output linearization method. The nonlinear class under review is assumed to be redefinition of the output so that the system can be appropriately linearized. Control design is performed for non-minimum phase nonlinear system with relative degree n, n is the dimension of the system. Meanwhile, [6] has developed modified steepest descent control to track output on non-minimum phase nonlinear system. The first step is to redefine the output so that the system can be precisely linearized or the system becomes minimum phase with the new output. Control design is performed for non-minimum phase nonlinear system with relative degree $n-1$, n is the dimension of the system. Next Ho et al. in [7] has designed control SISO non-minimum phase nonlinear system with input-output linearization and output redefinition for system with relative degree one.

In this paper we will design a control law for a non-minimum phase nonlinear system having a relative degree two by using input-output linearization and output redefinition.

LINEARIZATION AND ZERO DYNAMICS

Consider the following affine SISO non-minimum phase nonlinear control systems:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control input and , $y \in R$ is the measured output. $f : R^n \to R^n$ is a smooth function with $f(0) = 0$, $g : R^n \to R^n$ and $h : R^n \to R$ are smooth functions. Assume also that $h(0) = 0$.

The derivative function $y = h(x)$ to t is

$$\dot{y} = \frac{\partial h}{\partial t} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial h}{\partial x}(f(x) + g(x)u) = L_f h(x) + L_g h(x) u$$

The Lie derivative of $h(x)$ along $f(x)$ is expressed as $L_f h(x) = \frac{\partial h}{\partial x} f(x)$.

In the following we write the definition of relative degree of the nonlinear system (1)-(2) according to Isidori [2].

Definition 1. The nonlinear system (1)-(2) is said to have a relative degree r at a point x_0, if

(i) $L_g L_f^k h(x) = 0$ for all x in a neighborhood of x_0 and $k = 0,1,2,...,r-2$,

(ii) $L_g L_f^{r-1} h(x_0) \neq 0$.

Definition 1 means that we need to differentiate the output y respect to t of a system r times for obtain an explicit relationship between the output y and input u. Let the relative degree of the system is r, where $r < n$. Then we have

$$y^{(r)}(t) = a(x(t)) + b(x(t))u, \tag{3}$$

where

$$a(x(t)) = L_f^r h(x(t)) \text{ and } b(x(t)) = L_g L_f^{r-1} h(x(t)).$$

The following lemma is referred from the Isidori [2].

Lemma 1. Suppose the non linear system (1)-(2) has relative degree r at x_0, $r \leq n$.

$$\begin{aligned} \phi_1(x) &= h(x) \\ \phi_2(x) &= L_f h(x) \\ &\vdots \\ \phi_r(x) &= L_f^{r-1} h(x). \end{aligned}$$

If r is strictly less than n, it is always possible to find $n-r$ more functions $\phi_{r+1}(x), \phi_{r+2}(x), \ldots, \phi_n(x)$ such that the mapping

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \\ \vdots \\ \phi_n(x) \end{pmatrix}$$

has a jacobian matrix which is nonsingular at x_0 and therefore qualifies as a local coordinates transformation in a neighborhood of x_0. The value at x_0 of these additional functions can be fixed arbitrarily. Moreover, it is always possible to choose $\phi_{r+1}(x), \phi_{r+2}(x), \ldots, \phi_n(x)$ such that

$L_g \phi_i(x) = 0$, $\forall r+1 \leq i \leq n$ and $\forall x$ around x_0.

The following theorem is referred from the Isidori [2].

Theorem 1. Suppose a nonlinear system (1)-(2) is given. The state space exact linearization problem is solvable near a point x_0 (i.e. there exists an output function $\mu(x)$ which the system has relative degree n at x_0) if only if the following conditions are satisfied

1. the matric $\left(g(x_0)\, ad_f g(x_0) \cdots ad_f^{n-2} g(x_0)\, ad_f^{n-1} g(x_0)\right)$ has rank n,
2. the distribution $D = span\{g, ad_f g, \cdots, ad_f^{n-2} g\}$ is involutive near x_0.

