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Implementing Van Hiele Theory on Circle Module

IsnaeniUmi Machromah[1,a)], Christina Kartika Sari[2,b)], Mega Eriska Rosaria Purnomo[3,c)]

[1,2,3] *Department of Mathematics Education, Universitas Muhammadiyah Surakarta, Indonesia*

[a)]isnaeniumi@ums.ac.id
[b)]christina.k.sari@ums.ac.id
[c)]mega.eriska@ums.ac.id

Abstract. Even though circle has been learned by students since they were in primary school, students have not mastered ability in solving circle problems well. The van Hiele theory offers learning phases as an effective way to be applied in geometry classes. Because there is the difference of students' ability in solving circle geometry, a module can be used to deal with. This article describes the developing process of circle module based on van Hiele theory. The developing model was Plomp model. The module was developed by five phases of van Hiele theory. The first was inquiry phase. In this phase, the module offered information related to students' tasks. For example, contextual problems related to area of circle table, information about the history of "Pi", or observation pedal circuit. The second was directed orientation that asked students to do guided tasks based on information in the first phase. In the third phase, explication, students identified concepts, principles, or procedures that they found in the previous phase. The next phase was free orientation, in which it gave an opportunity to do complex tasks by using different methods. The last phase was integration where students made conclusions based on their whole activities. For instance, drawing a concept map, solving a puzzle, finding daily problems related to the material, or playing snakes and ladders game about the material.

INTRODUCTION

Geometry is one object that was learned by students since elementary school until senior high school and university. There are many benefits for life and for others knowledge by implementing geometry. Geometry is mathematics literature which is important for life. Then, Jones says that suitable learning of geometry school will give a positive impact for student's successful at others mathematics literature [1]. In addition, geometry school is essential fundamental for learning literature of formal deductive geometry [2].

Based on that explanation, geometry school has a crucial role. They are, especially for developing student's thinking abilities and generally for others aspect of life. Then, geometry should be learned well for students. On the contrary, there are many students meet with difficulties in geometry learning. The previous research states that students faced difficulties while learning geometry in school [3]. In addition, data from national exam 2016 shows that students' understanding of geometry and measurement at junior high school (SMP) level is 50.39%. This result is lower than others material such as numbering (56.80%), algebra (53.42%), and statistics (52.12%). Therefore, learning geometry for SMP students has been reached optimum result yet. Another point states that there is some problems on geometry school including: (1) there is misunderstanding on geometry school process [4], (2) geometry school is inclined to give direction student's thinking by formal deductive as the object consist of axiom, definition, theorem, and proof, then it does not emphasize student's understanding, and (3) learning geometry in school does not reflect enough about the essential of geometry for students [5]

On the other hand, systematic instruction on geometry learning is convinced that will give positive impact to geometry thinking level of students [4], [6]. Moreover, it gives more influence than student's age [4]. Piere Marie Van Hiele and Dina Van Hiele-Geldof introduce how to organize geometry school learning. Students will pass 5 phases on learning geometry, they are namely recognition, analysis, order, deduction and rigor, which started from identifying

geometry object such as the form until understood geometry by formal deductive [3]. This geometry learning with Van Hiele theory is convinced that will be effective for learning geometry in school [7].

Based on information from a teacher in SMP Muhammadiyah Surakarta, when learning plane geometry, she usually uses speech method without manipulative model. She also uses worksheet (Lembar Kerja Siswa or LKS) that contain geometry problems. This worksheet has not refer at Van Hiele theory yet. Therefore, it is needed for developing Van Hiele theory-based module as a method for increasing student's abilities on geometry. In general, this research aims to develope Van Hiele-based module for learning geometry.

THEORETICAL REVIEW

Van Hiele Theory of Geometry

The best way for learning geometry is using Van Hiele theory by classified students based on their Van Hiele level geometry [8]. With this method, the objective and accomplishment each group of students can be determined and teacher can give special treatment appropriate with student's abilities on geometry. In addition, this research also shows that, on implementing Van Hiele theory for learning geometry, there is no diversification result significantly based on gender. It presents that Van Hiele theory-based learning for geometry can be applied well for men and female students. Besides, for increasing Van Hiele thinking level, teachers should make consideration about language, words, and statement that is used have to suitable for learning contents [9].

