5th ICRIEMS Proceedings
Published by Faculty Of Mathematics And Natural Sciences
Yogyakarta State University, ISBN 978-602-74529-3-0

Activated Scheme in Pattern Problems by Student with Low Mathematics Ability

Helti Lygia Mampouw[1, a)] Agung Lukito[1)], St. Suwarsono[2)]

[1]*Postgraduate Program of Mathematics Education, State University of Surabaya,*
[2] *Department of Mathematics Education, Sanata Dharma University, Yogyakarta*

[a)]Corresponding author: heltimampouw@mhs.unesa.ac.id

Abstract. This paper aims to describe activated scheme in solving problems about pattern by a low-math student. The subject was a female grade 7 junior high school with low mathematics-capability based on mathematical competency test. The subject was working on repeating and growing patterns problems that contain numbers and geometric shapes. Furthermore, based on interviews and written responses by the subject, an activated scheme was analyzed at the subjective operator level according to the theory of constructive operators. It was found that activated scheme of repeating patterns was constructed when recognizing various versions of basic elements. The basic element was used to find the terms in sequences of recurring numbers but was ignored when searching for a particular sequence of shapes in repeating geometric sequences. Therefore, constructed scheme was detected at the intuitive and informal level. Activated scheme of growing patterns were constructed through the identification of rules in sequence of numbers or geometric shapes. The subject successfully converted geometric shapes into a number of patterns and was able to determine unknown consecutive terms. The subject began to develop her formal thinking although failed to make a generalization. In the growing quadratic number patterns, the subject uses the difference of the last two terms in sequences and then applies it to determine the next term in that sequence. Such understanding may be subject to limitation of the cognitive capacity. These findings suggested that recognizing the students' cognitive capacity is important to design learning that stimulate critical thinking.

INTRODUCTION

Regularity in mathematics is known as pattern. At a simple level, the pattern can be demonstrated physically and through picture. A picture that changes following a regularity can be extracted numerically into a number table. The functional relationships identified in the number table can be generalized to formulas. Behavior of relationships between numbers in the table or numbers generated using the formula can be displayed through a graph. When expressed in word problems, language plays a role in identifying the said pattern. These various representations indicate hierarchical abstraction pattern.

The hierarchical representation of patterns has become focus attention in school's mathematics curriculum. NCTM 2000 contains pattern-related materials which are connected to relation and functional materials in algebraic standards. Referring to NCTM 2000, Vogel suggests pattern forming characteristics which are comprised of exploring, identifying, extending, reproducing, comparing, representing and describing [1]. The appearance of characters and their sequence strongly depends on the context and the problem of the pattern itself. The mathematics curriculum of 2013 includes pattern as one of basic competencies that serve as a means of shaping the ability to think logically, analytically, systematically, critically and creatively. The mathematics curriculum of 2013 implies that pattern learning starts from recognizing the pattern of drawing and the pattern of numbers up to making generalization of the patterns found in numbers sequence and objects sequence configurations. The ability to generalize patterns is the primary foundation in learning about functions.

The ability to generalize patterns requires the ability to connect the nature of correspondence between algebraic thinking and generalization using algebraic symbols. Some research show relationship algebraic thinking and

generalization pattern. For examples, visual representation is a bridge that links numerical representation to algebraic representation [2], generalization of patterns is done by recognizing the numbers which represents the quantity of objects being discussed, putting them in writing sequentially, and identifying any changes in the numbers [3]. When students generalize the pattern, it is not enough to simply state the general rule of the pattern sequence. There are differences in the process students underwent when generalizing patterns from that of Radford's findings [4]. Thus, the first step to take toward the generalization stage is to recognize regularities and represent them.

Students have different abilities in recognizing and representing patterns. According to Piaget, the difference has largely to do with cognition. In terms of age, the older they get, the more commands they have got over systematic arrangement and formation of schemes that allow the formation of organized thoughts and actions [5]. In the neo-Piaget group, Pascual-Leone connects cognitive capacity with the ability to process information [6]. Thus, the complexity, success or shortcoming, of a person in recognizing patterns and representing them in various ways is an ability to process information which is due to fulfilled cognitive capacity.

Information processing of a child relates to activated schemes arising from the information it receives. In their growth, children are able to co-ordinate an increasing number of schemes [7]. The new scheme is being formed since there are old schemes being combined together or because there is a new information being processed together with the old scheme. Old schemes or old condition then will generate new schemes since there are operators that construct them. Scheme construction is the foundation of the Theory of Constructive Operators (TCO) developed by Pascual-Leone.

