5th ICRIEMS Proceedings
Published by Faculty Of Mathematics And Natural Sciences
Yogyakarta State University, ISBN 978-602-74529-3-0

Learning to Think Mathematically Through Reasoning and Problem Solving in Secondary School Mathematics: A Literature Review

Nanang Ade Putra Yaman[1, a)] and Jailani[2, b)]

[1]*Mathematics Education, Postgraduate Mathematics Education Program, Yogyakarta State University, Karangmalang, Depok, Sleman, Yogyakarta, Indonesia.*
[2]*Department of Mathematics Education, Faculty of Mathematics and Natural Science, Yogyakarta State University, Karangmalang, Depok, Sleman, Yogyakarta, Indonesia.*

[a)] nanang0475pasca2016@student.uny.ac.id
[b)] jailani@uny.ac.id

Abstract. A major part of the problem in secondary school mathematics is that students start off believing that they cannot do well, so they find it difficult to construct their sense of mathematics. However, encouraging students to learn mathematics to think mathematically is a good way to increase belief systems and a sense of themselves in mathematics. Mathematical thinking is a process which, by enabling students to increase the complexity of ideas they can handle, extends their understanding. If students' ability to think mathematically is an important outcome in math class, then it is clear that mathematical thinking must feature prominently in math lessons. Giving students the opportunity to come to enjoy mathematics through reasoning and to see themselves as successful, confident, mathematical problem solvers is a situation that supports students to think mathematically. This article investigate the role of reasoning and problem solving in learning to think mathematically in secondary school mathematics. In reasoning and problem solving, students engage in processes of understanding, generalizing, guessing, drawing conclusions, justifying, and evaluating. Students who do reasoning and problem solving in secondary school mathematics require skills in using mathematical knowledge, using mathematical reasoning, using heuristic strategies, and controlling the implementation of mathematical knowledge and strategy. Each process allows students to get some hands-on experience of mathematics in action, both cognitive and affective experiences so that they will able to build and develop mathematical thinking skills

INTRODUCTION

Learning mathematics can be a struggle for some students and the methods that educators use in the classroom can make a huge impact on the level of understanding of student [1], and this situation can affect students in thinking mathematically. Research results convinces us that all students can think mathematically and all young students must learn to think mathematically [2]. Making mathematical representations and representing mathematical ideas, making explicit connections between mathematics to other maths and with other content areas is a tool for expanding students' mathematical thinking capacity and helping them learn to think mathematically [3]. According to Mason, Burton, and Stacey [4], students 'ability to think mathematically or students' mathematical thinking can be improved by practice with reflection. Having experience in problem solving and reasoning is some form of practice with reflection that can improve mathematical thinking skills in secondary school students. If there is a lack of focus on mathematical reasoning, mathematical thinking and problem solving in the students' learning environment, it is English [5] contention that it may not be unrealistic to expect even high achieving students in some situations to focus on surface and not structural features of mathematical problems, even though deep mathematical understanding and structural features of mathematical problems enable students to be good mathematical thinkers.

MATHEMATICAL THINKING

Burton [6] defines thinking mathematically as "the style of processing which supports an inquiry which might lead to the learning of some mathematics but equally might lead to learning in other subject areas". He stated "we need to think about mathematical thinking skills in the area". Based on that definition, mathematical thinking is a process not only used to understand content in mathematics, but also in other fields outside mathematics although more widely used in such subject areas as Burton [7] states that "like the scientific method, which are not necessarily concerned with science alone, mathematical thinking is used when dealing with appropriate problems in every sphere of context, although the question of mathematical nature may be better prepared to expose such thoughts ".

As well as the definition, mathematical thinking is also defined as a dynamic mathematical process, by enabling students to increase the complexity of ideas we can handle, expands our understanding [4]. Thinking mathematically according to Schoenfeld [8], means "developing a mathematical point of view and applying the competence of the instruments of the trade, and those tools in the service of the goal of understanding structure - mathematical sense-making". Both definitions show that mathematical thinking is a dynamic process, which allows for the expansion or development of ideas, understandings, competences of mathematics that make sense.

