

Analysis of Metacognitive Skills of Undergraduate Students in Solving Math Problems

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Abstract. This study aimed to reveal metacognitive skills of undergraduate students in mathematics education major, in solving mathematics problems. Metacognitive skills prefer to procedural knowledge needed to control one's learning habits. This skill can help in planning several effective strategies and find the most efficient strategy to achieve the goal or make a right decision to solve the problems. To understand and analysis metacognitive skills in mathematics, this research used math test which was examined 16 undergraduate students in mathematics education major between around 20 – 21 years old in 6th semester from an University in Bandung by followed interviews. The topics in the test were about trigonometry, triangle, and algebra. The other sources of the data are from questionnaire of metacognitive skills. After that, the qualitative research method was used to analyse the data. The result shows that the majority approached the questions by some various strategies. About 83,5% participants could make plan to find solution by several attempts using various formulas. They performed identifying and formulating the problems, drawing triangle, putting forward several suspicions or temporary solutions and the strategies used, and then finding the solution and make conclusion about the most effective strategy to solve the problems.

INTRODUCTION

Cognitive development on transition stage from teenagers to adulthood may be characterized by skill in reflective thinking or abstract thought [1]. In the transition stage, there are possibly happen change on various intellegences, personality, and particularly in healthy and social behaviour [2]. Besides that, the way of new thinking of the adult is denoted by pascaformal thought, combining logics with emotions and practical experience in solving confused problems[2]. This is similarly the definition of critical thinking process. Critical thinking process is a skill to solve problems rationally according to logical steps and provide solutions more efficient [3]. Critical thinking process is a part of ability very necessary for facing the advanced technology in the modern ages to make proper and fast decision which depends on logical and rational thinking and considering some different aspects. One of the way to improve critical thinking skill is by metacognition. Research about metacognition and its implication for learning and instruction have become a central issue in education [10]. Metacognition is very required to understand a problem and how to solve it. Metacognition in particular is essential because it impacts achievement, understanding, storing, and applying what have been learned, also it impacts effectivity of learning, critical thinking, and solving problem [4].

Metacognitive skills pertain to the acquired ability of monitoring, guiding, steering, and controlling one's learning and problem-solving behavior [5]. The skills is assumed as a metacognitive skills are considered to be an organized set of metacognitive self-instructions for the monitoring of and control over cognitive activity [6]. The advantage of the metacognitive skills is an individu will be able to learn effectively for her/him sake to improve other abilities, like problem solving skill. Metacognitive skills can help in planning strategy, determining goal and decision, predicting uncomplete data, making multiple deliberation, improving elaboration of a problem sollution and a plan [7]. Metacognitive skills in solving problem, specially math, will help an individual to make some

strategies which potentially will be effective to find a solution and the best strategy to achieve the goal or make a proper decision.

A research indicates that children were not good at describing their own abilities, meanwhile, they will be better when they grow up [9]. The pattern of children who have low metacognitive skills will impact selection of strategies regarding to failure in achieving the goal [9]. The lack of metacognitive skills will effect an individual to habits of doing many activities without knowing the goals and the reasons why they do [3]. Meanwhile, students with advanced metacognitive skills are those who are aware of what they have learned and what they do not know [10]. This study focus on metacognitive skills whose 20-21-year-old undergraduate students in 6th semester in mathematics education major of a University in Bandung, Indonesia. The research will analyse how their metacognitive skills in solving math problems.

THEORETICAL BACKGROUND

Metacognitive Skills

Metacognitive skills are abilities to understand one's own process thinking based on the reason why they think about. The metacognitive skills are the abilities to understand and analyze one's own learning is especially influenced by educational background and previous experience [16]. Metacognitive skills can help students to assess the demands of the task, evaluate their own knowledge and skills, plan their approach, monitor their progress, and adjust their strategies [17]. Meichenbaum [18] said that Metacognition refers to awareness of one's own knowledge—what one does and doesn't know—and one's ability to understand, control, and manipulate one's cognitive processes. Some said metacognitive skills as regulation of cognition. Regulation of cognition involves: setting goals and planning; monitoring and controlling learning; and evaluating one's own regulation (assessing results and strategies used) [19].

Metacognitive skills refer to procedural knowledge that is required for the regulation of control over one's learning behaviour [6]. Metacognitive knowledge and metacognitive skills have a distinction. Metacognitive knowledge refers to the knowledge about the cognitive system, while metacognitive skills concern the regulation of cognitive process [10]. Flavell emphasized the role that metacognitive skills play in the development of memory functioning in children, and proposed a conceptual framework that is much more extended than that which underlies cognitive-based research on metacognition [12]. M. Carr and G. Taasoobhirazi said that metacognitive skills needed to achieve elite status [13]. Also, they said that metacognitive skills and knowledge help jump start the early emergence of expertise and accelerate the process. It is clearly seen that metacognitive skills is useful for individual to be an expert in their field and accelerate the thinking process.