TCO has two operator levels that are subjective and metasubjective. Subjective operators put emphasize on the fact that the success of schema activation process is the result of interaction of individuals with their own experience. Pattern formation an individual make is connected with their cognitive development. At this level, the activated scheme places more emphasis on the different cognitive and operative types [7]. Metasubjective operators, the hidden operators who do not have their own information content, work in the brain that causes the schemes to be activated. For example, M-operators activate schemes relevantly concerning with tasks, L-operators describe automation on activation schemes, and I-operators inhibit / reduce scheme activation in certain situations. Each operator has its own task but works simultaneously when processing information in activating the scheme.

Scheme is as a dynamic coordination of the three components: functional, releasing and effecting [8]. The plan to execute a scheme is started from setting the goal and expected outcome, strategy to achieve it, and the amount of information that will be processed using the selected strategy in attempt to achieve the goal. Functional component is about affixing/attributing the core of a scheme and expectation of an outcome that are referring to anticipating the consequences of scheme activation. Releasing component relates to a situation sufficiently enough to encourage an activated scheme. In this case, the two types of schemes observed are figurative schemes and operative schemes in general and specific model [9]. Operative schemes contain parameters that are executed to achieve the expected goals and results. The goal itself is the result of activating the scheme which in actuality presents as the impact of the components. The relation between scheme types and scheme components is shown in **FIGURE 1**.

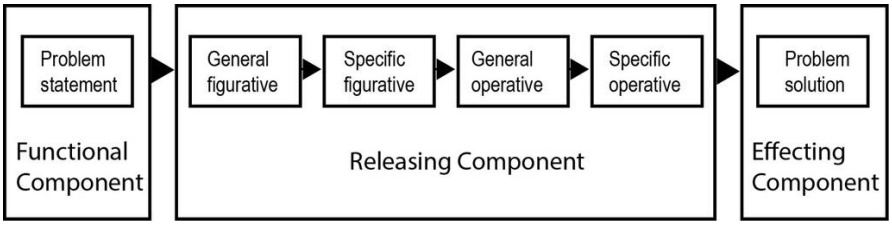

FIGURE 1. The process of achieving balance through activated schemes at subjective operator level

In the classroom, teachers are faced with differences in students' ability in forming patterns. To handle student differences, teachers can differentiate learning based on students' ability as a method of assisting students achieve the expected competencies [10]. Difference in students' math skills is perceived as difference in their capacity to information processing. Difference in mathematical abilities allows for different schemes being activated in pattern formation by students. Students with low math skills generally face more difficulty processing mathematical information than students with high and moderate math skills. Understanding activated schemes in low-math students in pattern formation is one of the attempts made to understand their mathematics learning process. This paper aims to describe an activated scheme in pattern formation by low-math students.

METHOD

This qualitative descriptive research aims to reveal the phenomenon of cognitive schemes that are being activated in pattern formation by low mathematics students viewed from the theory of constructive operators. The research subject is Ani (nickname), a junior high school student with low math skills and has never before studied pattern-related materials. Data is collected from written answers, oral answers, and gestures Ani demonstrated in the research during written tests, interviews and observation. Written tests and interview materials consist of repeating patterns and growing patterns, both of which are represented through geometric shapes and numbers. The collected data represents activated scheme by Ani, which contains figurative scheme and operative scheme in pattern formation of each character: exploring, identifying, extending, reproducing, comparing, representing and describing [1]. The result is analyzed using the functional, releasing and effecting components [8]. The process of achieving between balance through activated scheme at subjective operator level is shown in Fig.2.

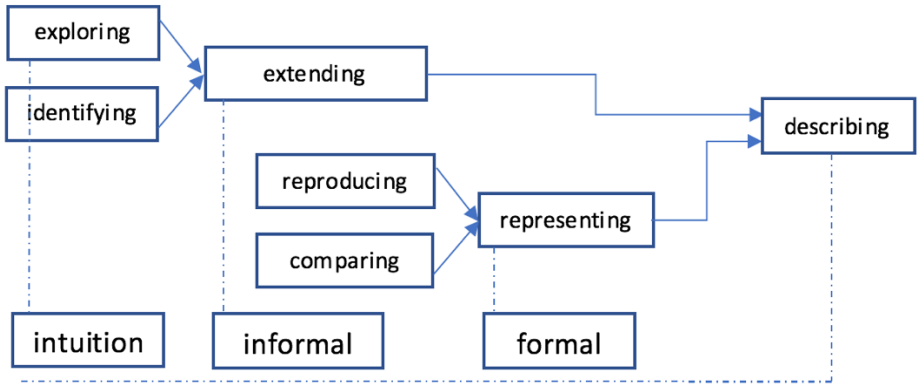

FIGURE 2. The characteristic relationship of pattern formation in thinking stages

RESULTS AND DISCUSSION

The activated scheme on the formation of repeating patterns

Problems of repeating patterns that contain images, letters, and numbers representation are used to trace the activated scheme in repeating pattern formation that Ani made. An example of repeating pattern of geometric images representation and Ani's answers is shown in Fig.3. An example of repeating pattern of letters or numbers representation and Ani's answers is shown in Fig.4. Based on these answers, an interview was conducted to explore the activated scheme in the formation of repeating patterns that Ani made.

