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Students' Geometry Skills Viewed from Van Hiele Thinking Level

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Abstract. The background of this study is geometry as one of the essential topics in mathematics. This study aims to know students' achievement in understanding geometry based on Van Hiele geometry thinking level, which beginning with level 1 (visualization) is to know the name of geometry shapes, level 2 (analysis) that is able to mention the properties of geometry shapes, level 3 (informal deduction) that is capable states a geometry shapes relationship of its components, and level 4 (deduction) that is able to make a deductive conclusion from a statement, this is in accordance with Van Hiele theory. The descriptive qualitative research was used in this study. Van Hiele geometry test was administered, followed by interviews. The subjects of the study were 40 students grade VII in Bandung. The data analysis technique of this study used three stages, that were reducing, displaying, and getting a conclusion of data. The results showed that 30 students or 75% are at level 1 (visualization), 8 students or 20% are at level 2 (analysis), 2 students or 5% are at level 3 (informal deduction) and no students have been able to reach level 4 (deduction) and level 5 (rigor). The characteristics of geometry skills including visual skills, descriptive skills, drawing skills, and logical skills at each level of Van Hiele thinking are different. Students who are at level 3 also master students' geometry skill in lower level. Based on the results, we suggest that mathematics teachers need to provide more practice questions that reinforce students' geometry skills.

INTRODUCTION

Mathematics plays an important role in daily life, especially to support the development of science and technology. Learning mathematics requires the process of thinking. One type of thinking needed in mathematics is geometry thinking. This is because the concept of geometry is closely related to the concepts of mathematics and other sciences. The learning of geometry is the basis for several topics in mathematics, such as division, measurement, probability, as well as numbers and operating systems [1]. Moreover, geometry is a mathematical concept that most closely related to human life. The concept of geometry is widely used in various areas of life, such as in the field of architectures, arts, urban plannings, and so forth.

Studying geometry is correlated to a theory of thinking related to the learning of geometry that is the Van Hiele theory. The theory divides a person's level of geometric thinking into 5 levels: Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction), Level 4 (Deduction), and Level 5 (Rigor) [2]. These levels indicate how one thinks and what type of geometry ideas are thought, rather than indicates how much knowledge the student possesses [3]. The Van Hiele thinking level possessed by a person affects their geometry skills. The geometry skills in question are the students' skill in learning geometry which according to Hoffer [4] consists of 5 skills, they are visual skill, descriptive skill, drawing skill, logical skill, and applied skill.

The geometric thinking level that students achieve does not only play a role in the geometry material. The geometric thinking level has an important role in learning other mathematical material as well. For example, in trigonometry, students at level 1 Van Hiele are able to recognize right triangles in different situations and are able to

distinguish between a scalene triangle and an isosceles right triangle. Students at level 2 Van Hiele, are able to understand that the right triangle has a fixed right angle although it has different sizes [5].

Based on the description above, it can be inferred that studying geometry is very important to equip learners with a mathematical mindset in facing problems both in mathematics itself and in everyday life. However, the reality shows that students are still struggling in studying geometry. Based on the report by Ozerem [6], there are several difficulties in studying the geometry of plane and the lack of knowledge related to geometry in seventh-grade students. One of the reason is that students only memorize the formula of a geometric plane without understanding the concept of the plane itself. Other research by Clements [7] found that junior high school students failed to learn the basic concepts of geometry and geometry problem-solving. The cause of this failure is because students are not ready during the learning process.

The problem on geometric thinking not only occur in the other part of the world but also in Indonesia. Based on the results of research conducted by Nursyam, it was found that the geometry comprehension of junior high school grade VII is still low [8]. It can be seen from the results of research indicating that there were 48 students out of 97 subjects (49.48%) failed the test because the grades of the 48 students did not reach the standard. Meanwhile, based on the Van Hiele level achieved, 61.86% of students were at level 0 (visualization level), 11.34% of students reached level 1 (level of analysis), and the rest could not be determined because there were students who reached level 3 but did not meet the previous level. It concluded that the junior high school students' understanding of geometry is still low, far from what was expected.