How To Measure Metacognitive Skills

Akturt et al said that simultaneous measurement of metacognition is implemented using tools that cause considerable loss of time and require that participants be evaluated individually, students tell you verbally how they handle a certain problem [10]. He also said that the questionnaire is one of the most frequently used tools for measuring metacognition [10,14]. Because of that, this study is not only math problem to measure metacognitive skills of undergraduate students, but also the questionnaire of metacognitive skills. This research also use interview to strengthen the findings and understanding the thinking process of students in solving mathematics problems. Interviews are useful in that they enable an in-depth investigation of students' ideas. Interviews have the power to demand the students to expand on the answers that they have given if they have responded to the interview questions in the form of "yes" and no [9].

Research Questions

Talking all above into account, as mentioned in the introduction that this research concerns on how undergraduate students' metacognitive skills in solving math problems, this study proposes the research questions as follows:

1. How do undergraduate students use their metacognitive skills in solving a math problem about trigonometry?
2. How do undergraduate students use their metacognitive skills in solving a math problem about algebra?

METHODOLOGY

To answer the research question, I offered an individual written test followed by metacognitive questionnaire and interviews based on the student's work. The research method used is qualitative research method [8].

Sample

Participants in this study are 16 Indonesian undergraduate students in mathematics education major which are between 20 and 21 years old in the 6th semester. The selection of the participants depends on researcher's views that their experience about learning mathematics for secondary school student last semester and their preparation to conduct teaching practicum in school next semester. It means that they should have strong grasp about mathematics for secondary student.

Data Collection

The data were collected from a public university in Bandung, Indonesia in early 2018. Before the test will be conducted, the students were told that it is for research about metacognitive skills and the researcher explain in short way about the skills and how the test will be assessed. First, the students were given a sheet containing one math problem about algebra. They should solve the problem in 30 minutes. After the test, they were given a metacognitive questionnaire about how they understand the problem, they think, and they solve it. Second, they were given a math problem about trigonometry. They should solve the problem 40 minutes. After the test, like before, they were given a metacognitive questionnaire. This steps are for making easier researcher to collect data about how their metacognitive skills in solving each of math problems. It also facilitate the participants in order to remember what they think when they face each of math problems. The math problem were modified slightly from Hendriana et [7] which were used for instruments of mathematics education research. It means that the validation and reliability has conducted. The indicators of the problems can be seen on Table 1 below.

TABLE 1. The Indicators of the math problems

Indicators	Math Problems
To arrange the steps in calculating each side of triangle and the concepts used	There is a triangle ABC and the sum of the length of two sides is ten and the angles of the triangle are 60° , 75° , dan 45° . Write down steps to calculate the length of each sides and explain concepts you use in every step.
To arrange the possibilities in solving a problem	There are three bottles, respectively measuring 70 ml, 80 ml, and 90 ml. Write down possible ways to measure 1150 ml of water by using 2 different bottles as much as fifteen times.

As mentioned in the previous paragraph, one of the data collection techniques used in this study is questionnaire. The questionnaire aims to explore more in-depth information about metacognitive abilities. The questionnaire used in this study is a combination and development of the questionnaire used in research Simamora et al [3] and Hendriana et al [7]. This questionnaire is usually called Likert scale that is used to measure the approval and disapproval of a person to an object.

After the test and questionnaire, they were interviewed to explore the reason behind the students' work. The students were encouraged to show their reasoning without interventions. There were several question such as, "how do you do to understand the problem?", "why you choose this strategy?", and "what the best strategy can solve this problem? And why?" during the interview. The results of the interview are used as a reference to strengthen the assessment of the subject's metacognitive ability in solving the problem.

Possible Responses

Based on the validation test combining with the findings from other researches the prediction of students' respond was made as seen on Table 3 based on the indicators of metacognitive skills [17, 18].

TABLE 3. The Prediction of Students' Respond

Question	Possible Answers
There are three bottles, respectively measuring 70 ml, 80 ml, and 90 ml. Write down possible ways to measure 1150 ml of water by using 2 different bottles as much as fifteen times.	Identifying problem
	Formulating Problem
	Stating suggestions or temporary solution and strategies used
	Finding the best solutions based on some sources
	Fluency in solving problem

Data Analysis

The data of this study contain of students' worksheets, students' questionnaire, and students' interview. In general, I analyzed these data using constant comparative methods [7]. Firstly, students' worksheets are analyzed by the indicators of metacognitive skills which are modified from some references about metacognition [17, 18, 19] as following: 1) identifying problem; 2) formulating problem; 3) stating suggestions or temporary solution and also strategies used; 4) Finding the best solutions from some sources; 5) Fluency in solving the problem. After that the result of questionnaire and interviews are analyzed to strengthen the findings of this research[7].

RESULTS AND DISCUSSION

Generally, most of students could solve the math problems as well. It can clearly seen from bar chart (Fig. 1) that most of participants fulfill every indicators of metacognitive skills. Averagely, about 83, 5% of participants can make participants could make plan to find solution by several attempts using various formulas. They performed identifying and formulating the problems, drawing triangle, putting forward several suspicions or temporary solutions and the strategies used, and then finding the solution and make conclusion about the most effective strategy to solve the problems.