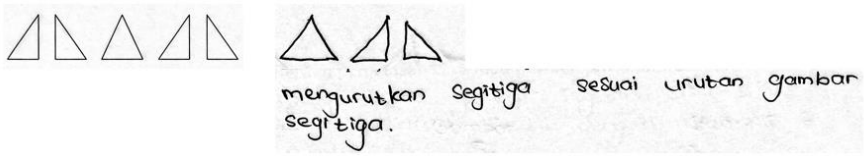

FIGURE 3. Ani's written answers in repeating pattern problems of geometric shapes representation

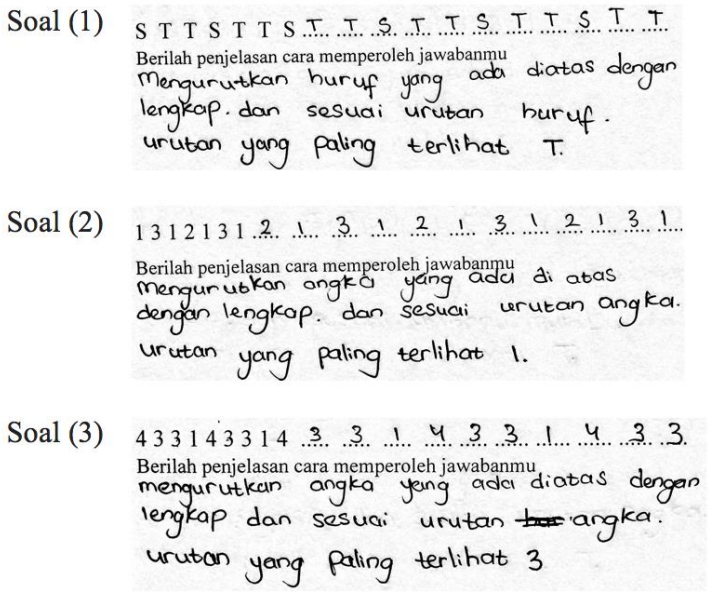

FIGURE 4. Ani's written answer in repeating pattern problems of numbers representation

Functional component. Ani explores repeating patterns using reading and understanding problems skill and activating common figurative schemes of letters and numbers, knowledge about two-dimensional shapes, and understanding of sequences. Ani takes longer time to understand the sequence of geometric shapes than she does to understand the sequence of letters or numbers since she needs to pay attention to the forms of the two-dimensional shapes. Nevertheless, Ani considers repeating pattern problems, either in geometric image sequence or letters or numbers sequence, were easy to do. After reading the problems, Ani can narrate the questions set forth in the problems.

Releasing component. In exploring and identifying repeating patterns of geometric shapes representation, the specific figurative schemes activated by Ani are 2 types of right triangles, isosceles triangles, geometric shapes which develop a sequence. Ani also recognizes the order of appearances of these geometric shapes so she can determine the sequence regularity, and then use it to continue the sequence. To continue the sequence, Ani uses "repetition" principle based on the regularity that she recognizes. Ani faces difficulty in rebuilding the geometric shapes sequence when the sequence starts in order different from the one shown in the problems. For example, Ani tries to use different method other than "repeating". Consequently, the specific activated operative schemes on the sixth through the tenth term are the repetition of the first through fifth term.

In exploring the repeating number pattern, a specific activated figurative scheme would constitute some letters or numbers configuration that forms a sequence. For example, from Fig.4 problem 1, there is a configuration of S and T, to which Ani thinks of ST, TT, TS and from problem 2, where there is a configuration of number 1,2,3, to which makes Ani think of 12, 21, 13, 31. Interestingly, in addition to identifying sequence regularity, Ani also describes the objects which made most frequent appearance. Likewise, with geometric shapes pattern, the next terms in number sequence are obtained by Ani by continuing the sequence using repeating sequence principle. Thus, the specific operative scheme that Ani has in the repeating pattern of image representation as well as letters or numbers representation is the process of repeating the images based on regularity that she recognizes.

