

Students' Fraction Magnitude Knowledge in Solving Equation Word Problems

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Abstract. Knowledge of fraction magnitude becomes a crucial in pre-algebra. This study examined students' fraction knowledge in converting the word problem into an appropriately equation. The sample was the 7th grade students of the VII B class of the Unesa's Labschool. All students had to answer two tests, i.e. the algebraic equation problems and the estimating-comparing fractions. Four volunteer students were selected as the research subjects. Semi-structured interview were conducted to all subjects to reveal students' thinking process when solving fractions and algebraic tests. Our result strengthened that students with low fraction ability cannot understand fraction as a number, which noticed numerator and denominator as two unrelated numbers. They used symbol on algebra test even though they did not yet understand it. They were not able to state equation writing to solve the given word problems. In contrast, students with high and medium fraction ability described any fraction as a relation between numerator and denominator. They were able to utilize any symbol correctly. However, student with high fraction ability addressed more than one equation for the same word problem, but student with medium fraction ability could not. The students' ability in writing algebraic equations is mainly influenced by students' fraction magnitude knowledge.

INTRODUCTION

Fractions are important in our daily life. At the elementary level, fractions were defined as numbers that were represented by the ordered pair of a/b , where a and b are natural numbers with $b \neq 0$ [1]. Understanding the concept of fractions, as well as its implementation, was important basis for studying higher concepts in algebra involving fractions [2, 3, 4, 5, 6]. Students' abilities in algebra were influenced by several things including working memory [7], fractions comparison [8], and fractions estimation [8, 9]. Since working memory has not significantly impacted in algebraic performance [8], then fractions comparison and fractions estimation become an important predictor for this context.

The concept of fractions were still difficult for students, who could not clearly understand the fraction magnitudes [2, 6, 10, 11]. It was underlined that some errors performed by students of Belgium, aged 10 and 11 years, in understanding the concept of fractions [2]. The errors indicated that students did not comprehend such relationships between numerator and denominator, or students had a low ability about the fraction magnitude knowledge. Understanding of fraction magnitude was a necessary and inevitable step of understanding fraction [6]. Difficulties in understanding fractions might affect students' beliefs to learn more about mathematics in advanced level [2]. Therefore, it was very important for students to have an understanding of fractions correctly. Another mistake students made in comparing two fractions such as $1/7$ and $1/3$. Many students answer $1/7 > 1/3$ because 7 is greater than 3 [2]. In addition, research on 15 elementary students of Madiun in comparing two

fractions were still low. There were only 4 students completed in the fraction comparison task, suppose comparing $\frac{2}{5}$ and $\frac{1}{4}$ [10]. This showed that students do not understand about fraction magnitude.

Research on 668 students of the 7th and 8th-grade in Texas and Delaware in symbolizing algebra described that there were only 9% of students answered correctly on three assigned tasks [12]. Similar research on the understanding of symbols and signs of similarity were made against 373 of the 6th – 8th grade students in America. The result indicated that students did not understand the signs of similarity, and less than 50% of 6th grade students understood symbols [13]. In addition, research on 32 8th grade students in Magelang indicated that in algebra, one of the students' difficulties was solved story problems and understood of symbols [14]. Therefore, students still have difficulties in symbolizing or modeling the word problems.

An understanding of fraction magnitude was central to mathematical development [15], and influenced students' development in mathematics. In particular, the ability to estimate fraction affected to algebra avility [9, 16, 17]. An understanding of fraction magnitude is a necessary structure for learning the concept of algebra [18] and learning algebraic procedures [19]. A lack of understanding of fraction magnitude caused students to be unable to estimate the answer even if it faced with only simple algebraic problems [11]. Suppose a student who does not understand the fraction magnitude, it is likely that he will not understand that in equation $\frac{1}{3}x = \frac{2}{3}y$, the value of x should be twice of y . However, in the equation $\frac{3}{4}x = 6$ the value of x must be greater than 6. There was a relationship between the understanding of fraction magnitude, operations on fractions, and pre-algebra or algebraic knowledge between 6th to 8th grades [11]. Mastery of fraction and algebra are an important competency for the 7th grade of students [20]. In the competence, the 7th grade students was required to solve math problems with fractions and algebra separately or comprehensively.

One form of representation in algebra is a symbol. Students faced difficulties in applying algebraic symbols to express word problems in appropriately equations that included understanding the letters used in symbolizing algebra, changing statements in the form of equations, understanding the language structure of stories and relationships between quantities and how they relate [21]. Hence, this study examined the ability of students in solving algebra word problems in terms of students' understanding of fraction magnitude.