Another point is that geometry learning which adopted from Van Hiele instruction can give great influence on learning outcomes [10]. Besides, it is suggested that Van Hiele theory-based learning for geometry could become consideration on curriculum [10]. Moreover, there is a research about the influences of implementation Van Hiele model toward geometry understanding of junior high school students at first level [11]. The results that related to Van Hiele theory-based learning show: 1) instruction that used learning model of Van Hiele was meaningful learning which based on student's understanding and relation of ideas on geometry, 2) there is diversification result significantly between students that taught by learning model of Van Hiele and students that taught by transferring of knowledge, and 3) student's concept understanding of geometry give positive impact for students and their abilities can be used on their life. Another research about the effectiveness of learning equipment based on Van Hiele theory toward student's achievement at senior high school states: 1) the rate of student's achievement at experiment class reach the learning objective appropriately, 2) score of pre-test and post-test increase significantly at experiment class and it is different with control class which the score of pre-test and post-test is not different significantly [12].

There is research development about learning activities use software GSP (Geometry Sketch Pad) based on learning phase of Van Hiele and it get positive respond from experts [5]. Besides, there is other research development about instrument to detect errors on understanding geometry based on Van Hiele learning model and it can be applied on classical and IT based-learning [13].

Van Hiele [3] states that students had different abilities to understand on learning geometry, then students have variety of thinking level. In addition, Van Hiele theory consists of three aspect, namely existence of levels, properties of levels, and movement from one level to the next [14].

There are 5 levels on understanding geometry which students can not reach a level without pass the previous level. The levels are recognition, analysis, order, deduction, and rigor. At the first level, recognition, students are identifying, giving name, comparing, and doing operation on geometry. At analysis level, students are analyzing about relation between components or attributes and classifying the object. Then, at order level, students can connect the attributes with informal argument. The next, at deduction level, students can prove theorem deductively and determine the relation between theorems. The last, at rigor level, student are explaining theorem as different system of postulate and analyzing or comparing the system.

Understanding geometry by Van Hiele theory needed to pass the level consecutively and it has properties of levels that consist of fixed sequences, adjacency, distinction, separation, and attainment. Fixed sequences mean that students can not reach a level without passed the n-1 level. Adjacency property mean that what is intrinsic on the previous levels become extrinsic at the next level, while distinction property mean that every level has its symbols and relation among the symbols. Then, separation property mean that although two people at the same level, they can not understand each other. The last, attainment property mean that process of learning brought to higher understanding and has 5 ordinary level, they are inquiry, directed orientation, explanation, free orientation, and integration.

Learning Module

Van Hiele explains the detail how teacher should teach from one level to the next level so that students understand. It is explained at 5 phases of Van Hiele theory, Those are inquiry, directed orientation, explanation, free orientation, and integration. At inquiry phase, students are recognizing concept that will be learned. Then, at directed orientation phase, students are doing some tasks that used relation from different concept, while at explanation phases, students are understanding about relation and trying to figure as statement based on topic. After that, at free orientation phase, students are studying to do complex task and determining the properties of concept that they learned. The last, students are resuming the material, figuring relation among the concept, and implementing at mathematics problems.

Module is an equipment of learning resources that presented systematically, then users can study with or without instructor or teacher [15]. The main purpose of writing module is students can study autonomous with or without guidance from teacher. Steffen-Peter Ballstaedt [15] explains that there are some concerning on developing module. (1) layout arrangement on module should easy organized, the title was concise, there is list of content, the cognitive structure is clear, there is resume, and there is task for readers. (2) The language is easy to understand. (3) there is content to examine the readers. (4) It is recommended to give any stimulation that push the reader for thinking. (5) Module should easy to read, such the font is not too small or large, text structure, the font was easy to read. (6) Content of module including material and worksheet should appropriate with the aim of learning.

On the other hand, there are 5 characteristics that make module can motivate students for learning. Those are self instruction, self contained, stand alone, adaptive, and user friendly [16]. In addition, developing module is done consecutively with the phases are (1) analyzing main and basic competition, (2) determining the title of module, (3) giving module code to make organize easier, (4) writing module that consist of formulating basic competition, determining instrument of evaluation, material, sequences of learning, and completeness of module structure [15]. Besides, quality of module should be evaluated based on 4 components, namely properness of content, validity, presentation, and graphic [15].