Effecting component. The final answers shown in Fig.3 and Fig.4 constitute as the impact of implementing specific operative scheme on specific figurative scheme. Ani executes pattern problems by continuing the next terms on the sequence and writing the reasons for each of those answers. By applying the same way, Ani is able to find tenth term of geometric shape sequences and thirtieth terms on the sequences.

The activated scheme on the formation of growing pattern

Problems and Ani's answers on growing patterns problems of geometric shape or number representation are used to trace the scheme that is being activated in Ani's growing pattern formation. Problems and answers of growing patterns of geometric shape representation are shown in Fig.5. Problems and answers of growing patterns of number

representation are shown in Fig.6. These answers form the basis for the interviews to explore the activated scheme on Ani's pattern formation.

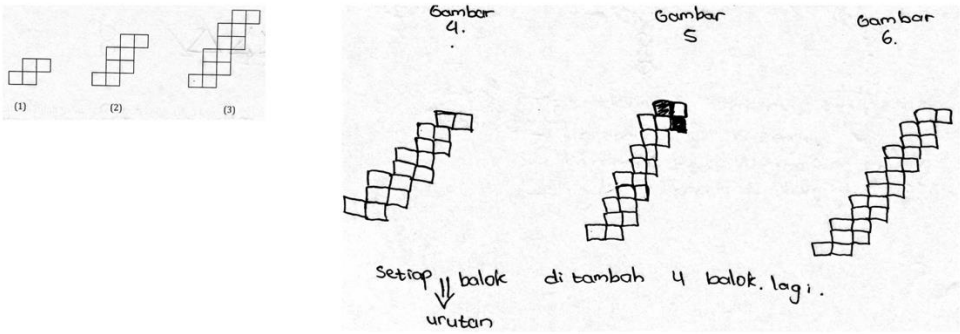

FIGURE 5. Ani's written answers to growing patterns of geometric shape representation problems

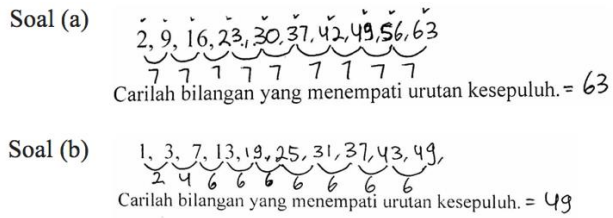

FIGURE 6. Ani's written answers to growing patterns of number representation problems

Functional component. The reading and comprehension skills, prior knowledge of two-dimensional shapes and numbers, play a role in activating the figurative scheme of geometric shapes and numbers used in growing pattern problems. To understand the problems, Ani observes the sequence of geometric shapes, and then reads the problem completely. Finally, she looks back at the geometric shapes to find the core of the pattern. For Ani, the problems of growing pattern are relatively easy. Furthermore, Ani can narrate the questions set forth in the problems.

Releasing component. A specific figurative scheme that is activated in growing pattern is developed based on the images appearing in each of the respective questions. In Fig.5, Ani recognizes the regularity of the image □ to develop  as the core of the geometric shape pattern. The specific operator scheme being constructed is the number of squares (Ani call them: cubes) ‘add by 4’. Ani can also apply add by 4 principle, numerically, without drawing them first. In Fig.6 problem a, number 2,9,16 are being activated and specific operation scheme of add by 7 is being constructed. In Fig.6 problem b, number 1,3,7,13 are being activated and specific operator scheme of add by 6 is being constructed. Ani executes problem a and problem b using the same method, whereas the types of the problems are different. In problem b, Ani takes the difference of the last two terms to determine sequence regularity. Furthermore, this regularity is applied to find the next terms on the sequence. Thus, Ani's operative scheme is still at the level of linear thinking because she uses an arithmetic sequence approach on geometric growing patterns.

Effecting component. The implementation of specific operator scheme produces final answers shown in Fig.5 and Fig.6. Ani executes the problem by writing the next terms on the sequence and writing the reason for the answer. By applying this method, Ani was able to find tenth terms on the sequence of geometric shape and fifteenth terms on the sequences of number sequences.

The role of metasubjective operators

The activated scheme of patterns problem is a balance achieved after through the process of adaptation of cognitive conflicts that occur when reading the problem. Ani’s concept of the pattern is not always the same as the concept of the pattern that other people think. This shows the operation of metasubjective operators in directing Ani's answer.

Here are the principles of the operators according to Mora, et.al [7]. The way each carrier works is still searched further.

