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Study Ethnomathematics: Classification of Geometrical Aspects of Traditional Timor Woven Fabrics by Ornamental Group

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Abstract. Study about Indonesian culture has been done by many researchers but still few who discuss that in terms of mathematical aspects of geometry. Aspects of geometry can be found in traditional Indonesian woven fabrics. By taking the research area in the Timor Island consisting of 39 fabrics from different regions. This study has the objective to find out the classification of Indonesian traditional woven fabrics from the ornamental group aspects. The results of the studies are expected to be used as the foundation in the making of new patterns for more varied variations. The ornamental group referred to in this studies are rosette group, frieze group and wallpaper group. The results showed that rosette group used were C_1 in three regions, C_2 in six regions, D_1 in nine regions, D_2 in 10 regions, D_4 in four regions, and D_6 in one region. While frieze group used were $\mathcal{F}_1$ in seven regions, $\mathcal{F}_2$ in eight regions, $\mathcal{F}_1^1$ in five regions, $\mathcal{F}_1^2$ in five regions, $\mathcal{F}_2^1$ in ten regions, $\mathcal{F}_2^2$ in one region, and $\mathcal{F}_1^3$ in one region. While wallpaper group used were W_1 in one region, W_2 in one region, W_1^3 in one region, W_2^1 in three regions, W_2^2 in six regions, W_2^3 in one region, W_4^1 in one region, W_6^1 in one region. Many ornamental groups have not been found in traditional woven fabrics on Timor Island.

INTRODUCTION

Indonesia has diverse culture ranging from traditional houses, traditional clothes, traditional weapons, traditional fabrics, to local languages. Each region has a different cultural diversity with certain characteristics. In Timor Island East Nusa Tenggara, traditional woven fabrics different in every village. It is interesting to be observed farther in terms of science, especially mathematics. Part of mathematics that studies a particular culture as the object of mathematical research is called Ethnomathematics. Ethnomathematics is the field of study which examines the way peoples from other culture understand, articulate and use concepts and practices which are from their culture and which the researcher describes as mathematical (Barton in Verawati, 2014: 9). Ethnomathematics has the six activities to develop mathematical idea in a culture that is counting, locating, measuring, designing, playing, and explaining (Bishop in Muhafidin, 2016: 19).

The frieze group is a mathematical concept to classify a two dimensional surface repeatable design in one direction, based on symmetry on the pattern. A two dimension frieze group that is initially one direction over and over again so that a complex group of wallpaper classifies repeating patterns in both directions. The former researcher studied Indonesian traditional fabrics "*batik*". Classical batik is composed of repetitive basic patterns (Gustyarini, 2012: 20). This research has the objective to classify traditional woven fabrics which compose of repetitive basic patterns. Therefore, the author want to do research to classify aspects of traditional Indonesian woven fabrics for ornamental group.

THEORETICAL MODEL

Ornamental Group

Theorem 1 (Leonardo's Theorem/Rosette group) (Martin, 1982: 67)
A finite group of isometrics is either a cyclic group C_n or a dihedral group D_n.

Lemma 1 (Rosette group) (Holme, 2010: 450)
Assume first G is a rosette group. Let G be a rosette group and denote by C the subgroup of rotations in G. Then there exist an integer n such that $C = C_n = \{S, S^2, \dots, S^{n-1}, S^n = I\}$ where S is the rotation about a fixed point O by the angle $\frac{2\pi}{n}$.
Theorem 2 (Frieze group) (Martin, 1982: 82)
Let $\mathcal{F}$ be a frieze group with center c whose translations from the group generated by translation τ. If $\mathcal{F}$ contains a half turn, suppose $\mathcal{F}$ contains σ_A; if $\mathcal{F}$ contains a reflection in a line perpendicular to c, suppose $\mathcal{F}$ contains σ_a with $a \perp c$. Let γ be the glide reflection with axis c such that $\gamma^2 = \tau$. Then, $\mathcal{F}$ is one of the seven different groups defined as follow
$\mathcal{F}_1 = \langle \tau \rangle$, $\mathcal{F}_1^1 = \langle \tau, \sigma_c \rangle$, $\mathcal{F}_1^2 = \langle \tau, \sigma_a \rangle$, $\mathcal{F}_1^3 = \langle \gamma \rangle$,
$\mathcal{F}_2 = \langle \tau, \sigma_A \rangle$, $\mathcal{F}_2^1 = \langle \tau, \sigma_A, \sigma_c \rangle$, $\mathcal{F}_2^2 = \langle \gamma, \sigma_A \rangle$.
Theorem 3 (Wallpaper group) (Holme, 2010: 460-468)
If W is a wallpaper group, then there are points and lines such that W is one of the 17 groups