Following problems above, almost all the research finds difficulties in learning geometry and the cause is because students do not master the concept of geometry so that they only memorize it. Students do not master the concept is an indication of the low ability on geometric thinking. One of the causes of the problem is the unsuitable method used in the classroom. The learning method is not adapted to the students' geometric thinking level so that students at low levels will feel depressed if the learning takes on the ability to think geometry higher than the students' level. The problem indicates that the learning process has not been optimal so that students have not been able to develop their geometric thinking ability. Learning geometry at school assumes that students think at a formal deduction level [9]. However, the reality shows otherwise and students' understanding of geometry is still lacking. This lack leads to a gap between the students' level of thinking and what is needed in the geometry that students expect to learn.

Given a great deal of the distribution of geometry at the Junior High School, which is 41% of the entire field of mathematics studies, the low ability of students to think geometry becomes a problem that must be solved. One way to overcome this problem is to apply the most suitable learning method. One of the solution so that students do not have difficulty in learning geometry is by considering the students' geometric thinking level [10]. By knowing the geometric thinking level of the students, it can help math teachers and mathematics educators because it has implications for teaching in geometry [11]. Teachers can arrange learning that is appropriate to the students' level of thinking if the teacher already knows the students geometric thinking level so that students do not feel depressed in learning geometry. Therefore, an analysis is needed to find out the students' geometry skills viewed from their Van Hiele levels. The research question of this research is: how is the students' geometry skills viewed from Van Hiele thinking level?

RESEARCH METHOD

To answer the research question, we conducted qualitative research through individual written tests and interviews. The subjects were 40 students of grade VII in Bandung, and 6 of them were interviewed afterward. The instruments consist of Van Hiele geometry tests and interview manual. The Van Hiele geometry test used was developed by the Cognitive Development and Achievement in Secondary School Geometry projects (CDASSG) in the form of multiple choices containing 25 items with 5 questions in each level thinking of Van Hiele. The Van Hiele geometry test is used to know the level of Van Hiele geometry thinking that students achieve. With Van Hiele geometry test, it can be identified which students reach level 1, level 2, level 3, level 4, to level 5. Two students were selected at each level to be interviewed. Interviews conducted were task-based interviews. This interview was conducted to identify students' geometry skills included visual skills, descriptive skills, drawing skills, logical skills, and applied skills.

RESULTS AND DISCUSSION

Based on the results of the Van Hiele geometry test, it was found that 30 students were at level 1 (visualization), 8 students were at level 2 (analysis), 2 students are at level 3 (informal deduction) and no students could reach level 4 (deduction) and level 5 (rigor). Here is the recapitulation of the achievement the Van Hiele level of geometric thinking.

TABLE 1. Recapitulation of Van Hiele Level Achievement

Group	Achievement Level	Percentage
High	Informal Deduction	5%
	Analysis	20%
	Visualization	75%
Medium	Visualization	
Low	Visualization	

Based on the table, it can be inferred that most of the students' achievement at the Van Hiele level of geometric thinking is level 1 (visualization). The best achievement of the Van Hiele thinking level that can be achieved by students is level 3 (informal deduction). No students were able to reach level 4 (deduction) and level 5 (rigor). It is consistent with other research findings such as research conducted by Burger & Shaughnessy [12] which stated that the highest level of thinking of junior high students in learning geometry is level 2 (informal deduction) and most of them are at level 1 (visualization). This statement was also supported by Van de Walle [3] that most of the junior high school students are between visualization level to informal deduction level. Based on the interview, no students could reach level 4 (deduction) and level 5 (rigor) because they had not yet studied the material on the questions for these levels. In other words, the Van Hiele level is related to the amount and type of geometric experiences and learning activities that they have. This is not due to age because the Van Hiele level does not depend on age in terms of Piaget's developmental stage [12], [13], [14].

Table 1 shows that students who reach level 2 (analysis) and level 3 (informal deduction) are students with the high mathematical proficiency level. This indicates that the Van Hiele level achieved by students is a good indicator to demonstrate their proficiency and skills in mathematics in general. In addition, Van Hiele level achieved by students also shows how the process of thinking and proficiency the students possess. This statement was also supported by Alex and Mammen [15] that the students' geometric thinking level influences their general mathematical understanding and geometric thinking ability in particular.