According to the Theorem 1, if the nonlinear system (1)-(2) can be linearized exactly, there is an output function $\mu(x)$ such that the nonlinear system

$$\dot{x} = f(x) + g(x)u, \qquad x \in R^n,\ u \in R \tag{4}$$

$$y = h(x), \qquad y \in R \tag{5}$$

can be transformed to

$$\begin{aligned} \dot{z}_k &= z_{k+1}, \qquad k = 1,2,...,n-1 \\ \dot{z}_n &= a(z) + b(z)u \\ y &= z_1 = \mu(x) \end{aligned} \tag{6}$$

where

$$a(z) = L_f^n h(x) \text{ and } b(z) = L_g L_f^{n-1} h(x).$$

If the nonlinear system (1)-(2) have a relative degree r at x_0, where $r < n$, then system (1)-(2) can be transformed to

$$\dot{z}_k = z_{k+1}, \qquad k = 1,2,...,r-1 \tag{7}$$

$$\dot{z}_r = a(z,\eta) + b(z,\eta)u \tag{8}$$

$$\dot{\eta} = q(z,\eta) \tag{9}$$

$$y = z_1$$

where

$$z_1 = h(x),\ z_2 = L_f h(x) = \frac{\partial h}{\partial x} f(x),\ ...,\ z_r = L_f{}^{r-1} h(x) = L_f L_f{}^{r-2} h(x) = \frac{\partial (L_f{}^{r-2} h)}{\partial x} f(x),$$

$a(z,\eta) = L_f{}^r h(x) = L_f L_f{}^{r-1} h(x)$, $b(z,\eta) = L_g L_f{}^{r-1} h(x) = \dfrac{\partial (L_f{}^{r-1} h)}{\partial x} g(x)$

$(z,\eta) = (z_1, z_2, ..., z_r,\ \eta_1, \eta_2, ..., \eta_{n-r})$, and $\eta_1(x), \eta_2(x), ..., \eta_{n-r}(x)$ are choosen such that $\dfrac{\partial \eta_i(x)}{\partial x} g(x) = L_g \eta_i(x) = 0,\ i = 1,2,...,n-r$.

Next the system (7), (8) and (9) is called normal form of the system (1)-(2), with the internal dynamics $\dot{\eta} = q(z,\eta)$.

Consider normal form system (1)-(2). If $y(t) = 0,\ \forall t$, then $\dot{z}_1(t) = \dot{z}_2(t) = ... = \dot{z}_r(t) = 0$. Such that $z(t) = 0,\ \forall t$. Consequently (9) become

$$\dot{\eta} = q(0,\eta). \tag{10}$$

If internal dynamics with value $z = 0$, then equation (10) is called zero dynamics of the system (1)-(2). If zero dynamics of the system (1)-(2) stable, then system(1)-(2) is called weak minimum phase, and if zero dynamics of the system (1)-(2) asymptotic stable, then system (1)-(2) is called minimum phase. While if zero dynamics of the system (1)-(2) unstable, then system(1)-(2) is called non-minimum phase.

RESULTS

In this section we will design the control law for non-minimum phase nonlinear systems (1)-(2) with relative degree two using the new output

$$y_s = h_s(x) = \theta^T x \tag{11}$$

where θ is output vectors.

Consider the nonlinear system (1)-(2). The first derivative of y_s to t is:

$$\begin{aligned} \dot{y}_s &= \frac{\partial h_s}{\partial t} = \frac{\partial h_s}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial h_s}{\partial x}(f(x) + g(x)u) \\ &= L_f h_s(x) + L_g h_s(x)u = \theta^T f(x) + \theta^T g(x)u \end{aligned} \tag{12}$$

If $L_g h_s(x) = 0$, then $\dot{y}_s = L_f h_s(x) = \theta^T f(x)$ is not depends u explicitly. By differentiate $\dot{y}$ respect to t, obtained

$$\begin{aligned}\ddot{y}_s &= y_s^{(2)} = \frac{\partial L_f h_s}{\partial x}(f(x)+g(x)u)\\ &= \theta^T f'(x)(f(x)+g(x)u)\\ &= \theta^T f'(x) f(x) + \theta^T f'(x) g(x) u\end{aligned} \tag{13}$$