W_1	W_2	W_4	W_3	W_6
W_1^1	W_2^1	W_4^1	W_3^1	W_6^1
W_1^2	W_2^2	W_4^2	W_3^2	
W_1^3	W_2^3			
	W_2^4			

define below.
W_1: The simplest type of wallpaper groups, with only translations. There are neither reflections nor glide reflections.
W_2: The wallpaper group with only translations and 2-fold rotations. It has no reflections nor glide reflections.
W_3: The wallpaper group with only translations and 3-fold rotations. It has no reflections nor glide reflections. All vertices are trigonal centers of rotation.
W_4: The wallpaper group with translations and 4-fold rotations. It also has rotations of order 2, with centers midway among the 4-fold centers. There are no reflections nor glide reflections.
W_6: Besides the translations, it contains 6-fold rotations and also 2-fold and 3-fold ones. It has no reflections nor glide reflections.
W_1^1: These wallpaper groups are generated by two translations and a reflection in a line bisecting the angle among the translations.
W_1^2: These wallpaper groups are generated by two orthogonal translations and reflection in a line parallel to one translation.
W_1^3: The wallpaper group generated by a glide reflection and a translation orthogonal to the glide axis. There are neither rotation nor reflections.
W_2^1: Two sets of parallel mirror lines, mutually perpendicular. Running horizontally is a set of parallel glide lines. The group thus has reflections in two perpendicular directions, and a rotation of order two whose center is not on a reflection axis. It also has the two rotations whose centers are on a reflection axis.
W_2^2: It obtained by enlarging a rectangular group of type W_2 by a reflection in a line passing through a diad. By rotation this gives a second, perpendicular, line of reflection. The group contains perpendicular axes of reflection, with 2-fold centers of rotation where the axes intersect.
W_2^3: Here there is a line of reflection which does not contain diads. There are glide reflections perpendicular to the lines of reflection.
W_2^4: A glide reflection in a line passing through a diad leads to this symmetry type with two sets of parallel glide lines and 2-fold rotations.
W_3^1: It has 3-fold rotations, all their centers lie on the reflection axes. The lines of reflections, inclined $\frac{\pi}{3}$ to one another, contain the shorter diagonals of the hexagon.
W_3^2: It contains reflections, with lines inclined $\frac{\pi}{3}$ to each other and rotations of order 3. The lines of reflection contain the longer diagonals of the hexagon.
W_4^1: The group has both order 2 and order 4 rotations. The four axes of reflection contain a tetrad, a center of 4-gonal rotation. Every rotation center lies on some reflection axes.

W_4^2: The group contains reflections and rotations of orders 2 and 4. There are two perpendicular reflection lines passing through each center of order 2 rotation. But the axes of reflection do not contain a tetrad. There are four directions of glide reflections.
W_6^1: W_6 enlarged by a reflection.