METHOD

The sample of this study was the 7th grade students of the Unesa's Labschool Surabaya, Indonesia. The class 7B was purposively chosen from the existing two classes, and it consisted of 20 boys and 8 girls. All students were given the fraction test to examine their understanding in comparing two fractions and estimating fractions on a line number. Then, students were given the algebraic test focusing on equation writing or modeling algebraic word problems. Based on the results of two tests, students were categorized in three groups, namely high, medium and low mathematics abilities, respectively. Four volunteer students were selected as the research subjects, one student from the high ability, one student of the medium ability, and two students of the low ability, respectively. The semi-structured interview was conducted to the subjects to assess the subjects' thinking process in the equation writing tests. The classification of students' fractions understanding was based on conceptual and procedural understanding. High-ability subject allowed to use the procedure and investigated the reasons for using the chosen procedures [2, 6]. The indicators of algebraic equation writing test, or word problems, were described in Table 1.

TABLE 1. The Indicator of Students' Ability in Equation Writing

Indicators
Use symbols in formulating / modeling word problems.
Operate the symbol used
Explain the meaning of the symbol used (lateral symbol)
Model situations using more than one equivalent equality
Solve problems based on the equations that have been established

RESULTS AND DISCUSSION

Subject PAF (Low Fraction Ability)

Subject PAF was a girl and used an incorrect procedure to compare two fractions. PAF multiplied numerator and denominator for the same fraction. In the semi-structured interview, PAF explained that she did not know the reason for the procedure. She just applied what it had been taught in the class.

- Q : How did you solve the problem?
 S1 : I multiplied between numerator and denominator.
 Then generated which ever is greater
 Q : Why did you use that way?
 S1 : I don't know, I was taught at the class like that.

In the fraction estimation task, PAF worked out by looking at the numerator of the fraction, the denominator has no effect. If the numerator is smaller than it was placed at the first on the number line. PAF had thought to convert the fractions into decimals but the procedure used was the wrong procedure. PAF directly divided the denominator with the numerator, so the result is greater than one. A part of the interview is as follows.

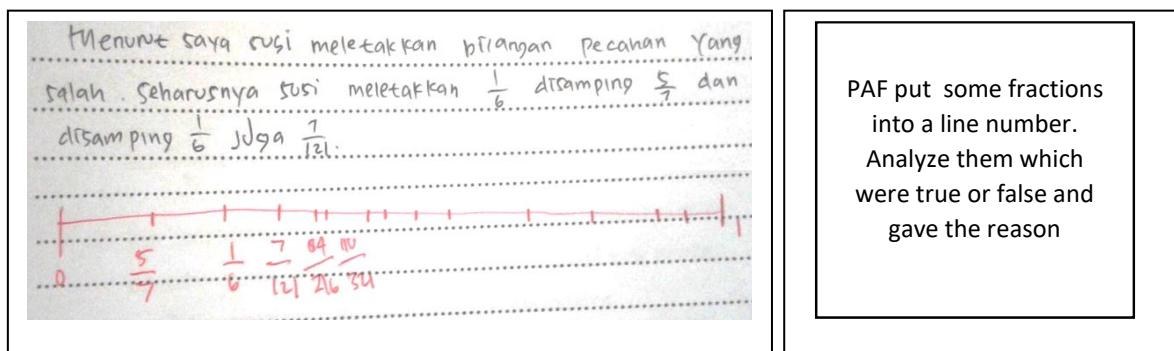


FIGURE 1. PAF's Answer in Estimation Fraction Test

- Q : Could you calmly explain how did you do this?
 S1 : I checked that numerator.
 If the numerator was greater than the others, then placed it on the line number first.
 P : So why did you put it here?
 Why did not you put it in the middle of the line number?
 S1 : I did not have any reason about that.
 I just thought it must be started from the beginning.

In the algebraic test, PAF utilized symbols to model the one variable word problem. However, she did not understand the meaning of the symbol. PAF was not able to identify the variables used in the problem. Therefore she was not able to model the problem. Because of she was not able to model the word problem, she also was not able to form another equivalent equation and solve it. Here, the resulting test was given below.

Bagas buys a pair of shoes Rp220.000,00. This price is four fifth from the initial price

Write the equation



Write the equivalent equation



What is the initial price?



FIGURE 2. PAF's Answer in Algebraic Test

The above answer were justified with the result of the interview as follows

- Q : What did x mean, here?
 S1 : I just assumed it.
 Q : What for was it?
 S1 : Oh it was wrong, the correct one was I did not use x.
 P : So what was the right one?

PAF explained a different answer during the interview, and did not understand what was given and asked in the problem. Then, she had difficulties in forming equations that were equivalent to the first equation. There were some reasons why she had difficulties in forming equations. First, she did not understand the symbol, therefore she did not use it. Second, she did not understand what equation is. Both of them were the basic concept of algebra that must be known by students [13, 22]. And also the understanding of fraction was low. It supported the theory that fraction magnitude was one of the predictors to mastery algebra, especially in estimating fractions [9,16,17]. In modeling two variables of the word problem, PAF faced in some difficulties. She used concept of ratio to model it and used only one variable. PAF's answer was shown below.