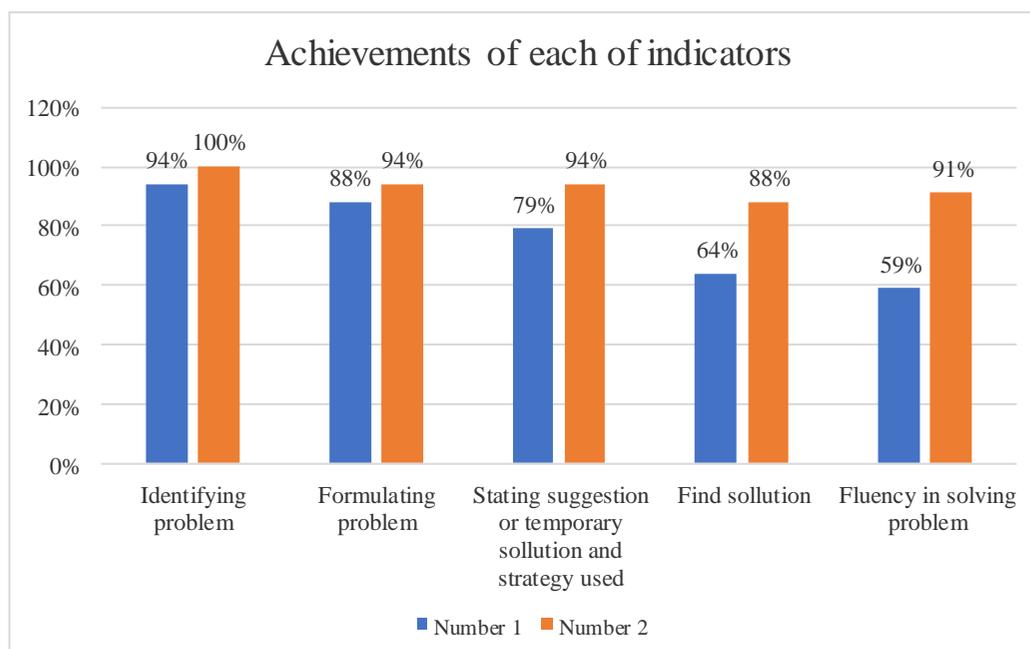


FIGURE 1. Achievements of Each of Metacognitive Skills' Indicators

Generally, students with advanced metacognitive skills may monitor their own learning, express their opinions about the information, update their knowledge and develop and implement new learning strategies to learn more [10].

Students' Answers

In this section, it will be discussed about metacognitive skills of undergraduate students according to result of students' worksheet each problems and began by problem number 1. To explain the students' typical answers on solving math problems in more detail, each of five students' answer types will be explained. It is provided with the example of students' works in each problems.

1. The Results of Analysis of Metacognitive Skills on Problem Number 1

In this problem, the concept needed are trigonometry and triangle material. The trigonometry material have been studied when they were high school students and triangle material what they learned in junior high school. The problem was selected from a book [7] which it was been used as instrument for mathematics education research. It assumes that the validation and reliability of the problem has been assessed. The problem number 1 can be seen in Fig. 2.

There is a triangle ABC and the sum of the length of two sides is ten and the angles of the triangle are 60° , 75° , dan 45° . Write down steps to calculate the length of each sides and explain concepts you use in every step.

FIGURE 2. The Math Problem Number 1

a) Identifying problem

In identifying problem, participants do activate reading the problem and identifying the information/concept they will use for further stage to achieve goal and how to solve the problem. 94%

participants could identify problem, information, and understand what the question. They began from drawing triangle ABC such as on Fig 3 below. After that, they make assumption about the sum of the length of two sides is ten.

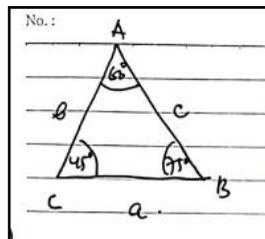


FIGURE 3. Participant's Drawing of the ABC Triangle

It is like Georgey said about metacognition in math that someone having metacognitive skills in math has ability in thinking based on the goal, understanding important concepts and predicting or make suggestions [12]. Participants also stated in questionnaire that he/she have learned about concepts who exist in problem number 1 and know the goal to find solution in this problem. They also said in interview session that they understand what were asked in the problem and how to solve that.

Scoenfeld [9] stated that researches about metacognition in math has a focus on three intellectual-habit categories which are different but interconnected. One of the categories is individual's knowledge about their own thinking process. How accurate someone in describing their own individual thinking. To know how participants in this study identify the problem, this research uses interview to clarify it. In interview session, most of participants know what the goal of the problem number one. They could explain what information or data they got from the problem too. The interview can be seen on Fig. 4.