RESULT

This research was sourced from Acjadi about woven fabrics from Timor Island, East Nusa Tenggara, Indonesia (Acjadi, 2015: 22-92). Research was conducted on 39 fabrics with ten different motives from 16 regions. The fabrics are

(1) Tais. North Amanatun, South Central Timor.
(2) Tais. North Amanatun, South Central Timor.
(3) Tais. Amanatun, South Central Timor.
(4) Tais. Amanatun, South Central Timor.
(5) Mau. Amanatun, South Central Timor.
(6) Netpala, Molo, South Central Timor.
(7) Netpala, Molo, South Central Timor.
(8) Netpala, Molo, South Central Timor.
(9) Tais. Napi, Amanuban, South Central Timor.
(10) Tais. Amanuban, South Central Timor.
(11) Tais. Ainuit, Insana, North Central Timor.
(12) Tais Feto. Malaka, Belu.
(13) Tais. Biboki, North Central Timor.
(14) Tais. Oekabiti, Amarasi, Kupang.
(15) Mau. Boti, Insana, North Central Timor.
(16) Insana, North Central Timor.
(17) Beti. Insana, North Central Timor.
(18) Beti. Oekabiti, Amarasi, Kupang.
(19) Beti. Molo/Fatuleu.
(20) Tais Feto. Manulea, Malaka, Belu.
(21) Tais Feto. Fatuaruin, Belu.
(22) Tais Panaalon. Kewar, Belu.
(23) Tais Runat. Kupang.
(24) Tais Feto. Belu.
(25) Tais Feto. Malaka, Belu.
(26) TAIS MANE. Belu.
(27) TAIS MANE. Malaka, Belu.
(28) Tais. Insana, North Central Timor.
(29) Tais. Insana, North Central Timor.
(30) Tais. Insana, North Central Timor.
(31) Tais. North Central Timor.
(32) Tais. North Central Timor.
(33) Tais. Fatuleu, Kupang.
(34) Tais. Molo, South Central Timor.
(35) Tais. Oinlasi, South Central Timor.
(36) Tais. Oinlasi, South Central Timor.
(37) Tais. Molo, South Central Timor.
(38) Tais. Amanatun, South Central Timor.
(39) Tais. Amanatun, South Central Timor.

Rosette group G has the two cases to consider. First that consists of only rotations, no reflections, so $G = C = C_n$. Thus $G = \{S, S^2, \ldots, S^{n-1}, S^n = I\}$, the group of *n*-fold rotations, for some integer *n*. Next consider the case when the group G of transformations contain rotations and reflections. The rotations are all about the same center and are powers of the rotations S by $\frac{2\pi}{n}$ for some integer *n*.

The rosette group C_1 consists of only rotation with 1-fold rotation. In other way, figure back to origin place when rotate 360°. It is found in three regions: Oinlasi, Amanatun, and North Central Timor with four different fabrics. The motives are shown below.


FIGURE 1. Tais (36)


FIGURE 2. Tais (38)


FIGURE 3. Tais (35)


FIGURE 4. Tais (32)

The rosette group C_2 consists of only rotation with 2-fold rotation. The angles of rotation are showed in black line and the centers of rotation in red. It is found in six regions: Amanuban, Amanatun, Insana, Belu, Oekabiti, and Biboki with 11 different fabrics. The motives are shown below.


FIGURE 5. Tais (14)


FIGURE 6. Tais (10)


FIGURE 7. Mau (5)


FIGURE 8. Tais (29)

The rosette group D_1 consists of 1-fold rotation and reflection. Line symmetries are showed in green line. It is found in nine regions: Amanuban, Amanatun, Netpala, Belu, North Central Timor, Insana, Oinlasi, Oekabiti, and Molo with 18 different fabrics. The motives are shown below.