RESEARCH METHOD

This research was research and development which define Van Hiele-based module for learning geometry. The subjects were teachers and students grade 8 at SMP Muhammadiyah 7 Surakarta. This model of this research was design by [17] that consisted of preliminary research phase, development or prototyping phase, and assessment phase. At this preliminary research phase, the research did by identifying and investigate about student's condition, module of geometry learning at school, and student's abilities on geometry. The next phase consisted of design prototype and do formative evaluation, but this article only described the process of designing prototype of circle module. The data was collected by observation, interview, and documentation process.

RESULT AND DISCUSSION

The first step in this research was identifying and studying of curriculum, geometry material, students' condition, and mathematics learning method in school. In Muhammadiyah schools in Surakarta still used school-based curricula, namely KTSP (*Kurikulum Tingkat Satuan Pendidikan*), one of them was SMP Muhammadiyah 7 Surakarta. It used Curriculum 2013 for the 7th grade, while the others still used KTSP. This school needed instructional package based on Curriculum 2013 for supporting the implementation of this curriculum comprehensively in all grades. Mathematics teacher and the students in this school used mathematics book from the government which called *Buku Sekolah Elektronik* (BSE) and could be accessed online. These books represented contextual problems poorly. They were less communicative and offered less daily lives [18]. Meanwhile, the students had difficulties in geometry, such as identifying the objects rectangular in the daily life [19]. Moreover**,** BSE did not give opportunity to students for learning materials independently and assessing their abilities in solving problems. According to the result of national exam 2016 in Indonesia, the geometry mastering of students in SMP Muhammadiyah 7 Surakarta reached 33,88%. Meanwhile, the rate of geometry mastering of all students in Surakarta was 50,39% and it was 40,26% in Central Java. In particular, it was 35,43% on circle material. It showed that the student ability in geometry, in particular circle material was not optimum. Furthermore, based on information from the teacher, most students had difficulties on

solving non-routine problems in daily life. However, Curriculum 2013 asked students to apply what they had learn in real life. Thus, this research offered a solution by design a module based on Curriculum 2013 and relevant to KTSP.

Learning method that often used in this school was lecturing. Based on observations and interviews, the teacher used teacher-centered instruction that dominated the activities in the class. Students' focus, students' activity and students' attention were still lacking although the teacher did learning in groups and discussions. This effort did not work well to improve students' competence and activity. In global era, this learning style was not recommended anymore. Many research showed that students-centered was effective for learning. Based on [20], changing perspective from teacher-centered to learner-centered give positive impact. Their classroom environments changed as they allowed students to take more ownership in the learning process, they had a positive effect on students' effort to achieve, and improved relationships with and among their students. McNeil, Dewey, and Rovai explained that learning process should consist of learned centered, authentic, individual and group work. In addition, teacher should be collaborator, facilitator, encourager, and community builder, while students should active, collaborate, construct the knowledge, and monitor themselves [21]. One of learning activities which accommodated that condition was Van Hiele theory of learning. The research which done by [22] showed that Van Hiele's phase-based learning can be applied in classroom in order to help students achieve better level of geometric thinking.

Depending on explanation above, the implementation of geometry module based on Van Hiele theory was expected to be a bridge for students to be able to construct concepts in geometry. Besides, it became motivation for students to be more interested in learning mathematics and by this module, students were expected to learn independently and reach optimum results.

The aim of identifying and studying material content of circle was analyzing, specifying, and arranging systematically the materials that would developed in Van Hiele module. After identifying the content of circle at the junior high school level, analyzing the objectives and indicators of learning outcomes for this material was done. They were 11 learning objectives in circle material, namely: 1) explain the elements and parts of the circle; 2) determining the value of Pi; 3) determine the perimeter and area of circles; 4) calculating the perimeter and area of circles; 5) explain the relation between central angle and inscribed angle with same arc; 6) determine inscribed angles with same arc; 7) determine arc length, area of sector and area of segment; 8) using the relation among central angle, arc length, and area of sector in problem solving; 9) find the properties that formed by tangents and lines through the center point; 10) explain direct common tangent of two circles and indirect common tangent of two circles; and 11) determine the length of direct common tangent of two circles and the length of indirect common tangent of two circles.