Researcher: Hello.
 Participant1: Hello, Miss.
 Researcher: I want to ask you about the problem number 1. What information you get from this problem?
 Participant1: I got the information that the angles are 60° , 75° , dan 45° . But I do not know whether 60° , 75° , dan 45° is respectively in the vertex A, B, C. And I assumed that it is respectively in A, B, and C. And in the problem it is said that the sum of length of the two sides is ten. It can be seen that it will exist three possible conditions.
 Researcher: How about the question? Do you understand what the problem ask?
 Participant1: Of course. It is asked the length of each sides of triangle.

FIGURE 4. One of a Participant's Interview

b) Formulating problem

After identifying problem, participants formulate or make mathematical models from what information they get from the problem. It is useful to guide and help them to think based on the goal or what the question of the problem was given. 88% participants can formulate the problem in mathematical models and connect them to relative concepts. Most of participants use the laws of sines or cosinus in trigonometry material . This can be presented by one of participant's work which can be seen in Fig5.

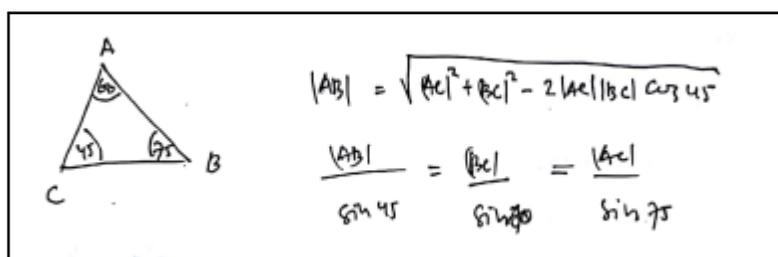


FIGURE 5. One of participant's work

Schoenfeld [9] suggested that one of indicators of metacognitive skills in math are belief and intuition. It means that what math ideas can be used when you do mathematical procedural and how to use them when you do math. This can be seen when participants also use information that the sum of length of two sides is ten for choosing mathematical concepts which are related to this problem. It is also strengthened from the result of questionnaires of participants' metacognitive skills. Participants considered the steps or strategies for solving problem number one so they can solve it on time. To understand the problem, in questionnaire, participants agreed that they read the problem more than once. They also agreed that they pay attention carefully the characteristics of data existing before solving the math problem.

In interviews, most of them can explain clearly about what steps they take when they have understood the problem, the following:

- 1) Draw a triangle and give notation A, B, and C in each vertex and write the angles.
 - 2) Write any mathematical formulas (math ideas) can be used to solve the problem.
 - 3) Make some potential suggestions from the information about the sum of the length of two sides is ten
- c) Stating suggestions and temporary solution and strategy used

In this stage, participants started to suggest assumptions or temporary solution which are be able to be used in solving the problem. Participants started to consider any information what needs attention carefully and various strategies to help them in solving the problem. Schoenfeld [9] stated that one of categories in metacognitive skills in math is to control or self regulate. It means that how good someone utilize data informed from observation or the problem to guide them in taking action in problem solving.

In this problem, information which need to be well rememberd is the sum of the length of two sides is ten when in this problem there is no explanation where sides are meant. Therefore, 79% participants make three possible conditions. The example of participants' work can be seen on Fig. 6.

Participants try to use different strategies on their worksheet before they choose the best solution

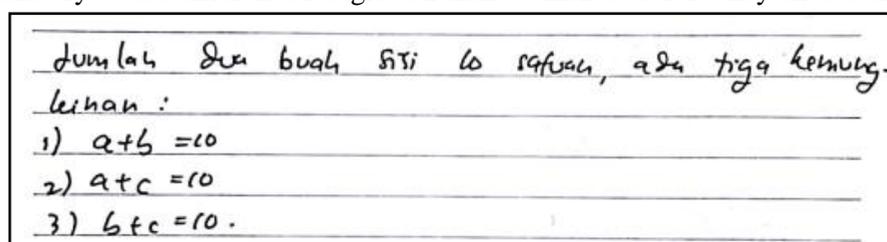


FIGURE 6. A Participant Made Three Possible Conditions

and strategies can be used to solve the problem. It can be seen on Fig 7 the left side. A participant seem not finish her calculation after using the law of cosines. The participant also make suggestions

for using the law of sines at first for obtaining the length of each side of triangle. It can be seen on Fig 7 in the right side.

kemungkinan :

1) $Ac + Bc = 10$
 $Ac = 10 - Bc$

$$|AB| = \sqrt{(10 - Bc)^2 + Bc^2 - 2(10 - Bc)Bc \frac{1}{2}\sqrt{3}}$$

$$= \sqrt{100 + 2Bc + Bc^2 + Bc^2 - 10\sqrt{3}Bc + \sqrt{3}Bc^2}$$

$$=$$

$|Ac| + |Bc| = 10$
 $A - c = 10 - |Bc|$
 $\frac{|Bc|}{\frac{\sqrt{3}}{2}} = \frac{10 - |Bc|}{\frac{\sqrt{3} + \sqrt{3}}{4}}$
 $\sin(30 + 45) = \sin 30 \cos 45 + \cos 30 \sin 45$
 $= \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2}$
 $= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$
 $= \frac{\sqrt{2} + \sqrt{6}}{4}$
 $\frac{\sqrt{2} + \sqrt{6}}{4} \cdot |Bc| = \frac{1}{2}\sqrt{3} - \frac{\sqrt{3}}{2}|Bc|$
 $\frac{\sqrt{2} + \sqrt{6} + 2\sqrt{3}}{4} |Bc| = \frac{1}{2}\sqrt{3}$
 $|Bc| = \frac{2\sqrt{3}}{\sqrt{2} + \sqrt{6} + 2\sqrt{3}}$