From the previous description of mathematical thinking, it can be said that the students need to think about mathematical thinking skills in the area. Processes that occur in mathematical thinking consist of strategies such as classifying, ordering, sub-setting and enumerating; specializing, conjecturing testing and generalising; and the many different uses of logical processing such as if. . then, and analogy, this is similar to that [6]. They demand familiarity with asking "what if ...?" as well as "what if not.?" and being ready to use trial and improvement methods, to justify an argument to fall, or be amended, as a result of one counter-example. Specializing (looking for patterns and relationships), conjecturing (predicting relationships and results), and convincing (finding and communicating reasons why something is true) is the basis for mathematical thinking which is the processes which underlies mathematical thinking [7,4].

To develop the ability to think in mathematics need to be noticed what can support it. Mathematical thinking ability can be supported by creating an atmosphere where confidence can grow is necessary, but not sufficient. To flourish, mathematical thinking requires not only nurture, but also extension. Three components in particular create such an atmosphere. the three atmosphere are questioning, challenging, and reflective [4]. Questioning is visible from our attitude towards questions like: can i make conjectures ?, can I justifying or falsifying arguments ?, can I check, modify, and alter?. Challenging is seen from our attitude towards questions such as: can I dentify questions for investigation ?, can I negotiate meanings of terms ?. Reflective view of our attitude towards questions such as: can I be self-critical ?, can I expect and assess different approaches ?, can I shift, renegotiate, change direction ?.

TOWARD LEARNING TO THINK MATHEMATICALLY THROUGH REASONING

Mathematical Reasoning

Reasoning has been defined as the processes of drawing conclusions or inferences [9][10] based on facts and evidence after a causal and rational analysis of the problem [11]. These processes refer to cognitive processes that start with information and arrive at assertions and conclusions [12], that go beyond that information [13]. This process can also be described as a mental recognition of cause and effect relationship. According to Skinner, it may be the prediction of an event from an observation cause or the inference of a cause of an observed event [14]. From this description, reasoning can be described as certain cognitive processes in an attempt to transform or develop a particular situation or information into a reasonable and justifiable conclusion.

The description of reasoning tells us that when we reason, we may develop a number of purposes to convince ourselves or others of a particular claim; to solve a problem; or to integrate a number of ideas into a whole coherent. Two processes are important to reasoning by Ball and Bass [15]; first, that the different steps or moves in the line of reasoning are connected with each other; and second, that these links are somehow "reasoned", there are reasons why a number of moves come together to form an argument or to solve a problem. Therefore, the argument resulting from acceptable conclusion is the product of reasoning. As discussed above, mathematical reasoning is about the conviction that comes with a justified argument, which you can communicate to others.

In mathematics, mathematical reasoning certainly does reasoning about and with the objects of mathematics [16]. Therefore it can be said that mathematical reasoning is the reasoning about and with the objects of mathematics. Based on the previous description of reasoning, then the process involved in mathematical reasoning needs some elaboration. In mathematics, reasoning is a basic skill [15] and is necessary for a number of purposes, eg to understand mathematical concepts or to use mathematical ideas and procedures flexibly, and forgot mathematical knowledge. Kilpatrick et al [2] defines the five strands of mathematical skills required by students in learning mathematics as follows:

- conceptual understanding-comprehension of mathematical concepts, operations, and relations
- procedural fluency-skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence-ability to formulate, represent, and solve mathematical problems
- adaptive reasoning-capacity for logical thought, reflection, explanation, and justification
- productive disposition-habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Although all the strands are important and mutually influential, for kilpatrick et al., "adaptive reasoning is the glue that holds everything together" [2]. Central to adaptive reasoning is the justification of claims and development of arguments. Justification allows mathematicians and mathematics teachers and learners to make connections between different ideas and parts of an argument, to provide warrant for claims and conjectures, to settle disputes, and to develop new mathematical ideas. A justified argument makes sense because it is justified. Therefore, learning mathematics should provide an opportunity for students to be able to justify and explain ideas in order to make their reasoning clear, hone their reasoning skills, and improve their conceptual understanding [2].