The Van Hiele level achievement between medium and low mathematical proficiency shows similar results. Both groups are only able to achieve visualization level. All students in high, medium, or low group reach the same level that is visualization level. It suggests that students with high, medium and low proficiency may experience similar difficulties. Students with high, medium or low proficiency can experience learning difficulties. In this research, the difficulty in question is the difficulty in geometry.

Students with medium and low mathematical proficiency still struggle in analyzing the properties of the quadrilaterals and triangles, analyzing the relationships between the quadrilaterals, and the difficulty in drawing conclusions deductively. The difficulties experienced by students with high level of mathematical proficiency vary. High-proficiency students who only achieve the level of visualization experience the same difficulty as students with medium and low proficiency. However, high-proficiency students who reach the level of analysis, find it difficult to draw conclusion deductively and find the relationship among the quadrilaterals. It is the contrast to high-proficiency students who reached the level of informal deduction. Students who reached this level have difficulty in proving theorems and axioms. Thus, the difficulties students experienced in geometry vary depending on the level of geometric thinking they achieved. Based on the results of the geometry proficiency analysis described above, it can be concluded that geometry skills at three levels of thinking based on Van Hiele theory has different characteristics.

Students' Geometry Skill

Geometry skills in this research include students' ability to recognize the kinds of plane geometry, describe plane geometry, draw plane geometry, recognize the differences and similarity of plane geometry, apply plane geometry

into the real world, and solve real problems which are related to plane geometry. Following the data analysis of research results, the identification of the students' geometry skills viewed Van Hiele level can be seen as follows.

Geometry Skills of Students at Level 1 (Visualization)

The first geometry skill identified is the visual skill. Researchers provide triangle and quadrilateral image to identify students' visual skills. In visual skills, students who are at level 1 (visualization) can only determine the type of quadrilaterals and triangles based on their global appearance. Students are not able to specifically explain the properties of quadrilaterals and triangles such as the values of the angles and the position of the sides. They are more focused on the number of sides and the number of angles on the quadrilaterals and triangles. For example, the students know that a plane is a rectangle because it has four sides but the students do not know the relationship between the sides. At the time of interviews, students are given figures of quadrilaterals and triangles that presented in Figure 1 below:

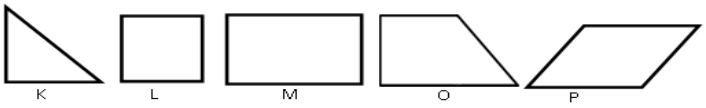

FIGURE 1. Question about the names of figures

Students who are at level 1 can determine the names of figures are given. They answer that figure K is a triangle, figure L is a square, figure M is a rectangle, figure O is a trapezoid, and figure P is a parallelogram. But, they can not explain properties of each plane. Students cannot identify that K is a right triangle. They cannot distinguish types of triangle. Anything that looks like a triangle is a triangle for them. This is because they identify shapes according to their concrete examples [14].

The second geometry skill is the descriptive skill. In descriptive skill, students can give the correct names for the given quadrilateral images. However, the students have not been able to define a quadrilateral based on their properties such as the angle's values, the length of the side, and the relationship between sides of the quadrilateral. Students at this level are also still difficult to distinguish the quadrilaterals because of the similar properties among the quadrilaterals. As an illustration, students' answer presented in Figure 2 below:

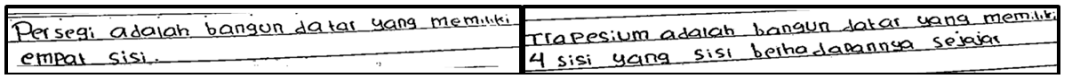

FIGURE 2. The answer of S1 for the second question

The next geometry skill identified is drawing skill. In drawing skills, students are only able to create a quadrilateral by labeling. However, at this level students have not been to construct images in accordance with the characteristics and properties given such as two perpendicular lines and two parallel lines. As an illustration, when students are asked to drawing square ABCD and drawing a figure which has properties opposite angles are congruent, all four sides are the same length, diagonals bisect vertex angles, and diagonals are perpendicular, students' answer presented in Figure 3 below:

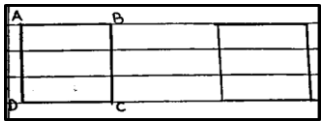

FIGURE 3. The answer of S1 for the third question

The fourth skill is the logical skill. In logical skills, students are able to understand the conservation of quadrilateral images in various positions by mentioning the type of each image. Example, when students are given figures of the parallelogram in various positions as Figure 4 below:

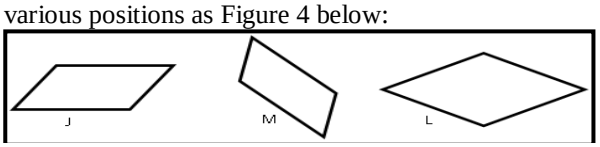

FIGURE 4. The fourth question about parallelogram in various positions

They know that all are the parallelogram. But they can not determine similarities and the differences properties of some quadrilateral, in which all of them have four sides. For example, they can not determine the similarity between square and rectangle where both of them have similarities such as opposite sides are congruent, all angles are right angles, diagonals bisect each other, and the diagonals are congruent.

The last skill identified is the applied skill. In applied skills, students at this level not yet able to relate information from a given physical object and develop them into a geometry model. When they are given the fifth question "Pak Amir has a rectangular shaped garden, the circumference is 100 m and the garden's width is ten meters lack of the length. In the garden will be planted with vegetables. To facilitate the maintenance of vegetables on each edge of the garden made a road that is 1 meter wide. Make a sketch of the above problems then calculate the area of the garden where planted with vegetables". As an illustration, students' answer presented in Figure 5 below:

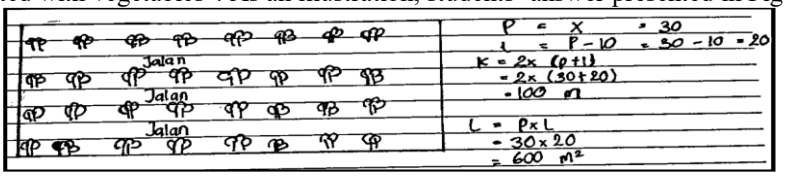

FIGURE 5. The answer of S4 for the fifth question

Gometry Skills of Students at Level 2 (Analysis)

Visual skill is the first geometry skill identified. In visual skills, students who reach level 2 (analysis) are able to determine the type of quadrilaterals based on the shape. In addition, they also have been able to determine the type of planes based on the properties. They are able to explain the specific properties of quadrilaterals and triangles that include the number of sides, the length of sides, angle's value, the relationship between sides, and the relationship between two opposite angles. When students are given figures of quadrilaterals and triangles that presented in Figure 1, they also can determine the names of figures are given correctly. Moreover, they can explain properties of each plane. For example, when the figure of a square is given, they are able to tell that the four sides are the same length and the four angles have the same value of 90°. According to the characteristi of level 2, Students can identify shapes according to their properties [14]

The second geometry skill to be identified is the descriptive skill. In the descriptive skills, students who reach level 2 (analysis) are able to define the planes either quadrilaterals or triangles based on their properties correctly. They are able to provide definitions based on the number of sides, the length of sides, angle's value, the number of angles, and the relationship between sides. For example, when asked about the definition of a square, they explain that a square is a quadrilateral plane where side lengths are all equal and it has 4 equal-angles. As an illustration, students' answer about the definition of square and trapezoid in Figure 6 below:

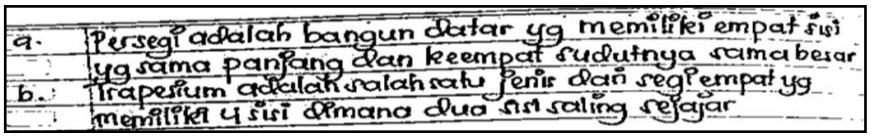

FIGURE 6. The answer of S2 for the second question

The next geometry skill is drawing skill. In drawing skills, students at this level are able to construct images according to the characteristics and properties given such as two parallel lines, two perpendicular lines, and determined a vertex in a line. But, they are not yet able to give information on the figure correctly include give a symbol of the right angle, the same length sides, and two parallel sides. As an illustration, students' answer presented in Figure 7:

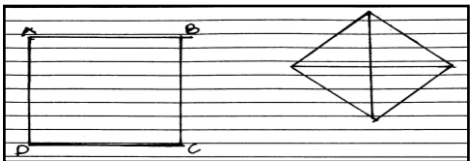

FIGURE 7. The answer of S2 for the third question

Logical skill is the next geometry skill to be identified. In logical skills, students are aware of the similarities of some quadrilateral images. They realize that square, rectangular, parallelogram, trapezoid, kite, and rhombus are types of the quadrilateral which have 4 sides. Although they are aware of the similarities, they are still able to tell the differences between various types of quadrilaterals and triangles. They realize that properties can be used to distinguish between quadrilaterals and triangles, starting from the length of sides, angles' value and the number of parallel sides. For example between rectangles and squares, they know that the square has four sides of the same length while the rectangle is not. Besides that they know that both have some similarities, such as opposite sides are congruent, all angles are right angles, diagonals bisect each other, and the diagonals are congruent.

The last skill identified is the applied skill. In applied skills, students are able to create a geometry model based on the information provided. In addition, students are also able to use the model to solve the given geometric problems but not completely true. Students' answer for the fifth question presented below:

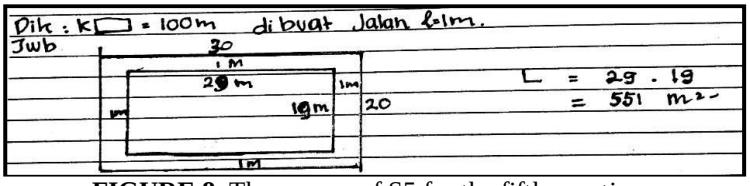

FIGURE 8. The answer of S5 for the fifth question

Geometry Skills of Students at Level 3 (Informal Deduction)

The first geometry skill identified is visual skill. In visual skills, students at level 3 (informal deduction) are able to explain specifically the properties of quadrilaterals and triangles based on the images, including the number of sides, the length of sides, the relationship between sides, angles' values, the relationship between two opposite angles, the number of angles, and the relationship between adjacent angles add up to 180º. In addition, students have been able to explain the interrelationships between different types of quadrilateral images based on the properties of each image. When students are given figures of quadrilaterals and triangles that presented in Figure 1, they can determine the names of figures are given and explain properties of each plane correctly. Besides that, students know that a square is a rectangle which four sides are equal in length. In other words, students know the properties of rectangular are also present on the square. According to characteristics of informal deduction level, students at this level understand the relationship between different shapes [14].

The next geometry skill to be identified is descriptive skill. In descriptive skill, students were able to define quadrilaterals based on the properties they possess from the number of sides, sides'values, angle's values, the number of angles, sides alignments, the relationship between two opposite angles, and the relationship between the adjacent angles add up to 180º. In addition, in line with their visual skills, students at this level are able to formulate a sentence that shows the relationship among the quadrilaterals based on the common properties. In defining the shapes, they use their own language correctly. As an illustration, students' answer for the second question presented in Figure 9 below:

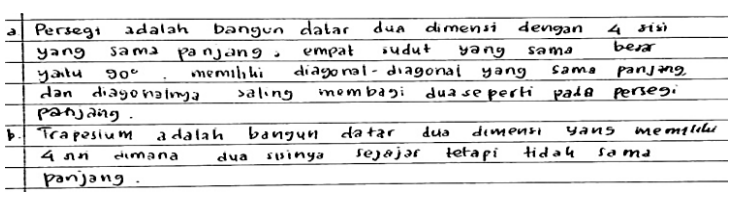

FIGURE 9. The answer of S3 for the second question

Another geometry skill to be identified is drawing skill. In drawing skills, students at level 3 are able to draw shapes based on the properties are given and provide information on the figure correctly. They are able to sketch drawings and label correctly. They also can draw another quadrilateral based on the image provided and even able to create supporting lines to form a new quadrilateral. They also able to explain the properties that they sketch. As an illustration, students' answer for the third question presented in figure 10 below:

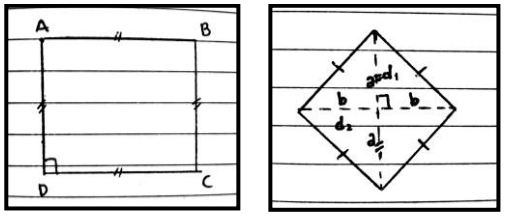

FIGURE 10. The answer of S3 for the third question

Logical skill is the next geometry skill to be identified. In logical skills, like students are at level 2, students are at level 3 also can determine similarities and differences of some quadrilateral based on their properties correctly. They are able to use the properties of a quadrilateral image to determine a quadrilateral class contained within another quadrilateral class.