FIGURE 7. A Participant Try To Use Different Strategies

It can be deduced that participants can make suggestions which approximately can help them to solve the problem. This showed that participants have fulfill the indicator of metacognitive skills in stating suggestions or temporary solution as well.

d) Finding the best solution from some sources

After making some suggestions, participants choose mathematical ideas or formulas to find each length of sides of triangle. Most of them use the law of sines or cosines or combines of both in the trigonometry concept to tackle the problem. The law of sines can be seen on the left side of Fig. 8 and the right one is the law of cosines.

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	$c^2 = a^2 + b^2 - 2ab \cos C$
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FIGURE 8. The law of sines (left side) and The law of cosines (right side)

Participants make three suggestions to find the length of three sides of triangle, it can be seen from Fig. 6, as following:

- a) $a + b = 10$
- b) $b + c = 10$
- c) $b + c = 10$

After that, participants did mathematical procedurals by using the law of sines or cosines. For example, a participants denoted a, b, and c with x, y, and z. In first suggestion, Making three suggestions, she changed one equality to different form, such as $x + y = 10$, so that $y = 10 - x$. Then, the participant used the law of sines to obtain the length of x and also y. After that, she used another proportions of the law of sines to get the length of z. This way be also used in other suggestions. Mathematical procedurals used by the participant in the first, second, and third suggestion can be seen on Fig. 9.

It is seen that there are three different length of each sides of triangle based on every of the assumptions. It because there are three different condition, as following: when $x + y = 10$, $y + z = 10$, and $x + z = 10$. Also, all participants admitted that they felt difficulty to do algebra operations for obtaining the length of the sides because the root form in interview section. Many of them did not simplify the final answer because the square root form. 64% participants can attain this stage as well.

Handwritten mathematical work for three conditions (I, II, III) solving a system of equations with square roots. Each condition leads to a final expression for x , y , and z .

Condition I:

$$y + z = 10 \rightarrow y = 10 - z$$

$$\frac{y}{\sqrt{3}} = \frac{z}{\sqrt{2}}$$

$$\sqrt{2}y = \sqrt{3}z$$

$$\sqrt{2}(10 - z) = \sqrt{3}z$$

$$10\sqrt{2} - \sqrt{2}z = \sqrt{3}z$$

$$\sqrt{3}z + \sqrt{2}z = 10\sqrt{2}$$

$$(\sqrt{3} + \sqrt{2})z = 10\sqrt{2}$$

$$z = \frac{10\sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$y = 10 - \frac{10\sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$= \frac{10(\sqrt{3} + \sqrt{2}) - 10\sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$y = \frac{10\sqrt{3}}{\sqrt{3} + \sqrt{2}}$$

$$\frac{x}{\frac{1}{2}(\sqrt{6} + \sqrt{2})} = \frac{y}{\frac{1}{2}\sqrt{3}}$$

$$\frac{x}{\frac{1}{2}(\sqrt{6} + \sqrt{2})} = \frac{10\sqrt{3}}{\sqrt{3} + \sqrt{2}}$$

$$\frac{x}{\frac{1}{2}(\sqrt{6} + \sqrt{2})} = \frac{10}{\sqrt{3} + \sqrt{2}} \left(x = \frac{5(\sqrt{6} + \sqrt{2})}{\sqrt{3} + \sqrt{2}} \right)$$

Condition II:

$$x + z = 10 \rightarrow x = 10 - z$$

$$\frac{x}{\frac{1}{2}(\sqrt{6} + \sqrt{2})} = \frac{z}{\sqrt{2}}$$

$$\sqrt{2}x = \frac{z}{2}(\sqrt{6} + \sqrt{2})$$

$$2\sqrt{2}x = z(\sqrt{6} + \sqrt{2})$$

$$2\sqrt{2}(10 - z) = z(\sqrt{6} + \sqrt{2})$$

$$20\sqrt{2} - 2\sqrt{2}z = z(\sqrt{6} + \sqrt{2})$$

$$20\sqrt{2} - 2\sqrt{2}z = z(\sqrt{6} + \sqrt{2}) + 2\sqrt{2}z$$

$$20\sqrt{2} = z(\sqrt{6} + \sqrt{2} + 2\sqrt{2})$$

$$z = \frac{20\sqrt{2}}{\sqrt{6} + \sqrt{2} + 2\sqrt{2}}$$

$$x = 10 - \frac{20\sqrt{2}}{\sqrt{6} + \sqrt{2} + 2\sqrt{2}}$$

$$= \frac{10(\sqrt{6} + \sqrt{2} + 2\sqrt{2}) - 20\sqrt{2}}{\sqrt{6} + \sqrt{2} + 2\sqrt{2}}$$