FIGURE 9. Tais (10)

FIGURE 10. Tais (2)

FIGURE 11. Netpala (6)

FIGURE 12. Tais Mane (26)

FIGURE 13. Tais Feto (20)

FIGURE 14. Tais (31)

FIGURE 15. Tais Feto (21)

FIGURE 16. Tais (4)

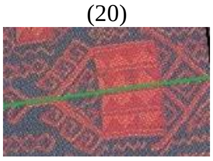


FIGURE 17. Tais (38)

FIGURE 18. Tais (37)

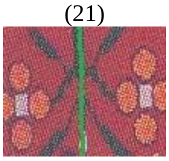


FIGURE 19. Tais (35)

FIGURE 20. Tais (32)

The rosette group D_2 consists of 2-fold rotation and reflection. Line symmetries are showed in green line, angles of rotation are showed in black line and the centers of rotation in red. It is found in ten regions: Amanatun, Insana, Amanuban, Kupang, Molo, North Central Timor, Belu, Oekabiti, Oinlasi, and Fatuleu with 34 different fabrics. The motives are shown below.

FIGURE 21. Tais (2)

FIGURE 22. Netpala (8)

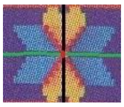


FIGURE 23. Tais (36)

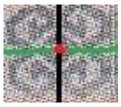


FIGURE 24. Mau (15)

FIGURE 25. Tais (4)

FIGURE 26. Tais (10)

FIGURE 27. Tais (33)

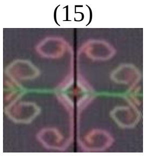


FIGURE 28. Insana (16)

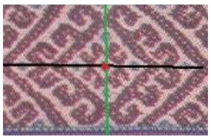


FIGURE 29. Tais (14)

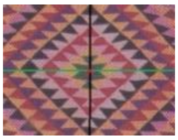


FIGURE 30. Netpala (7)

FIGURE 31. Beti (17)

FIGURE 32. Tais (3)

The rosette group D_4 consists of 4-fold rotation and reflection. Line symmetries are showed in green line, angles of rotation are showed in black line and the centers of rotation in green. It is found in four regions: Amanatun, Oinlasi, Molo, and Insana with eight different fabrics. The motives are shown below.

FIGURE 33. Tais (3)

FIGURE 34. Beti (19)

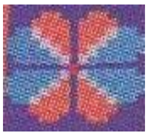


FIGURE 35. Tais (36)

FIGURE 36. Netpala (6)

The rosette group D_6 consists of 6-fold rotation and reflection. In other way, figure back to origin when rotate 60°. Line symmetries are showed in green line, angles of rotation are showed in black line and the centers of rotation in pink. It is found in one region: Belu. The motive is shown below.

FIGURE 37. Tais Feto (25)

A group of isometrics that fixed a given line c and whose translations form an infinite cyclic group is frieze group with center c. The frieze group $\mathcal{F}_1$ is only generated by translation $\langle\tau\rangle$. It has no point of symmetry, has no line of symmetry, and is not fixed by glide reflection. Two consecutive black line shows unit cell. It is found in seven regions: Kupang, Insana, Amanatun, Oekabiti, Belu, Amanuban, and Oinlasi with 14 different fabrics. The motives are shown below.

FIGURE 38. Tais (36)

FIGURE 39. Tais (35)

FIGURE 40. Tais Runat (23)

The frieze group $\mathcal{F}_2$ is generated by translation and half turn $\langle\tau, \sigma_A\rangle$. It has a point of symmetry but no line of symmetry. It is found in eight regions: Belu, Oinlasi, Insana, Amanatun, Amanuban, Oekabiti, North Central Timor, and Molo with 16 different fabrics. The motives are shown below.

FIGURE 41. Tais (30)

FIGURE 42. Tais (35)

FIGURE 43. Tais (32)

The frieze group $\mathcal{F}_1^1$ is generated by translation and reflection $\langle\tau, \sigma_c\rangle$. It has no point of symmetry and the center is a line of symmetry. Two consecutive black line show unit cell and green line shows line of symmetry on the center. It is found in five regions: Belu, Oinlasi, Amanatun, Molo, and North Central Timor with 11 different fabrics. The motives are shown below.

