Understanding How Blind Student Learn Rigorous Mathematical Thinking on Two-Dimensional Shapes

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Abstract. The rigorous mathematical thinking of blind student is blind student's mental procedure with synthesis and utilization of cognitive tools for mathematical conceptual formation. This procedure is done by a learning experience that involves cognitive functions. There are three level cognitive functions in rigorous mathematical thinking i.e. level 1, 2, and 3. This cognitive functions can't be viewed as a natural maturational path but must be actively constructed during the learning process. The objective of this research is to understand how blind student learn rigorous mathematical thinking when given geometry models. This research was an exploratory study using a qualitative approach. The subject of this research was the fourth-grade student of an Elementary School for exceptional children in Sidoarjo-East Java, who was totally blind by birth. The researcher interviewed the subject about his rigorous mathematical thinking on two-dimensional shape's concept. This rigorous mathematical thinking based on rigorous mathematical thinking model of Kinard and Konzulin. The results of this study show: (1) almost all blind student's cognitive functions are at levels 1 and level 2; (2) some of his cognitive functions are at level 3; (3) there is a uniqueness of his thinking representation on two-dimensional shapes.

Keyword: blind, rigorous mathematical thinking, two-dimensional shapes

INTRODUCTION

The learning concept of mathematics at the elementary school level is done by using concrete to the abstract procedure. According to [1], the basic object of mathematics is abstract in nature because this is a mental object. Indeed, this learning not easy to do by teachers. They are demanded to minimize the characteristic of abstract within the concept mathematics according to the level of students' development. Whereas not all of them are able to optimize their learning ability, particularly students with special needs who have a limitation in acquiring certain sensory knowledge such as blind students.

Blindness can lead to limited number and types of experience, mobility, and interaction with the surrounding environment. Such limitations affect their cognition, language, and social development. The major impact of blindness on students' development of conceptions and cognition is that they will visualize objects with tactual experience, while normal students will do so visually (Lowenfeld in [2]). Furthermore, [3] stated that the development of blind conception depends on the level of blindness and the age when it occurred. In line with this opinion, [4] also stated that there is a difference between the perception of someone who blinds by birth and sudden loss of vision. Blind students who have visual experience before the blindness will be able to use such experience to understand the concept they learn.

Loss of visual information access demands blind students to substitute the sense of their eyesight with other working-senses. Without this substitution, they cannot feel the objects around them both in terms of size and movement ([5], [6]).

According to [7], blind people use the sense of touch to adjust and manipulate objects around them. This sense of touch is active and informative for students' perceptual system [8]. Through this sense of touch, blind students are able to maximize their tactical ability which is affected by synthetic and analytical perception [9]. Nevertheless, it has a limitation of range and it demands greater conscious effort to use it. In general, tactical ability will work only if it is used for cognition needs. Meanwhile, sight is active as long as the eyes are open.

Since the cognitive function can't work properly without activation, so the teacher should apply rigorous mathematical thinking approach. It is based on a theory of physiological tools and a concept of students' learning experience. Through this approach, students are able to construct their mathematical conceptual formation based on their experience. With this approach, the student not only in linking the knowledge that has been previously owned with the problems that are facing but also develop new ideas in finding an appropriate solution in solving mathematical problems ([10], [11]).

As stated by [12], in fact, considering that the structure of mathematics which is not appropriate for blind students' needs, they tend to have problems when learning this subject. One branch of mathematics that has difficulties potential for taught to blind students is geometry. In geometry learning many physical models used to understand and explain concepts related to the measurement of length, surface area, and volume [13]. Although the environment and the object of play during their development stage use a lot of geometry models, there are still many students who consider geometry as a complicated subject because geometry has ideal material ([14], [15]). In addition to the ideal material problem, according to the results of a study from [16], it is also known that students with visual impairments have special obstacles in understanding geometric images.

In another side, also many teachers who have difficulty in giving instructions on how to learn geometry for blind students even though they use physical models [17]. According to [18], this difficulty of teaching geometry is because blind students need a long time to construct mental representations of their spatial concepts.

Various obstacles and difficulties experienced by the blind, does not mean closed the access of the blindness to learn geometry because geometry is a fundamental skill that students need to understand [19]. In fact, blind students need more support in accessing various public facilities, transportation, and education ([20], [21],[22]).