$$x = \frac{10\sqrt{6} + 10\sqrt{2}}{\sqrt{6} + \sqrt{2} + 2\sqrt{2}}$$

$$\frac{y}{\sqrt{3}} = \frac{z}{\sqrt{2}}$$

$$\frac{y}{\sqrt{3}} = \frac{\frac{20\sqrt{2}}{\sqrt{6} + \sqrt{2} + 2\sqrt{2}}}{\sqrt{2}}$$

$$\frac{y}{\sqrt{3}} = \frac{20}{\sqrt{6} + \sqrt{2} + 2\sqrt{2}}$$

Condition III:

$$x + y = 10 \rightarrow x = 10 - y$$

$$\sqrt{3}x = \frac{y}{2}(\sqrt{6} + \sqrt{2})$$

$$2\sqrt{3}x = y(\sqrt{6} + \sqrt{2})$$

$$2\sqrt{3}(10 - y) = y(\sqrt{6} + \sqrt{2})$$

$$20\sqrt{3} - 2\sqrt{3}y = \sqrt{6}y + \sqrt{2}y$$

$$20\sqrt{3} = \sqrt{6}y + \sqrt{2}y + 2\sqrt{3}y$$

$$20\sqrt{3} = y(\sqrt{6} + \sqrt{2} + 2\sqrt{3})$$

$$y = \frac{20\sqrt{3}}{\sqrt{6} + \sqrt{2} + 2\sqrt{3}}$$

$$x = 10 - \frac{20\sqrt{3}}{\sqrt{6} + \sqrt{2} + 2\sqrt{3}}$$

$$= \frac{10(\sqrt{6} + \sqrt{2} + 2\sqrt{3}) - 20\sqrt{3}}{\sqrt{6} + \sqrt{2} + 2\sqrt{3}}$$

$$x = \frac{10\sqrt{6} + 10\sqrt{2}}{\sqrt{6} + \sqrt{2} + 2\sqrt{3}}$$

$$\frac{y}{\sqrt{3}} = \frac{z}{\sqrt{2}}$$

$$\frac{\frac{20\sqrt{3}}{\sqrt{6} + \sqrt{2} + 2\sqrt{3}}}{\sqrt{3}} = \frac{z}{\sqrt{2}}$$

$$\frac{20}{\sqrt{6} + \sqrt{2} + 2\sqrt{3}} = \frac{z}{\sqrt{2}}$$

$$z = \frac{20\sqrt{2}}{\sqrt{6} + \sqrt{2} + 2\sqrt{3}}$$

FIGURE 9. A Participant Use The Three Different Conditions and The Law of Sines

e) Fluency in solving problem

In this stage, the definition of fluency in solving problem is an individu can show accuracy in doing problem solving problem with predetermined time. The time given for solving the problem number 1 is 40 minutes. This time duration is based on the high difficulty level of the problem so that 40 minutes is estimated to be enough to solve the problem. However, many participants said in interview that they felt the time given was not enough to solve the problem. This is one of the factor many participants did not simplify their final answer of this problem.

2. The Results of Analysis of Metacognitive Skills on Problem Number 2

The problem number 2 contains about algebra, especially two-variable equation system. This material have been learned in junior high school stage. It needs metacognitive skills in predicting

phenomena and making some assumptions from information given and what the problem was asked. The problem can be seen in Fig. 10.

2. There are three bottles, respectively measuring 70 ml, 80 ml, and 90 ml. Write down possible ways to measure 1150 ml of water by using 2 different bottles as much as fifteen times.

FIGURE 10. The Math Problem Number 2

a) Identifying problem

Participants could understand as well the goal of the problem number 2. It can be seen from the bar chart (Fig 1.) that all of participants could identify the problem. According to participants in interview session, the problem number 2 and number 1 have similarity. They said that the uncompleted informations make the problem having three different and possible conditions, like the problem number 1. Participants were be able to understand the information given of the problem. It can be seen in Fig. 11 for Indonesian language on the left side and the translation to english on the right side.

<p>* 3 buah takaran dan ukurannya * 2 jenis takaran yang digunakan * banyaknya ml air yang harus ditakar * 15 kali menakar.</p>	<ul style="list-style-type: none"> • Three dozes and their sizes • Two kind doses used • Much ml of water which has to be measured • 15 times
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FIGURE 11. The Information The Participants Obtain From The Problem

Moreover, some participants also mentioned how to solve this problem. Many of participants drew the bottle to understand the problem (Fig. 12) Some participants mentioned in interview that mathematical formula which can be utilized in solving the math problem is algebra concept. Then in interview session the participants specialize the concept they used, two-variable equation sytem. Besides that, there are some participants using trial-error attempts and multiplication tables. All participants understood and were be able to identify the problem and information given. Therefore, it can be concluded that they can fulfill the indicator of metacognitive skills in identifying the problem number 1 as well.

