

Students' Understanding of Negatif Integers and Its Operations Based on Pseudo-Scaffolding

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Abstract. Negative integers play an important role for any student to solve daily problems. Many students have some difficulties to understand the negative integers concept. This study aims to investigate the ability of students in solving negative integers problems. A pseudo-scaffolding was implemented for students to resolve their mistakes. The sample was the students of 7th grade (6 girls, 9 boys) and 9th grade (3 girls, 8 boys) of the Winaf's private tutoring of Surabaya. All students had to answer the given problem which focused on negative integers with addition and subtraction operations. Six volunteer students were selected, namely two boys and one girl from each grade respectively. The interview and pseudo-scaffolding were implemented based on the subjects' answer. The results show clearly the pattern of students' errors and misconceptions when applying addition and subtraction involving negative integers. One grade seven and two grade nine students were confused with addition and subtraction when operating together in negative integers. This study underlined that the pseudo-scaffolding may give direction to students for correcting their errors and misconceptions. Furthermore, it can help students for improving and understanding the concept negative integers. This study can be extended to other operations of integers and regularly classes.

INTRODUCTION

Integers is one of mathematics lessons which has to be taught in Junior High Schools. Integers have actually been introduced at the elementary school level, because it is a very important topic to be learned as a prerequisite subject for an advanced mathematics. Integers become the earliest concept for students to understand an advanced concepts in arithmetic [1, 2]. Many researches have been underlined that there are many students with some difficulties in understanding negative integers. The grade seven students in the Philippines had difficulty for understanding the basic concepts and principles of integers [3]. Similarly, the grade seven students in Manila have often difficulties with related to the concept of integers, from which they were struggle when solving algebraic equations [4].

Moreover, the grade seven students is still difficult to learn integers because numbers, as well as integers, are an abstract concept. In particularly, negative integers may create such difficulties for students as they had always tried to make sense based on their presuppositions of natural numbers, by assuming they already knew that natural numbers holds for integers [5]. It cannot be allowed if many students have difficulties in understanding negative integers. Take into account the notation "-" may have three meanings, namely unary (negative), binary (subtraction), and symmetric (opposite) [6]. The above difficulties are need to be addressed and improved, so that students can appropriately understand the concept of negative integers. Hence, a special understanding is needed to assist students, since integers comprehension become an essential for students to succeed in mathematics [1]. In addition, mathematical understanding is very important in the mathematics learning process. Mathematical ability is one goal of the educational curriculum, because mathematical understanding may strongly support to other mathematical abilities, such as mathematical communication, reasoning, connection, representation, and problem solving [7].

Ones help students to understand mathematics is to implement the so-called scaffolding. Scaffolding is a teaching strategy that provides individual support based on the Learning Zone of Proximal Development [8]. According to [9] scaffolding is a support strategy for pupil's convergent development areas. Scaffolding can help students to correct misconception made by students while working on the problem. It is based on controlled support offered by an able adult to amend the cognitive difficulty faced by children when they cannot solve problems with their existing level of development [9]. While *pseudo-scaffolding* is a way for helping students by simply giving some partly hints, explanations, or directions, but not fully guiding students in every detail step. It is applied to fix and for correcting any mistake or misconception which is performed by students, so that the student's self belief can be improved. Thus, this study is to investigate students' understanding of negative integers and its operations, as well as an appropriate treatment to help students for correcting their errors and misconceptions in solving the integer problem.

METHOD

This study used a descriptive-qualitative approach that aimed to investigate students' understanding of negative integer and its operations based on pseudo-scaffolding. This study was conducted in the Win After Study (Winaf) of Surabaya. The Winaf is one private tutoring which provides some helps and guidances to both elementary and secondary students. Most students of the Winaf private tutoring come from the public schools which were located in South and West of Surabaya. Two mainly motivations for involving students of the Winaf were because students exactly needed extra help to achieve a better mathematics ability and also strategically prepared for regular examination as well as the national examination. The study focused on two privately groups, namely grade seven and grade nine students' groups. The 7th grade students (consists of 6 girls and 9 boys) were obviously chosen because they are in a transition phase from the elementary level to earlier the secondary level. On the other hand, the 9th grade students (consists of 3 girls and 12 boys) were involved because they were seriously in the preparation phase of the national examination. All students had to complete the given test which focused on negative integers with addition and subtraction operations. The results were ranked to indicate the students mathematics' ability, and virtued on students' mistakes, incompleted answer, misconceptions or miscalculations. Three students of the 7th grade group (consists of 2 boys and 1 girl) and three students of the 9th grade group (consists of 2 boys and 1 girl) were chosen purposively, as the research subjects, based on the different mathematical abilities. All subjects were individually interviewed in a comfortably room and were given an appropriately pseudo-scaffolding by considering each answer sheet. The pseudo-scaffolding was intended to help the subject for correcting and resolving every misconception or mistake occurred in solving on the given test.