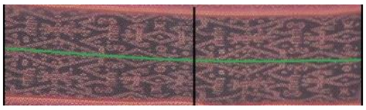


FIGURE 44. Tais Feto (21)

FIGURE 45. Tais (38)

FIGURE 46. Tais (35)

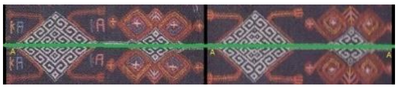


FIGURE 47. Tais (1)

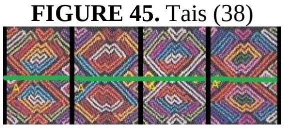


FIGURE 48. Tais (3)

FIGURE 49. Mau (5)

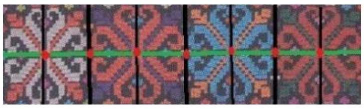
The frieze group $\mathcal{F}_1^2$ is generated by translation and reflection $\langle\tau, \sigma_a\rangle$. It has no point of symmetry, has a line of symmetry, but the center is not a line of symmetry. Two consecutive black line show unit cell and green line shows line of symmetry perpendicular with the center. It is found in five regions: Amanatun, Belu, Insana, Molo, and North Central Timor with seven different fabrics. The motives are shown below.

FIGURE 50. Tais (32)

FIGURE 51. Beti (17)

FIGURE 52. Netpala (6)

The frieze group $\mathcal{F}_2^1$ is generated by translation, half turn and reflection $\langle\tau, \sigma_A, \sigma_c\rangle$. It has a point of symmetry and the center is a line of symmetry. Green line shows line of symmetry on the center, red polygon shows center of half turn. It is found in ten regions: North Central Timor, Belu, Amanatun, Insana, Fatuleu, Oekabiti, Molo, Amanuban, Oinlasi, and Kupang with 32 different fabrics. The motives are shown below.



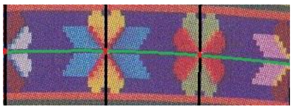


FIGURE 53. Netpala (6) **FIGURE 54.** Tais (36) **FIGURE 55.** Insana (16)



FIGURE 56. Beti (17) **FIGURE 57.** Tais (37) **FIGURE 58.** Tais (33)

The frieze group $\mathcal{F}_2^2$ is generated by glide reflection and half turn $\langle \gamma, \sigma_A \rangle$. It has a point of symmetry, has a line of symmetry, but the center is not a line of symmetry. Green line shows line of symmetry, red polygon shows center of half turn, two consecutive black line show unit cell. It is found in one region: Insana. The motive is shown below.

FIGURE 59. Tais (29)

The frieze group $\mathcal{F}_1^3$ is generated by glide reflection $\langle \gamma \rangle$. It has no point of symmetry, has no line of symmetry, but is fixed by glide reflection. It is found in one region: Amanatun. The motive is shown below.

FIGURE 60. Tais (1)

The wallpaper group W is group of isometric whose translations in two directions. The wallpaper group W_1 is generated by only translations. There are neither reflections, nor glide reflections. It is found in one region: Oinlasi with two different fabrics. The motives are shown below.

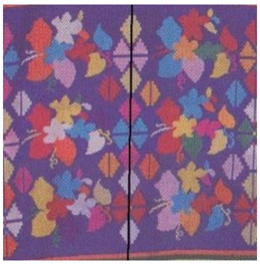


FIGURE 61. Tais (36)

FIGURE 62. Tais (35)

The wallpaper group W_2 is generated by translations and 2-fold rotations. It has no reflections nor glide reflections. It is found in one region: Belu. The motive is shown below.

FIGURE 63. Tais Feto (12)

The wallpaper group W_1^3 generated by a glide reflection and a translation orthogonal to the glide axis. There are neither rotation nor reflections. Grey area shows rectangle unit cell, white area in unit cell shows base for wallpaper, green line shows line of symmetry. It is found in one region: Belu. The motive is shown below.