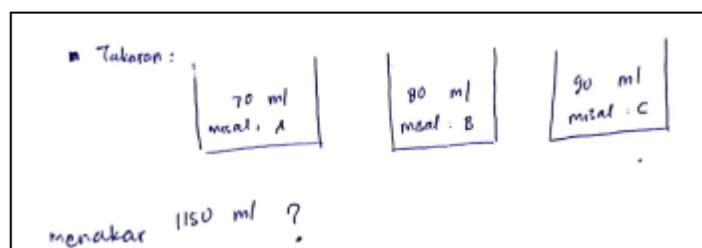


FIGURE 12. A Participant Draw The Three Bottle

b) Formulating problem

One of the steps used by participants in solving this problem can be seen in Fig. 13. It can be seen that the first thing they did was making possible condition which could tackle the proble number 2. This

because on the problem number 2 was not mentioned which the kind bottles were used in measuring 1150 ml of water. Then, participants utilize algebra concept of two-variable equation system in finding the sum of using each of two different bottles. It can be seen in Fig. 14.

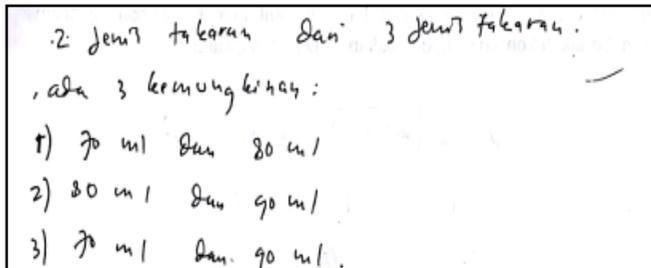


FIGURE 13. The Information The Participants Obtain From The Problem

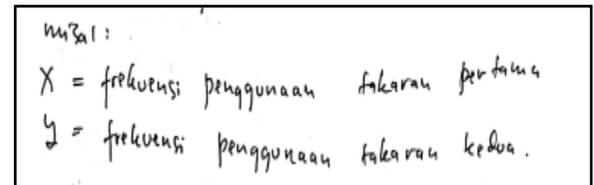


FIGURE 14. The Participant Try To Define Two Variables He Used

The next step is that participants do experiments in each conditions which have been structured in the first step to find solution by utilizing two-variable linear equation system concepts. This showed that participants have been aware of the steps they will do in solving the problem so it can be concluded that participants have fulfill the indicator of metacognitive skills in formulating the problem in mathematical model.

c) Stating suggestions and temporary solution and strategy used

Participants mentioned that there are three conditions or temporary solution in solving the problem. To confirm these conditions, participants used various approach to find the solution which are able to measure 1150 ml of water use two different bottles by the sum of using is 15 times. The three temporary conditions are as following:

- a) 70 ml and 80 ml
- b) 80 ml and 90 ml
- c) 70 ml and 90 ml

After that, some participants utilized the concept of two-variable linear equation system to find the sum of using each of two different bottles in the three conditions by substitution and elimination. Other used multiplication table to find the sum. The various strategy the participants used to solve this problem can be seen in Fig. 16, Fig. 17, and Fig. 18.

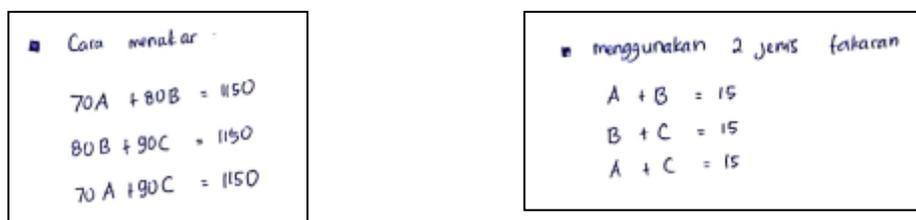


FIGURE 15. The Participant Make Two-variable Linear Equation

In the first condition, participants used the concept of elimination and substitution to obtain the sum of using 70 ml bottle and 80 ml bottle in measuring 1150 ml water. Most of them did not make mistake in their mathematical procedures. 88% participants enabled to find the way to measure 1150 ml of water by using 70 ml bottle and 80 ml bottle, 5 times for 70 ml bottle and 10 times for 80 ml

bottle. For the second condition, participants did not find the possible solution to measure 1150 ml of water by using 80 ml bottle and 90 ml bottle. In the third possibility, the concept used is the same as the concept used in the previous possibility, namely substitution and elimination. The bottles used to measure 1150 ml of water 15 times is 10 times of 70 ml bottle and 5 times of 90 ml bottle. Participants performed mathematical procedures well without any errors. So the solution of the third possibility obtained is correct based on the mathematical procedure that has been done. Based on this it can be concluded that the participants can express the allegations or temporary solutions in solving the problem well. Other approach to find the solution are multiplication table and trial-error attempts (Fig 17. and Fig 18). This indicates that the participants fulfill the indicator in presenting the alleged or temporary solution well.