Viewing the importance of student's rigorous mathematical thinking, the limitations and difficulties of blind students in understanding geometry, therefore the researcher is interested to conduct research about how blind student rigorous mathematical thinking on geometry, especially two-dimensional shapes. In regard to this topic, there are many interesting things obtained, such as how blind students recognize two-dimensional shapes they never see so that they will rely on their limited experience and tactile ability.

METHOD

This research was an exploratory study with a qualitative approach which aims to understand how blind student learn rigorous mathematical thinking when given geometry models. The question of interviews consisted of questions about the subject's thinking of two-dimensional shapes especially square and rectangle. The Subject chosen was employed as a source of qualitative data, and he is one of the fourth-grade students who experienced a total blind by birth from Elementary School for exceptional children in Sidoarjo-East Java.

Data on how blind student learn rigorous mathematical thinking on the two-dimensional shapes were collected by means of task-assisted interview and the interviews were carried out two times by using equivalent problems on two different times. A subject is assigned to recognize models of the two-dimensional shapes with a different size that given researcher. Then, he is assigned to explain or to illustrate his thinking about the shapes that his recognize in their own way, for example by describing the attribute of this shapes.

The data collected from the interviews were then transcribed, classified, reduced and validated by using times triangulation methods to yield credible data. The credible data were analyzed by using qualitative research analysis data method including data display interpreting and conclusion drawing as [23] statement.

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RESULT AND DISCUSSION

The subject began the activity of thinking one of the two-dimensional shapes, there was square, by naming the models that given by researcher. A subject recognized the models of various size as a square based on shape's attribute, there was the length of a side and right angle. According to the subject, the critical attributes that belong to the square had the length of a side same and right angle. In this case, the subject was at the level 1 cognitive function of labeling-visualizing such as the level of cognitive functions accorded by [24]. The cognitive functions of rigorous mathematical thinking were the cognitive function of qualitative thinking or level 1, the cognitive function of quantitative thinking with thoroughness or level 2, and the cognitive functions of relational thinking. Although the square models had different sizes, it was recognized because it had four sides of the same length and four right angles (adjacent sides perpendicular to each other). In this case, the subject was at the level 1 cognitive function of comparing and the level 2 cognitive function of conserving constancy. By showing the total of four sides and the four corners of the models, the subject was also at the level 2 cognitive function of analyzing and integrating, but not the generalizing cognitive.

A subject identified the sides of each square models by used his tactile. The lengths of each side were measured by counted the total of times the finger inched along the side of the shape that was identified. For the right angle, a subject checked the perpendicular to a pair of adjacent sides of the model. The subject touched a pair of short sides (less than 5 cm) by used the thumb and index finger simultaneously, so that both fingertips met together at one point, as in the following figure 1.

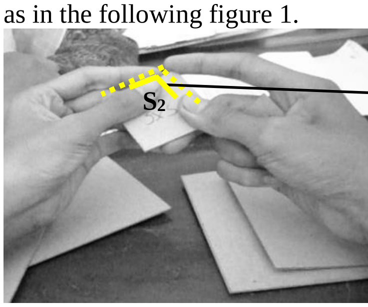

Figure 1. Both fingertips met together at one point on square

In this case, the subject was at the level 1 cognitive function of searching systematically to gather clear and complete information. Besides at the level 1, this activity also showed that the subject was at the cognitive function of the use of using more than one source of information because mentally he used more than one concept or point of view at the same time to identify the shapes, i.e. size, and shape of the models.

When the subject touched a pair of adjacent sides of the model actually, the subject was at the level 2 of quantifying space and spatial relationships because he used his previously owned spatial knowledge as a guide for analyzed spatial relations and spatial representations. In this activity actually, the subject was also at the level 3 cognitive function of activating prior mathematically related knowledge, that was knowledge related to line concept and kind of angle as characteristics which owned by two-dimensional shapes.

Then, the subject was asked to illustrate his explanation of the right angle attribute in the shape. Using both of his index fingers, the subject formed both of fingers in an upright position as a representation of the right angle, as in the following figure 2.

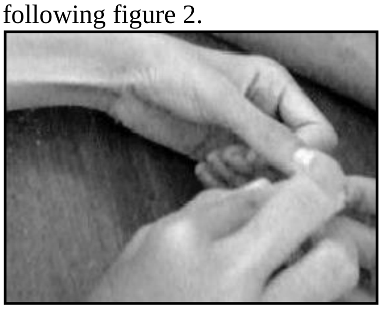

Figure 2. Representation of angle right

In this case, the subject was at the level 1 cognitive function of encoding- decoding.