Based on the objectives, circle module was developed in seven sections of module. Each sections, respectively, discusses the elements and parts of the circle; the value of Pi and the perimeter of circle; the area of circle; the central angle and the inscribed angle; arc length, area of sector and area of segment; the properties of tangents; and the direct common tangent of two circles and the indirect common tangent of two circles.

Circle module in this research was designed and structured with reference to van Hiele's theory. The learning module of geometry based on van Hiele theory was developed based on the geometry theory of van Hiele and the characteristics of module issued ministry of education. Based on the theory of van Hiele geometry by [4], the steps to achieve the higher level of van Hiele geometry were inquiry, directed orientation, explication, free orientation, and integration. Further, depend on [16] the characteristics of module were self-instruction, self-contained, stand alone, adaptive, and user friendly.

Implementation of van Hiele geometry theory in this module was realized through the phases contained in each module sections. The integrated van Hiele phases in each sub module are described as follow:

1. Inquiry

The inquiry phase introduced the students to the concept that will be learned. In this module, the inquiry phase was called by *inkuiri*. At this phase, the main activities were the introduction of concepts through the association of information with concrete objects, contextual issues related to the material, as well as information related to the concept. In addition, the inquiry phase can also be given through the introduction of the history of pi values contained in the pi value module section and the circumference of the circle, as well as the activities of classifying objects according to their type contained in the sub-module of the center angle and the circumference angle. This phase is suitable for [23], inquiry does not require simple question or right answer. This step contains information that leads to discussion and allows questions that could be explored to reach the topics [24]. Moreover, based on [22] some activities at this phase consist of introducing and asking students about geometrical shapes, identifying shapes from various figures, recognizing and drawing the shapes, or giving name based on student's description. Another activity at inquiry phase is exposing students with the application of the concepts [25] and also exploring certain structures of

holistic examples and non-examples, observing some concrete objects in their surroundings, and constructing geometric shapes [26]. Figure 1 below is an example of an activity in the *Inkuiri* phase of the section of Pi value and the circumference of the circle.

Inkuiri

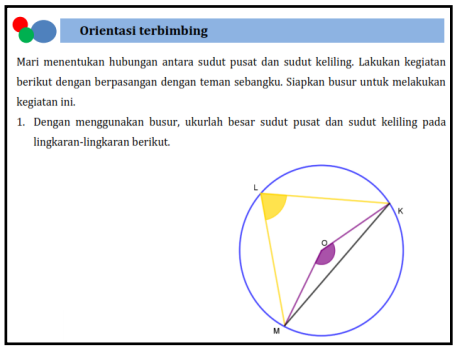
Bilangan π menunjukkan perbandingan keliling terhadap diameter lingkaran. Beberapa orang jaman dulu menggunakan bilangan 3 sebagai bilangan π. Namun. bilangan tersebut jauh dari keakuratan. Sekitar 250 sebelum masehi. matematikawan Yunani yang bernama Archimedes menggunakan polygon sebagai bantuan untuk menemukan nilai π yaitu antara $\frac{223}{71}$ dan $\frac{22}{7}$. Kemudian pada abad ke50. matematikawan China bernama Zu Chungzi menemukan bilangan π yang lebih akurat dari pada temuan Archimedes. Nilai π yaitu $\frac{355}{113}$ dan enam satuan desimal seperti yang sekarang digunakan. Sementara itu. Willian Jones. matematikawan Inggris memperkenalkan simbol "pi" pada tahun 1700. Simbol "π" dipilih karena pelafalan π di Yunani menyerupai huruf "pi" yang merupakan singkatan dari perimeter (keliling lingkaran). Bagaimana cara menentukan nilai "pi"?

FIGURE 1. The inquiry phase on the section the value of pi and circumference of circle

2. Directed Orientation

At this phase, students worked on tasks that use different network relationships. The directed orientation in this module was namely *orientasi terbimbing*. The main activities were related to construct students' concepts and basic concepts, find formulas, or deal with problem solving algorithms. The task was designed in a way so that students could find the characteristic of geometry concept gradually [4]. In directed orientation, students face with the task that offers sequential steps to find the concept. This phase is suitable with [26] which the students were asked to examine the properties of geometric shapes and [25] which the students learn concept of translation and explore the characters of translation. In addition, the activities at this phase based on [22] are exploring object to identifying the properties of the new geometric concept, classifying the figures, or in the topic of Circles, students were asked to measure the angles and state the relationship between the two angles.