Next we find the controller u by choosing γ and λ such that the solution of

$$\ddot{y}_s + \gamma \dot{y}_s + \lambda y_s = 0 \tag{14}$$

convergent to zero. By Substitution $\dot{y}_s$ and $\ddot{y}_s$ we have:

$$\theta^T f'(x) f(x) + \theta^T f'(x) g(x) u + \gamma \theta^T f(x) + \theta^T x = 0 \tag{15}$$

So obtained controller

$$u = -(\theta^T f'(x) g(x))^{-1} \theta^T (f'(x) f(x) + \gamma f(x) + \lambda x) \tag{16}$$

EXAMPLES

1. Consider SISO affine the nonlinear system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2\\ \dot{x}_2 &= 3x_2 + x_1^3 + (2+\sin^2(x_4))u\\ \dot{x}_3 &= 5x_1 - 2x_3\\ \dot{x}_4 &= -x_4 + x_3^2\end{aligned} \tag{17}$$

$$y = x_1 \tag{18}$$

The system (17) can be written in the form:

$$\dot{x} = \begin{pmatrix} -1 & 1 & 0 & 0\\ x_1^2 & 3 & 0 & 0\\ 5 & 0 & -2 & 0\\ 0 & 0 & x_3 & -1\end{pmatrix}\begin{pmatrix} x_1\\ x_2\\ x_3\\ x_4\end{pmatrix} + \begin{pmatrix} 0\\ 2+\sin^2(x_4)\\ 0\\ 0\end{pmatrix} u \tag{19}$$

The normal form system (17) with output $y = x_1$ is:

$$\begin{aligned}\dot{z}_1 &= \dot{y} = \dot{x}_1 = z_2\\ \dot{z}_2 &= -\dot{x}_1 + \dot{x}_2 = 3z_1 + 2z_2 + z_1^3 + (2+\sin^2(\eta_2))u\end{aligned} \tag{20}$$

where $z_1 = x_1$ and $\eta = \begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = \begin{pmatrix} x_3\\ x_4\end{pmatrix}$.

Since we have differentiated the output z_1 respect to t of a system 2 times, we have obtained an explicit relationship between the output z_1 and the input u, then the system (17) - (18) has a relative degree two.

Next $\eta = \begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = \begin{pmatrix} x_3\\ x_4\end{pmatrix}$ must satisfy $L_g\eta(x) = \frac{\partial \eta}{\partial x} g(x) = \begin{pmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix} 0\\ 2+\sin^2(x_4)\\ 0\\ 0\end{pmatrix} = \begin{pmatrix} 0\\ 0\end{pmatrix}$.

Since $L_g\eta(x) = 0$, then lemma 1 is satisfied.

Therefore we obtained internal dynamics:

$$\dot{\eta} = \begin{pmatrix}\dot{\eta}_1\\ \dot{\eta}_2\end{pmatrix} = \begin{pmatrix}\dot{x}_3\\ \dot{x}_4\end{pmatrix} = \begin{pmatrix} 5z_1 - 2\eta_1\\ -\eta_2 + \eta_1^2\end{pmatrix} \tag{21}$$

Next checked the zero dynamics, that is when $z = 0$. We get

$$\dot{\eta} = \begin{pmatrix} -2\eta_1 \\ -\eta_2 + \eta_1^{\,2} \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \eta_1^{\,2} \end{pmatrix} \tag{22}$$

Therefore all eigen values are negative, then the zero dynamics locally stable asymptotic . So that minimum phase system (17) with output $y = x_1$.

Next if we are choose the controller

$$u = \frac{1}{2 + \sin^2(\eta_2)}\left(-3z_1 - 2z_2 - z_1^{\,3} - c_0 z_1 - c_1 z_2\right)$$

then equation (20) can be written as

$$\dot{z} = Az \tag{23}$$

where $A = \begin{pmatrix} 0 & 1 \\ -c_0 & -c_1 \end{pmatrix}$.

The characteristic polynomial of A is $p(\lambda) = c_0 + c_1\lambda$ where λ is characteristic value of A. If we choose $c_0 > 0$ and $c_1 > 0$, then $\lambda < 0$ so that the system (23) locally stable asymptotic. This means external dynamics locally stable asymptotic.