The last geometry skill is the applied skill. In applied skills, aside from connecting information (physical objects) given and developing it into geometry model (by using the scale), students at this level are also able to use the concept of a mathematical model that represents the relationship between objects. As an illustration, students' answer for the fifth question presented in figure 11 below:

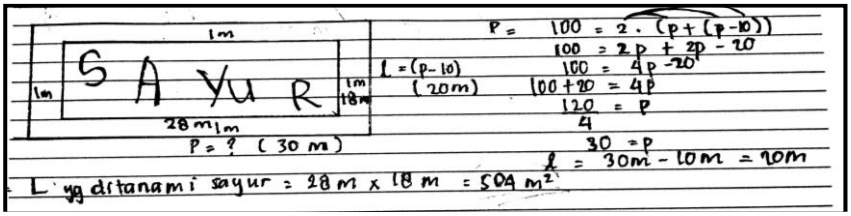

FIGURE 11. The answer of S6 for the fifth question

From the result of this research, we learn that the characteristic of geometry skills at each Van Hiele thinking level is different, where if students are at level 3 (informal deduction) means that the students also master geometry skills at level 2 (analysis) and level 1 (visualization), while the students who are at level 2 (analysis) also master geometry skills at level 1 (visualization) but do not master the geometric skills that exist at level 3 (informal deduction). So does students who are at level 1 (visualization) do not master the higher geometric skills i.e. level 2 (Analysis) and level 3 (informal deduction). Therefore, the fact that students who failed to reach the previous level, they also will fail to reach the next level. Students cannot operate with understanding one level without having been through the previous levels [14]. As proposed by Atebe and Schafer to reach one of Van Hiele level, it must go through the mastery of the level below it. [17]. Thus, the Van Hiele geometric thinking level is hierarchical, meaning that each student that studies geometry through these levels in the same order and there is no chance of a skipped level.

Based on interviews, students in different Van Hiele level also use different language in their explanations. Students at level 1 use descriptive imprecise language in most of their explanations. While students at level 2 and level 3 use precise language in their explanations. It means each level has its own language. This result seems to confirm Van Hiele's suggestion that two people operating at two different levels speak a very different language [18].

The differences appear in the way students answer questions or explain the way they think at every Van Hiele levels are supported by Van De Walle's opinion [3] that not everyone thinks about geometric ideas in the same way. Because everyone can produce and develop their ability to think in the context of geometry, so that teachers in the mathematics learning process, especially on teaching quadrilaterals, can consider the level of thinking that students possess, so that the students learn not only to memorize, but also to understand the materials. It coherent with Suherman's opinion [19] that the teacher needs to know the students' mental development level and how the lesson should be done according to the stages of the development.

CONCLUSION

Based on the results of this study, it could be concluded that from 40 students given Van Hiele geometry test, 30 students were level 1 (visualization), 8 students were level 2 (analysis), 2 students were level 3 (informal deduction)

and there were no students reach level 4 (deduction) and level 5 (rigor). The results of this study revealed that there is a correlation between Van Hiele level and students' geometry skills. The characteristics of geometry skills at each level of Van Hiele thinking are different. Students are at one Van Hiele level means that students also master geometry skills at the previous level. The Van Hiele levels of geometric thinking are hierarchical, meaning that each student learns geometry through these levels in the same order and to reach one of Van Hiele level, then it must go through the achievement of the level below.

Based on these results, it is suggested to mathematics' teachers to consider the students' Van Hiele level of geometric thinking in determining the proper and effective way of teaching, in accordance with the geometry skills possessed by the students. In addition, by knowing the Van Hiele level and geometry skills that students possess, teachers are expected to develop methods, strategies, and learning models that can improve students' geometric skills in learning geometry. Mathematics teachers also need to provide more practice questions that reinforce students' geometry skills.

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