The Role of Reasoning in Thinking Mathematically

No doubt that reasoning is one that is essential to understanding mathematics. In reasoning, it is possible for students to develop ideas, exploring phenomena, making conjectures and justifying results, and using mathematical conjectures in all content areas and sophistication-at all grade levels. This situation allows students to see and mathematics makes sense [3] and provokes a challenging atmosphere, which supports students to think mathematically. Reasoning is involved in many other situations besides making decisions. For example, Students might use reasoning to help solve problems. In solving problems through reasoning, students engage in situations that encourage them to understand the problems of students to gain experience to conduct mathematical investigations, to query ideas or assumptions, for negotiate meaning of terms, to arrange or expect and assume different approaches and to make the processes more flexible. These situations support students to think mathematically. As already discussed, mathematical reasoning is also about the conviction. It comes with knowing that the students have a justified argument, which they can communicate to others. Knowing and understanding how to justify conclusions is important for students. Justifying actions allow the students to sense and come from valid reasoning. Learning through reasoning enables the emergence of questions from students such as, what are you doing? or why does that make sense? [17]. Taking into account the similarity or closeness of processes of reasoning and mathematical thinking, then having high-quality standards of mathematical reasoning affects the improvement of students' mathematical thinking quality.

TOWARD LEARNING TO THINK MATHEMATICALLY THROUGH PROBLEM SOLVING

Problem Solving

It is important to define the problem before discussing what is problem solving and it is important to consider whether the situation or tasks, especially the tasks or situations in the mathematics classes, can be said to be a problem. In order to answer these questions it is necessary to discuss the literature relevant to the question. The problem is unknown to any situation that a person seeks to fulfill a need or accomplish a goal [18]. The emergence of "felt need" in situations that indicate a problem [19]. Not only that, problems are problems when there are obstacles that need solutions but how the path or path to the solution can not be known immediately [20]. Hence situations or tasks can be regarded as problems when the solution of these situations or tasks is not known before or not routine, can not be known immediately and the desire or motivation to solve them.

To solve the problem, it takes certain processes to solve it or find a solution to solve it. The whole process of dealing with a problem in attempting to solve it is called problem solving [21]. According to Jonassen [22], problem solving is "any goal directed sequence of cognitive operations" directed at finding that unknown. The sequence of cognitive operations means a number of multiple-step processes where schema and the problem at hand and then act upon a solution [23]. According to the nature of the problem in problem solving, problem solving is a complex activity, involving problem solving necessarily engages a variety of cognitive components, such as propositional information, concepts, rules, and principles (domain knowledge); information networking, semantic mapping / conceptual networking and mental models (structural knowledge); constructing / applying arguments, analogizing, and inferencing (ampliative skills); goal setting, allocating cognitive resources, assessing prior knowledge, and assessing progress/error checking (metacognitive skills) in the learner [18].

As discussed earlier that problem solving is a complex activity, it is not wrong if problem solving is also defined as cognitive activity involving processes and strategies [24]. Processes and strategies in solving problems can be stages starting from understanding the problem to finding solutions to solve problems. George Polya in his book "How to Solve it" discusses heuristic as a problem-solving plan. Heuristics is the process by which problem solvers try to try different approaches to find problem solving [20]. Polya [25] mentions 4 steps or stages in solving the problem that includes:

- understand the problem, ie create an internal representation of the problem
- plan the completion, ie create a pattern or formulate a mathematical model that can be used to solve the problem
- implement the plan, using the model or method that has been planned and formulated
- re-examine the solution, ie look back at the steps almost the same as polya,

Garofalo & Lester [26] also mentions problem-solving steps that are, problem oriented, organizing problems (identifying solving goals and planning settlement actions), implementing plans, and verifying.

The Role of Problem Solving in Thinking Mathematically

Learning through problem solving means that students learn mathematics through real contexts, problems, and the other problematic situations. When students are placed in the problem-solving process, they are subject to solving problems. Providing students with the opportunity to carry out mathematical investigations, to ask ideas or assumptions, to negotiate the meaning of terms, to organize or expect and assume different approaches, to make the process more flexible and to be self-critical, is a situation that can support students to think mathematically.When they can find the problems they face in the mathematics class and beyond the math class [3] because they have their own skills and concepts they have learn before. Students who are used to be solving new problems without explicit instructions on how to overcome them. In problem solving, students learn mathematics by doing math. Therefore, through problem solving, mathematical skills or ideas are the outcomes of problem solving [27]. There are good ideas in teaching or learning through problem solving [28], focuses students' attention on ideas and sense making.