When the subject was asked about the meaning of the symbol perpendicular to his two fingers, the subject explained that the two fingers represented as the right angles were not in a sloping position or form a pointed angle. In this case, the subject is at level 3 of the cognitive function of providing also articulating mathematical logical evidence.

The subject began a rectangular thinking activity by naming the model given by the researcher. Subject recognized the model of various sizes as rectangles based on form attributes, there were side lengths and right angles. According to the subject, the critical attributes that belong to the rectangle had the length sides, the short sides, and the right angle. In this case, the subject was at the level 1 cognitive functions of labeling-visualization such as the level of the cognitive function given by Kinard and Konzulin. Although the rectangle model had different sizes, it was recognized because it had two sides that the same length, two sides that the same short and four right angles (adjacent sides perpendicular to each other). In this case, the subject was at the level 1 cognitive function of comparing and the level 2 cognitive functions of conserving constancy. By showing the total of two sides that the same length and the same short, also the four corners of the models, the subject was also at the level 2 cognitive function of analyzing and integrating, but not the generalizing cognitive.

The subject identified the sides of each rectangle models by used his tactile. The lengths of each side were measured by counted the total of times the finger inched along the side of the shape that was identified. For the right angle, a subject checked the perpendicular to a pair of adjacent sides of the model. The subject touched a pair of short sides (less than 5 cm) using the thumb and index finger simultaneously, so that both fingertips met together at one point, as in the following figure 3.

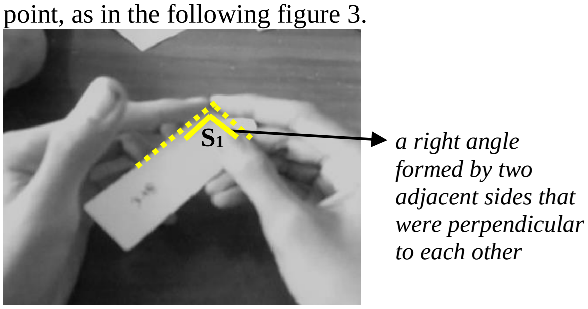

Figure 3. Both fingertips met together at one point on rectangle

In this case, the subject was at the level 1 cognitive function of searching systematically to gather clear and complete information. Besides at the level 1, this activity also showed that the subject was at the cognitive function of the use of using more than one source of information because mentally he used more than one concept or point of view at the same time to identify the shapes, i.e. size, and shape of the models.

When the subject touched a pair of adjacent sides of the model, actually the subject was at the level 2 of quantifying space and spatial relationships because he used his previously owned spatial knowledge as a guide for analyzed spatial relations and spatial representations. In this activity actually, the subject was also at the level 3 cognitive function of activating prior mathematically related knowledge, that was knowledge related to line concept and kind of angle as characteristics which owned by two-dimensional shapes.

Then, the subject was asked to illustrate his explanation of the right angle attribute in the shape. Using both of his index fingers, the subject formed both of fingers in an upright position as a representation of the right angle. In this case, the subject was at the level 1 cognitive function of encoding- decoding.

In thinking activity about the square and rectangle concepts, it turned out the subject focused on the attributes of the side and angle of the shape, so that the subject could be said to be at the level 2 cognitive function of being precise. However, the subject couldn't yet be formed a mathematical proposition and proved mathematically the truth of the proposition. Therefore, at the level 3 cognitive function of inferential- hypothetical thinking, projecting and restructuring relationships, as well as other higher cognitive functions hasn't been achieved by the subject.

From different activities above, it is found that the subject abstraction had appeared as he can recognize similarities characteristics model of two-dimensional shapes that had different sizes, even he could be collected the same characteristics of similar models of two-dimensional shape and related them with the formal definition of the shape. But in this abstraction activity, the subject just couldn't construct the relationship between the two-dimensional shapes.

CONCLUSION

The blind student began the activity of thinking on two-dimensional shapes, either square or rectangle, by naming the shape. Almost all blind student's cognitive functions in his rigorous mathematical thinking are at levels 1 and level 2. At the level 3, the blind student could achieve only a few cognitive functions. So, the rigorous mathematical thinking on two-dimensional shapes that has been achieved was at the cognitive function of qualitative thinking or level 1, the cognitive function of quantitative thinking with thoroughness or level 2, as well as some cognitive functions of relational thinking or level 3. There is a uniqueness of his rigorous mathematical thinking representation on two-dimensional shapes, i.e. utilization the tactile to identified sides and angles of two-dimensional shapes

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