Figure 2 below is an example of activity in the directed orientation phase in the section of the relation between central angle and inscribed angle.



Orientasi terbimbing

Mari menentukan hubungan antara sudut pusat dan sudut keliling. Lakukan kegiatan berikut dengan berpasangan dengan teman sebangku. Siapkan busur untuk melakukan kegiatan ini.

1. Dengan menggunakan busur, ukurlah besar sudut pusat dan sudut keliling pada lingkaran-lingkaran berikut.

FIGURE 2. The phase of directed orientation in the section of the relation between central angle and inscribed angle

3. Explication

At this phase, students were expected to be aware of the relationships in the given concept. This phase was called *penjelasan*. The content used in this phase was an explanation related activities to find concepts that students did by explaining the formula formally and explaining the material in depth. In this part, students could share their understanding based on the activities on the previous phase [24]**.** The activities of the phase of this research were suitable with [25] which students were taught to describe the polygon using appropriate mathematics language. Another activities were explaining the previous data from the second phase by students [26] and in the topic of circles, students discussed the relationship of the angles that they had explored in front of the class [22]. Figure 3 below was an example of explication in the section of the area of the circle.

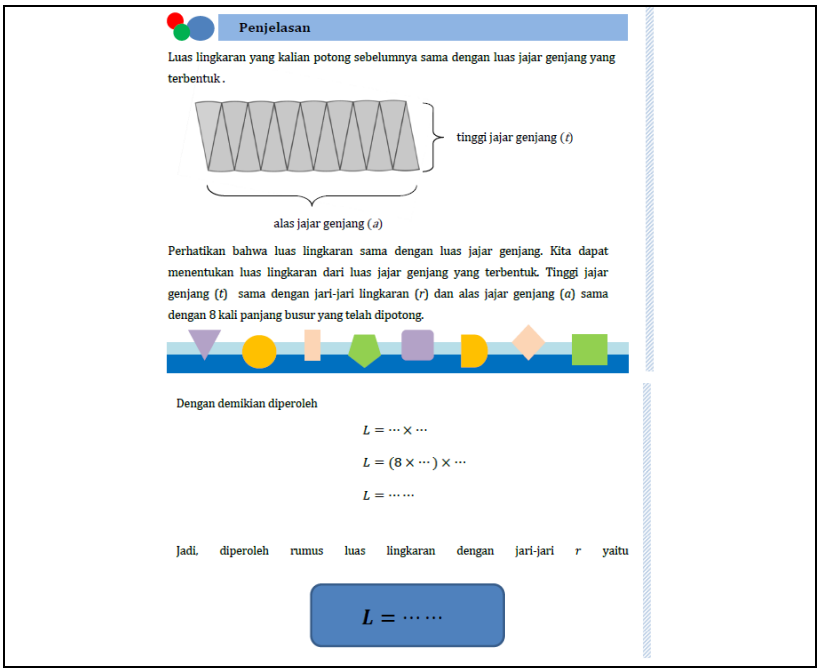


FIGURE 3. The explication phase in the section of the area of circle

4. Free Orientation

The activities provided were more complex or sustainable from the directed orientation phase. In this module, free orientation phase namely *orientasi bebas*. The goal of this phase was that students could find their own way in building the network of concept relationships that had been done in the previous phase. Activities at this phase were concept application in a given problem or further invention based on initial findings. Here, students were treated to solve problems related to geometry content [24]**.** In addition, this phase could also provide guidance or problem solving steps that appear at the inquiry phase, as in the section of the area of circle that shown in Figure 4 below. This activity was similar with [22] which students would carry out more complex tasks than in the directed orientation phase. In addition, students could unleash their creativity completely by doing composite mission students to find their own way in the network of relations [26] and students were free to build any diagram of relation between geometric shapes [25].