- Develops student confidence that they are capable of doing mathematics and that mathematics makes sense.
- Provides a context to help students build meaning for the concept.
- Allows an entry point for a wide range of students.
- Provides ongoing assessment data useful for making instructional decisions, helping students succeed, and informing parents.
- Allows for extensions and elaborations.
- Engages students so that there are fewer discipline problems.
- Develops "mathematical power."
- Is a lot of fun!

Of course, situations like this support the development of students' mathematical thinking.

CONCLUSION

To be successful in mathematics, students need to learn to think math, and understand how to support it. Through reasoning and problem solving, students may have experience to do mathematical inquiry, to ask ideas or assumptions, to negotiate the meaning of terms, to organize or expect and take different approaches, to make the process flexible and to make them critical learners. This situation can support students to think math and can improve the quality standards of students' reasoning. When the quality of students' reasoning qualities increases, their skills for mathematical thinking can also increase. Learning math through reasoning and problem solving also gives students the opportunity to develop their reasoning and problem-solving skills. when they can develop reasoned reasoning and solve problems, their skills and mathematical thinking can develop.

REFERENCES

1. M. Daniel, *International Journal of Research in Education and Science (IJRES)* **2** 295, (2016).
2. K. Jeremi, S. Jane and F. Bradford, *Adding it up: helping children learn mathematics* (Washington: National Academi Press, 2001)
3. National Council of Teachers of Mathematics, *Principles and standars for school mathematics* (Reston, VA: NCTM, 2000)
4. M. John, B. Leone and S. Kaye, *Thinking mathematically* (London: Pearson Education, 2010)
5. D. E. Lyn, *Mathematical and analogical reasoning young learner* (New Jersey: Lawrence Erlbaum Associates. Inc, 2004)
6. B. Leone, *Early Child Development and Care* 82 57, (1992).
7. B. Leone, *Journal for Research in Mathematics Education* 15 35, (1984).
8. H. S. Alan, *Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics* (New York: MacMillan, 1992)
9. P. Jacqueline and J. S. Robert, *The nature of reasoning* (UK: Cambridge University Press, 2004)
10. J. H. Keith and G. M. Robert, *The cambridge handbook thinking and reasoning* (New York: Cambridge University Press, 2005)
11. S. Maryam, A. Parvaneh and R. M. Mohsen, *International Scientific Publications and Consulting Services* 2013 1, (2013).
12. L. Johan, *Educational Study in Mathematics* 42 165, (2000)
13. E. B. Goldstein, *Cognitive psychology: connecting mind, research, and everyday experience* (Belmount: Wadsworth)
14. S. K. Fatima and B. R. Digumarti, *Reasoning ability of school students* (Delhi: Discovery Publishing House, 2007)
15. Ball and Bass, *National Council of Teachers of Mathematics* 27, (2003).
16. B. Karin, *Teaching mathematical reasoning in secondary schools* (New York: Springer, 2010)
17. G. Robert and B. R. Lauren, *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (Hillsdale, N.J: L. Erlbaum Associates, 1989)
18. H. J. David, *Springer* 45 65
19. P. K. Arlin, *Praeger* 229, 1989.
20. S. P. Alfred and K. Stephen, *Problem solving in mathematics, grades 3–6: powerful strategies to deepen understanding* (USA: Corwin, 2009)
21. B. Werner and N. Mogens, *Educational Study in Mathematics* 22 37, 1991.
22. H. J. David, *Learning to solve problem. an intsructional design guide* (San Fransisco USA: John Wiley & Sons, Inc, 2004)
23. M. Richard, *Thinking, problem solving, cognition* (New York: W.H. Freeman and Company, 1983)
24. K. James, *Principles for teaching problem solving* (Indiana University: PLATO Learning, Inc, 2003)
25. George, *How to solve it, a new aspect of mathematical method* (Princeton, New Jersey: Princeton University Press, 1945)

26. G. Joe and K. L. Frank, *Journal for Research in Mathematics Education* 6 163, (1985).
27. P. C. Thomas, et al. *Making sense: teaching and learning mathematics with understanding* (Portsmouth, NH: Heinemann, 1997)
28. A. V. John, *Elementary and middle school mathematics, teaching developmentaly* (Boston, USA: Pearson Education, Inc, 2010)