2. Consider SISO affine the nonlinear system

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= -3x_2 + x_1^{\,3} + (2 + \sin^2(x_4))u \\ \dot{x}_3 &= x_1 + 2x_3 \\ \dot{x}_4 &= x_4 + x_3^{\,2} \end{aligned} \tag{24}$$

$$y = x_1 \tag{25}$$

The system (24) can be written in the form:

$$\dot{x} = \begin{pmatrix} -1 & 1 & 0 & 0 \\ x_1^{\,2} & -3 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & x_3 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 + \sin^2(x_4) \\ 0 \\ 0 \end{pmatrix} u \tag{26}$$

The normal form system (24) with output $y = x_1$ is:

$$\begin{aligned} \dot{z}_1 &= \dot{y} = \dot{x}_1 = z_2 \\ \dot{z}_2 &= -\dot{x}_1 + \dot{x}_2 = z_1 - 4z_2 + z_1^{\,3} + (2 + \sin^2(\eta_2))u \end{aligned} \tag{27}$$

where $z_1 = x_1$ and $\eta = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix}$.

Since we have differentiated the output z_1 respect to t of a system 2 times, we have obtained an explicit relationship between the output z_1 and the input u, then the system (24) - (25) has a relative degree two.

Next $\eta = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix}$ must satisfy $L_g\eta(x) = \dfrac{\partial \eta}{\partial x} g(x) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 2 + \sin^2(x_4) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Since $L_g\eta(x) = 0$, then lemma 1 is satisfied. Therefore we obtained internal dinamics:

$$\dot{\eta} = \begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} z_1 + 2\eta_1 \\ \eta_2 + \eta_1^{\,2} \end{pmatrix} \tag{28}$$

Next checked the zero dynamics, that is when $z = 0$. We get

$$\dot{\eta} = \begin{pmatrix} 2\eta_1 \\ \eta_2 + \eta_1^{\,2} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \eta_1^{\,2} \end{pmatrix} \tag{29}$$

Therefore all eigen values are positive, then the zero dynamics unstable. So that non-minimum phase system (24) with output $y = x_1$.

Furthermore need to redefine the output of the system. If selected new output $\mu(x) = x_4$ from the system (24), then form normal is:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= a(z) + b(z)u \end{aligned} \tag{30}$$

where $z_1 = x_4$, $a(z) = x_4 + 8x_1^{\,2} + 85x_3^{\,2} + 6x_1x_2 + 54x_1x_3 + 10x_2x_3 + 2x_1^{\,3}x_3, b(z) = 2x_3 + x_3 \sin^2(x_4)$

Therefore the relative degree is 4. So that it is an exact linearization. Consequently (30) do not have zero dynamics, so that the system is stabilized. So that system (24) minimum phase with new output $\mu(x) = x_4$.

Next if we are choose the controller

$$u = \frac{1}{b(z)}\left(-a(z) - c_0 z_1 - c_1 z_2 - c_2 z_3 - c_3 z_4\right)$$

then equation (30) can be written as

$$\dot{z} = Az \tag{31}$$

where $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -c_0 & -c_1 & -c_2 & -c_3 \end{pmatrix}$.

The characteristic polynomial of A is $p(\lambda) = c_0 + c_1\lambda + c_2\lambda^2 + c_3\lambda^3$.

We use the Hurwitz Criterion method to find the roots of characteristic polynomial [8].

If we choose $c_0 > 0,\ c_1 > 0,\ c_2 > 0,\ c_3 > 0$ and $c_1 c_2 > c_0 c_3$, then obtained all the roots of the polynomial have negative real part so that the system (31) locally stable asymptotic. This means external dynamics locally stable asymptotic with new output $\mu(x) = x_4$.

CONCLUSION

In this research, the parameter value of θ for new output not yet determined as a method. But, based on the two examples, the parameter θ can be selected easily. By selecting the parameter θ the system becomes the minimum phase and by giving control, the external dynamic is locally stable asymptotic. In the future research we develope the method how to determine the parameter θ.

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