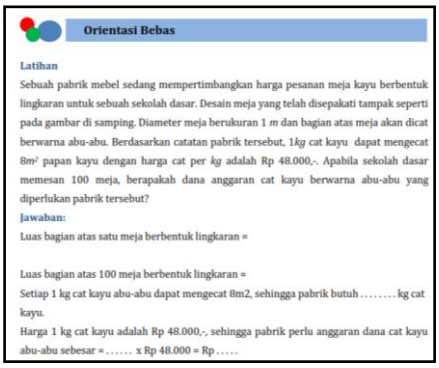


FIGURE 4. The phase of *Orientasi bebas* in the section of the area of cricle

5. Integration

Activities that help students to summarize or interpret the concept entirely, or integrate with other concepts. The integration phase namely *Integrasi*. At this phase, the activities of the students were summarizing the material, making concept maps, discussions, and presentations [4]**.** In addition, other activities provided to develop more complex material or related material such as the section of the central angle and the inscribed angle, as well as on the section of arc length, the area of sector and the area of segment. The integration phase could also offer a challenging and fun

exercise for students. As in the section of the properties of tangents, understanding the concept can be checked by solving the concept labyrinth as shown in Figure 6 as follow. This activity was summarising and integrating what students have learned and develop a new network of objects and relations [22], [25]–[27].

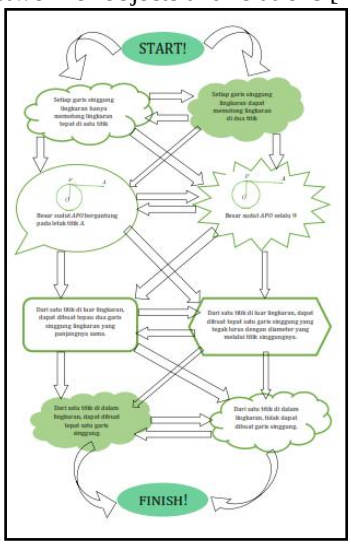


FIGURE 5 The integration phase on the section of tangent lines

In addition, this circle module was developed according to characteristics module issued by ministry of education. The characteristics were self instruction, self contained, stand alone, adaptive, and user friendly. In this module, the self instruction was showed that module contains steps to solve the sample problem and there was a key answer to the exercise questions, so it enabled students to learn independently. The self contained in this module was developed based on the syllabus and geometry material taught to junior high school students so that students could learn thoroughly all the junior geometry material. Stand alone in this module was arranged independently, so the use of the module did not depend on other reading sources or media. The module included contextual issues, materials, guided assignments, and practice questions. This module was adaptive because it had a good adaptability to the development of science, especially in geometry. Contextual issues were also presented in drawings that facilitate the understanding of junior high school students. In addition, GeoGebra software was also used to present geometric shapes to be precise [28], and also attractive. In some module sections, the content of integration phase was offered in games like concept labyrinth, snake ladder, and true-false games. This module was also user friendly to use because the instructions and information in this module use language that wass easy to understand by junior high school students, so that the students could understand the material easily. The use of terms in this module were flexible and friendly, such as: Let do the exercises, namely "Mari berlatih!" and Let check your answer, namely "Ayo cek jawabanmu!", were expected to make children more interested in reading the module. In addition, supportive images of contextual issues were also presented in each module section. Besides, geometric objects were depicted in order to support the attractive view. This characteristics was suitable with [29] which explained that elements of module should consist of title, preface, table of contents, the background, the standard of competence, a map concept, benefits, learning objectives, instructions for use of modules, basic competence, subject matter, description of materials, summaries, exercise / assignment , self-test, follow -up, hope, glossary, bibliography and an answer key.

CONCLUSION

This study is a series of development research with the main result was learning module based on van Hiele theory. This research had 3 phases on developing module, they were of the preliminary research phase, development or prototyping phase, and assessment phase. This study focused on preliminary research phase and development phase. The result was a van Hiele-based module on circle material. This module consisted of 7 sections and each sections consist of van Hiele phases namely inquiry, directed orientation, explanation, free orientation, and integration. At inquiry phase, the main activities were introducing toward concept by connected information with concrete material,

contextual problems, and real-life information about the concept. Besides, it could be explaining about the history of Pi and classifying various objects. Then, at directed orientation, the activities were constructing students concept and understanding such as basic concept, determining formula, and problem solving algorithm. After that, at explanation phase, the contents were explaining formula on mathematics model and explaining deep material. At free orientation phase, the activities were implementing concept on problems or discovering another concept based on the previous, also it could be given the guided solution of problems at inquiry phase. The last, at integration phase, the activities were resuming material, making mind-map, discussing, or presenting. Besides, other activities were developing complex material than previous or solving challenging problems by concept maze.

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