RESULTS AND DISCUSSION

All subjects were encoded as follows.

S₁H = the 7th grade subject with high ability; S₁M = 7th grade subject with medium ability;

S₁L = 7th grade subject with low ability; S₂H = 9th grade subject with high ability;

S₂M = 9th grade subject with medium ability; S₂L = 9th grade subject with low ability.

The result of the given test are presented in table 1 below.

TABLE 1. Results of Correct Answers of Grade (7 and 9) Subjects

Item test	% of correct answers subject	
	7 th grade	9 th grade
-3 + (-4)	66,67%	33,33%
(-17) - (-9)	33,33%	33,33%
(-18) + 7	100%	66,67%
5 + (-15)	100%	100%
(-5) + (-6)	33,33%	66,67%
(-23) + (-35)	33,33%	66,67%
17 - 13	66,67%	66,67%
(-15) + 7	33,33%	66,67%
(-4) + (-16) + (-7)	33,33%	66,67%
(-19) + 8 - (-11) + 4	0%	66,67%

Table 1 shows that the subjects' answers were correct and true whenever summing positive and negative integers, also when adding negative and positive numbers. The 7th grade subjects answered correctly all item tests. In contrary, the 9th grade subjects was only S₂M answered incorrectly. The most mistake is when the subjects executed a subtraction for two negative integers, and only S₁H and S₂H demonstrated their answer correctly. The 7th grade subjects had seriously misconceptions when working out for the item test number 10.

1.1 Result of the subject S₁H

Subject S₁H did not have much difficulty in solving the problem. S₁H made an error when solving the item test number 10. In the interview, S₁H described that he had forgotten when operating subtraction with negative integers. Here, how S₁H answered the item given problem.

FIGURE 1. Answered Item Test 1

FIGURE 2. Answered Item Test 3

In Figure 1 and 2, S₁H showed the way in understanding to solve the problem is to place a negative number under symbol "-" and positive number under symbol "+". This indicates that S₁H separated negative and positive integers. In Figure 1, S₁H performed instantly addition, because on item test 1 was the addition of two negative integers so that S₁H directly given negative sign "-" to final answer. While in figure 2, S₁H executed subtraction to the larger number first, because in item test 3 was the addition of negative integers and positive integers, and the larger negative integer subject directly subtraction and given a negative sign "-" in final answer.

1.2 Result of the subject S₁M

S₁M made an error when implementing addition and subtraction two negative integers, as well as counting the integers mixed operation. An error occurred also when S₁M answered the test item 6, and because S₁M ignored the "minus" sign in directly addition.

FIGURE 3. S₁M Answer for Test Item Test 6

From the interview, the subject S₁M did not understand the concept of addition of two negative integers, and S₁M totally ignored the "negative" symbol on the problem.

- R : Suppose you associated the negative sign as a "debt" and positive as "pay".
 Thus, how could you mention about the test item 6?
 S₁M : I had a debt 23 then I debt 35 again
 R : Please explain, how much were your debt now?
 S₁M : Of course, my debt was 58
 R : Could you tell me, was your answer true or wrong?
 (By pointing to the answer sheet of subject S₁M)
 S₁M : Wrong
 R : So what was the right answer now?
 S₁M : -58

From the given pseudo-scaffolding, by impelenting the idea of debt and pay, S₁M was able to implement the debt-pay context, which could facilitate S₁M better understanding the addition of two negative numbers.

1.3 Result of the subject S₁L

S₁L's made an error when adding two integers. S₁L did not understand the concept of negative integers. S₁L ignored totally the "negative" symbol and considered the addition in the negative integers were the same as the addition of the natural numbers. Integers, especially negative integers, may create such difficulties for students as they had always tried to make sense based on their presuppositions of natural numbers, and by assuming they already knew that natural numbers holds for integers [5].

$$(-23) + (-35) = 58$$

FIGURE 4. Answered Item Test 6 by S₁L

Here is a part of interviews with subject S₁L

- R : What was known on the problem?
 S₁L : Addition $(-23) + (-35)$
 R : How did you add it?
 S₁L : Directly summation.
 R : Why was 58 as your result?
 S₁L : Because $23 + 35 = 58$
 R : But these two numbers (pointing to numbers (-23) and (-35)) were negative numbers?
 S₁L : Yes, those were negative numbers.
 R : So how? Could them be added directly?
 S₁L : Maybe, because I think the same sums the natural number, so I added them directly

It can be seen from the above interview that S₁L performed the addition directly for the test item 6 and ignored the "negative" sign because S₁L considered the summation of negative integers was equal to the summation of the natural numbers, hence the answer of S₁L was incorrectly. Here, a part of interview as given below.