M-operator deals with one's cognitive capacity. Ani's answer to the questions about patterns reflects his cognitive capacity. Understanding the problem, finding the core of pattern, setting parameters and performing arithmetic operations to obtain the final answer is an example of the Ani's cognitive ability. Ani should have been able to activate five schemes simultaneously using his cognitive abilities [8]. In the repetitive pattern of geometric shapes representation, Ani has an executive scheme that plays a role in coordinating the five images on the problem (figurative scheme) to produce the core of pattern. Ani applies the core of pattern to reproduce terms pf the pattern as requested on the problem (operative scheme). Ani has answered all the questions that show her cognitive capacity.

Some Ani's answers are not in expected. This shows that Ani's cognitive capacity has not reached the expected level. In the pattern of repetitive geometric shapes pattern, Ani managed to transform the geometric shapes sequence to a numerical form. Ani successfully formulate the number of geometric shapes on the developing pattern with "$4 + 4 + 4 + \cdots$." but she has not reached the "$4 \times n$" form of multiplication. In Fig. 6 problem b, Ani can find "added number" but Ani has not been able to find the relation between numbers added. Therefore, the next terms are built with the rule of 'add 7'. Thus, Ani's cognitive ability is still limited to linear patterns. These results indicate Ani's L-operator has not been fully developed. Ani has established a new scheme but not a result of coordinating two or more simultaneous activated schemes [11].

I-operator completes the control function on M-operator. I-operator works to block or disable the scheme or deviant tendencies [7,11]. When Ani does not find the expected pattern then this means there are conditions that block or interrupt in Ani's mind. The results of I-operator are observed when Ani determines the tenth term in the representation of repeating geometric shape pattern. The five of geometric shape on the task in Fig.3 are too dominant to prevent Ani from finding the true geometric shape on the tenth term.

CONCLUSION

Pattern formation occurs on intuition level, both informal and formal. Ani can explain and give reasoning for every regularity she recognizes. Ani has an activated scheme on repeating patterns formation by means of two methods, namely repetition of terms in sequence according to regularity and Ani attempts of using regularity without registering the terms sequence. Both methods provide different results, where terms repetition in sequence has been in accordance with repeating patterns concept; whereas, regularity without registering the terms is different from repeating patterns concept. In growing pattern, Ani has an activated scheme using images and numerical. The selection of image regularity has been in accordance with the concept of growing patterns. Ani can transform figure pattern to the numeric where the next terms are generated from the result of numeric calculation without images. In the growing pattern of number representation, the activated scheme is at the level of linear thinking. As a result, the number pattern that increase nonlinear is solved by linear addition method.

The activated scheme of patterns by Ani is the coordinated work of her metasubjective operator. M-operator plays a role in the maturity of cognition and mental readiness of the subject completes the task, C-operator plays in the subject can receive and complete the tasks and I-operator hinder the occurrence of an expectation understanding of the repeating and growing pattern.

The results of this study are expected to enrich the insight of teachers in preparing low-math students to learn more about the pattern and other subject in mathematics.

ACKNOWLEDGEMENT

This work has been supported by grants from Ministry of Research, Technology and Higher Education Republic of Indonesia under the scheme of doctorate dissertation research program.

REFERENCES

1. R. Vogel, *ZDM* **37**, 5, 445-449 (2005).
2. B. Patton, E.D. Santos, *International Journal of Instruction* ***1***, Vol. 5, (2012).

3. L. Radford, "The Rhetoric of Generalization. A Cultural, Semiotic Approach to Students" Processes of Symbolizing" in *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, Haifa, Technion-Israel Institute of Technology, Vol.4, 89-96. (1998).
4. S. Inganah, Subanji. "Semiotik Dalam Proses Generalisasi Pola" in *Prosiding KNPM V* (Himpunan Matematika Indonesia, Juni 2013).
5. D.H. Schunk, *Learning Theories: An Educational Perspective edisi 6,* (Pustaka Pelajar, Yogyakarta, 2012).
6. J. Pascual-Leone, *Journal of Experimental Child Psychology* 77, 138-154 (2000).
7. S. Morra, C. Gobbo, Z. Marini, S Sheese, *Cognitive Development: Neo-piagetian Perspectives,* (Lawrence Erlbaum Associates., New York, 2008).
8. J. Pascual-Leone, E.M.R. Escobar, J. Johnson. *Logic: Development of Logical Operation*. *Encyclopedia of Human Behavior* edited by W. Hirstein (Elsevier, Toronto, 2012), 538-549.
9. M. Niaz, *Educational Psychology*, Vol.14, No.1, 23-43. (1994).
10. J. Hattie, "Visible learning for teacher: maximizing impact on learning" (Ruotledge, London, 2012), pp.25-38.
11. S. Morra, *Journal of Experimental Child Psychology* 75, 191-227 (2000).