FIGURE 64. Tais (28)

The wallpaper group W_2^1 has reflections in two perpendicular directions, and a rotation of order two whose center is not on a reflection axis. It also has the two rotations whose centers are on a reflection axis. Grey area shows rhombic unit cell, white area in unit cell shows base for wallpaper, green line shows line of symmetry, and red polygon shows order 2 rotation. It is found in three regions: Insana, Belu and Molo with four different fabrics. The motives are shown below.

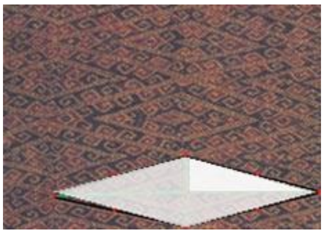


FIGURE 65. Tais Feto (12)

FIGURE 66. Tais (37)

The wallpaper group W_2^2 contains perpendicular axes of reflection, with 2-fold centers of rotation where the axes intersect. Grey area shows rectangle unit cell, white area in unit cell shows base for wallpaper, green line shows line of symmetry, and red polygon shows order 2 rotation. It is found in six regions: Molo, Oekabiti, Belu, Insana, North Central Timor, and Fatuleu with 12 different fabrics. The motives are shown below.

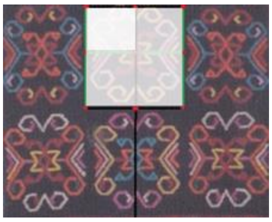


FIGURE 67. Insana (16)

FIGURE 68. Mau (15)

The wallpaper group W_2^3 has glide reflections perpendicular to the lines of reflection. It is found in region: Molo. The motive is shown below.

FIGURE 68. Tais (37)

The wallpaper group W_4^1 has both order 2 and order 4 rotations. The four axes of reflection contain a tetrad, a center of 4-gonal rotation. Every rotation center lies on some reflection axes. Grey area shows square unit cell, white area in unit cell shows base for wallpaper, green line shows line of symmetry, green polygon shows 4 order rotation, and red polygon shows order 2 rotation. It is found in one region: Molo with two different fabrics. The motives are shown below.

FIGURE 69. Netpala (6)

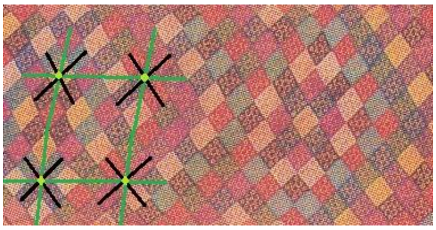


FIGURE 70. Beti (19)

The wallpaper group for W_6^1 has order 6 rotations and reflection. Grey area shows parallelogram unit cell, white area in unit cell show base for the wallpaper, green line shows line of symmetry, red polygon shows order 2 rotation, green polygon shows order 4 rotation, pink polygon showed order 6 rotation. It is found in region: Belu. The motive is shown below.

FIGURE 71. Tais Feto (25)

CONCLUSION

Indonesian traditional woven fabrics have the classification by ornamental group. Ornamental group consists of rosette group, frieze group and wallpaper group. The rosette groups found in this research was C_1 C_2, D_1, D_2, D_4 and D_6. The most widely used rosette group was D_2 of 34 fabrics from 10 regions. An undiscovered rosette groups were centered rotation of 3, 5, 7, so on. The interesting thing is the 3-centered rotation that is not found when it look easy to do. This can be used as research material next. Seven frieze groups are found in fabrics, with the most used was $\mathcal{F}_2^1$ of 32 fabrics from 10 regions. The rarely used types were $\mathcal{F}_2^2$ and $\mathcal{F}_1^3$.The wallpaper groups found in this research was W_1, W_2, W_1^3, W_2^1 , W_2^2 , W_2^3 , W_4^1 , and W_6^1 . The most used was W_2^2 of 12 fabrics from six regions. From 17 wallpaper groups still many that have not been used as a pattern on the fabrics in Timor Island, East Nusa Tenggara, Indonesia.

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