- R : How if you associated negative as a "debt" and positive as "pay".
 Thus, how could you mention about test item 6?
 S₁M : I had a debt 23 then I debt 35 again
 R : Please explain, how much was your debt now?
 S₁M : Of course, my debt was 58
 R : Could you tell me, your answer was true or wrong?
 (By pointing to the answer sheet of subject S₁M)
 S₁M : Wrong
 R : So what was the right answer now?
 S₁M : -58

In the interview, S₁L was given pseudo-scaffolding to fix the mistake occurred. The pseudo-scaffolding was implemented as in the previous treatment, and S₁L was able justify and to revise the wrong answer.

1.4 Result of the subject S₂H.

Subject S₂H had no difficulty in solving the problem. S₂H answered correctly. In the interview, S₂H responded no difficulty and had a good understanding concept of addition and subtraction involving negative integers.

1.5 Result of the subject S₂M.

Subject S₂M made a clearly mistake in solving the problem, namely S₂M computed two negative integers and addition of negative and positive integers.

$$-15 + 7 = 8$$

FIGURE 5. S₂M Answered for the Test Item 8

From the interview, the misconception occurred by S₂M and it was addressed the carelessness in calculating the addition of negative and positive integers. Consequently the S₂M answer was incorrectly one. Here a part of the interview with subject of S₂M.

- R : How did you answer item test 8?
 S₂M : Negative 15 added 7 result 8 (pointing to item test 8)
 R : Let you assume that negative as a "debt" and positive as a "pay"
 So how much was your debt now?
 S₂M : Oh yes, my answer was wrong.
 The result should be -8 .
 Therefore S₂M was able to do self justification for fixing the wrong answer.

1.6 Result of the subject S₂L.

Subject S₂L performed that the error occurred in solving the given problem. It could be identified when completing the addition and subtraction of negative integers. S₂L ignored the "minus" and "negative" symbols. When S₂L executed a subtraction involving two negative numbers, S₂L concluded that a subtraction with negative integer produces a negative integer.

The image shows a handwritten calculation on a piece of paper. The first line is $(-17) - (-9)$. The second line is $= -26$. There is a small arrow pointing to the right next to the equals sign.

FIGURE 6. S₂L Answered for the Test Item 2

Here a part the dialogue with subject S₂L

- R : What did you know about the test item 2?
 S₂L : Subtraction (-17) and (-9) (points to item test 2)
 R : How did you answer that question?
 S₂L : Directly subtraction
 R : Why was your result -26?
 S₂L : Because subtraction from $(-17) - (-9) = -26$
 R : Was there a subtraction? With the inverse of addition (point to - (-9))
 S₂L : Yes, in my opinion the reduction with the sum inverse produces minus
 So, I directly subtracted it.

From the above interview, it is clear that the subject S₂L ignored the "minus" and "negative" signs so that S₂L concluded that the subtraction with the inverse addition results in "minus", whereas the subtraction by the inverse addition results in "summation".

CONCLUSION

This study emphasizes that subjects with high ability have no problem in solving the given problems. The 7th grade subject with low medium ability made some errors in solving the problems, especially when adding two negative numbers and subtracting two negative numbers, as well as implementing negative integers in arithmetic operations. According to [10] subtraction operation involving negative integers may provide students difficulties in solving mathematical problems such as arithmetic and algebraic. On the other hand, the 9th grade subject with medium ability have still difficulties when adding two negative numbers as well as adding positive and negative numbers. The 9th grade subject with low ability was not able to execute both addition and subtraction two negative numbers, and addition of negative and positive numbers. While the subject of 9th grade with high ability answered all tests correctly when adding negative and positive numbers only. In contrast, high ability subjects have no problem for solving the given problem because they had understood the concept of addition and subtraction of negative integers. Subjects with medium ability often ignored the minus symbol on negative integers because they cannot differentiate between "minus" and "negative" symbols. Moreover, subjects with low ability were still confused when they are given the problem of addition and subtraction between positive and negative numbers. According [11] performing operations on integers involves signs of the numbers and the signs of required operation. This makes students get confused and struggle when asked to perform operations on integers.

The implemented pseudo-scaffolding could correct the errors and misconception, and also increase the subject's knowledge of negative integers. This facilitate easier helps for the subjects in understanding the addition and subtraction operation of negative integers. Pseudo-scaffolding were given to the 7th and 9th grade subjects was relatively similar, i.e. by using the debt-paying contexts. This could help easier the subjects in understanding the addition and subtraction operation of negative integers. Most preservice teachers (9 of 15; 9 of the 12 who gave answers) used a money/debt context for the expression, " $2-7=$ __" [12].

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