Cara I

$$\begin{array}{r} 70A + 80B = 1150 \quad \times 1 \\ A + B = 15 \quad \times 70 \\ \hline 70A + 80B = 1150 \\ 70A + 70B = 1050 \quad - \\ \hline 10B = 100 \\ B = 10 \\ A = 5 \end{array}$$

Cara II

$$\begin{array}{r} 80B + 90C = 1150 \quad \times 1 \\ B + C = 15 \quad \times 80 \quad 730 \\ \hline 80B + 90C = 1150 \\ 80B + 80C = 1200 \quad - \\ \hline 10C = -50 \\ C = -5 \end{array}$$

(ga mungkin)

Cara III

$$\begin{array}{r} 70A + 90C = 1150 \quad \times 1 \\ A + C = 15 \quad \times 70 \\ \hline 70A + 90C = 1150 \\ 70A + 70C = 1050 \quad - \\ \hline 20C = 100 \\ C = 5 \\ A = 10 \end{array}$$

FIGURE 16. The Participant Used The Two-variable Linear Equation System.

→ Cara 3, dengan tabel perkalian

jumlah tabung	70	80	90
1	70	80	90
2	140	160	180
3	210	240	270
4	280	320	360
5	350	400	450
6	420	480	540
7	490	560	630
8	560	640	720
9	630	720	810
10	700	800	900

11	770	880	990
12	840	960	1080
13	910	1040	1170

1) $10 \cdot 70 + 5 \cdot 90 = 1150$
 2) $10 \cdot 80 + 5 \cdot 70 = 1150$

FIGURE 17. The Participant Used Multiplication Tables.

Jawab:

→ Cara 1, dengan menggunakan 10 x takaran 80 ml & 5 x takaran 70 ml

$$1150 = 10 \cdot 80 + 5 \cdot 70$$

→ Cara 2, dengan menggunakan 10 x takaran 70 ml & 5 x takaran 90 ml

$$1150 = 10 \cdot 70 + 5 \cdot 90$$

FIGURE 18. The Participant Used Trial Error Attempts.

d) Finding the best solution from some sources

Based on their worksheets, Participants revealed in interviews that the best solution to measure water was 1150 ml in 15 times using two different bottles by using of 70 ml bottle and 80 ml bottle or with 70 ml bottle and 90 ml bottle. This solution is based on information gained from the mathematical problems and the concepts in two-variable linear equation system. The 80 ml bottle and 90 ml bottle were not be able to be used because according to Figure 12 the participant found that the value of x which represents the 80 ml bottle is obtained 20 and the y value representing the 90 ml bottle is -5. This is not possible because no quantity amount is negative and exceeds 15. Therefore, it can be concluded that most of participants can find the best solution or answer to solve this problem 2. This suggests that participants can meet indicators of metacognitive ability in finding the best solution. The conclusion the participants made can be seen on Fig 19.

Jadi, cara yang mungkin ada 2
yaitu (1) takaran A sebanyak 5 kali dan
takaran B sebanyak 10 kali
(2) takaran A sebanyak 10 kali dan
takaran C sebanyak 5 kali.

FIGURE 19. The Participant Used Trial Error Attempts.

e) Fluency in solving problem

Participants can solve the problem fluently, precisely, and faster than the number 1 problem. This is also due to the level of difficulty in question number 2 is being where the number 1 issue is high. The procedure used systematically, logically, and in accordance with the mathematical concepts that can be used in solving masalah problem 2 this. So it can be concluded that the participants fulfilled the indicators in solving the problem of number 2 fluently and precisely.

Therefore, based on the results and discussion of metacognitive ability on questions 1 and 2, it can be concluded that participants have high metacognitive ability. This is because participants fulfill every metacognitive capability indicator on each question. In addition the results of questionnaires, interviews, and questionnaire on the cognitive abilities of participants showed that participants can explain both in writing and verbally about the thinking process that he thinks in solving math problems.

DISCUSSION AND CONCLUSION

Based on the results of data analysis of mathematics tests, questionnaires, and interviews it can be concluded that participants who are between about 20 and 21 year old college students who are in semester 5 in Mathematics Education Department have high metacognitive ability. Participants were able to answer math problems with moderate difficulty levels, but for high-level math problems, participants can solve problems by 83.5%. This is consistent with the two-item test of mathematics used

in assessing metacognitive abilities in mathematics as well as metacognitive capability questionnaire as well as interviews that have been conducted. This showed that the participants of this study have high metacognitive skills. The background knowledge possessed by the students would be one of the factors of this.

However there are some cases which the researcher must consider. First, the limitation of this research is the test to investigate metacognitive skills in math only contains two problems. It means that the further research need to use many different math problem to investigate college students' metacognitive skills in other different mathematics topics. Second, the future researcher may give more time to solve problem which has high difficulty like the problem number 1. It because many participants argued that they were be able to simplify the square-root form if they were given more additional time. The researcher may use the instruments of this study if they want to measure metacognitive skills in other field, such as science education. The instruments mentioned are the interview instruments and the questionnaire about metacognitive skills. It is hoped that this research can be used as a reference for research on metacognition in mathematics or other realms.

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