Mr. Toni stand near the one year old child that we don't know the height. The height of Mr. Toni is tree times the height of this child

Write the equation

a. Tulis persamaan yang membuat
 misal : tinggi Pak Toni $3x$ tinggi dari Anak tersebut.
 malah : $1x$ jadi tinggi Pak Toni $3x$ dari Anak
 lalu jadi $3x : 1x$.

Write the equivalent
 equation

$1 \times \text{tinggi Anak} \times 3(\text{tinggi Pak Toni}) = 90 \text{ cm}$

What is the initial price?
 $\text{Pak Toni } 3x \text{ dari anak lho Jadi } 60 \times 3 = 180 \text{ cm}$
 karena tinggi Pak Toni Adalah $3x$ tinggi dari Anak
 a sebuah forum rapat terdapat aturan bahwa hasil

Figure 3. PAF's Answer in Two Variable Word Problem

Subject AAQ (Low Fraction Ability)

Subject AAQ used cross product in solving fraction comparison test. AAQ multiplied the numerator of the first fraction with the denominator of the second fraction and multiplied the numerator of the second fraction with the denominator of the first fraction. In the interview, AAQ did not know the reasons for using the method. Here are a part of the interview results is as follows.

- Q : How did you work out in the fraction comparison task?
 S2 : I used multiplication. Then the result is placed on the top of fraction.
 Larger results means bigger fractions.
 Q : How did you used that multiplication?
 S2 : I just multiplied the numerator of the first fraction with the denominator of the second fraction and multiplied the numerator of the second fraction with the denominator of the first fraction.
 Q : Why did you implement that method?
 S2 : I did not know, I was taught it from my teacher at the school.

AAQ applied only the procedure without knowing the reason for the use of the procedure, so that she was categorized as low in understanding the fraction comparison.

In the fraction estimation task, AAQ worked out by looking at the fraction numerator. If the numerator was larger than the others, then the fraction was the largest fraction. If the numerator was the same then the denominator was considered. The bigger denominator effected the bigger fraction. AAQ explanations were different from the test results. AAQ stated that the location of fractions are true, but she has the different opinion when interviewed. Here is the answer of AAQ when interviewing

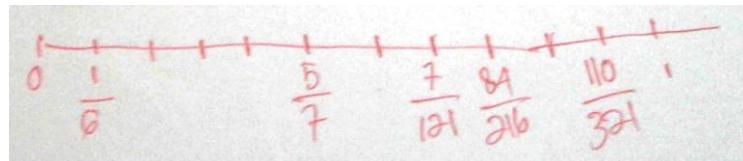


FIGURE 4. AAQ's Answer in Estimation Fraction Test

Algebra test results showed AAQ did not know what is asked and known in the problems so she was not able to determine the variable that will be used in modeling the word problems.

From the results of fraction test and interviews two subjects with low fraction ability did not understand fractions as number. They assumed that the numerator and denominator are two unrelated numbers. Hence, they did not understand the fraction magnitude correctly.

Subject MAR (Medium Fraction Ability)

Subject MAR used equivalence fraction for comparing two fractions and used the concept of whole-part for estimating fractions. A part of interviews with MAR

- Q : Would you mind to describe, how did you answer the question?
 S3 : I looked for the greatest common multiple of two denominators for comparing two fractions
 Q : Why did you use that?
 S3 : I used only the concept of equivalence fraction
 Q : How did you estimate fractions?
 S3 : I divided whole into 2 equivalence parts, then I have $\frac{1}{2}$. I divided $\frac{1}{2}$ into 2 equivalence parts then I have $\frac{1}{4}$. I divided again and again until I find the location of the fraction that I want.

From the interview result, MAR had exactly known the concept of fraction. He understood the procedure of finding the greatest common multiple is from the concept of equivalence fraction. However, he faced difficulties when estimated fraction. He knew the concept, but lack in procedural knowledge.

In the algebraic test, subject MAR was able to write an equation from the given word problem and solved it. He used symbol. But he faced difficulty in wrote more than one equation based on the same word problem. A part of interviews with MAR is as follow.

- Q : What did it mean by x ?
 S3 : It was only a symbol for representing the unknown. It might be changed by another, for example y , z , or a
 Q : So, in this problem, what did x mean?
 S3 : The initial price.

From the interview, subject MAR understood the used symbol. He stated this symbol to denote the unknown and might be changed by another. Subject MAR did not understand what the equivalence equation was. Here is a part of the interview result.

- Q : What did it mean by equation?
 S3 : Equation did mean the right side and the left side have the same value.
 Q : What did it mean by equivalence equation?
 S3 : I don't know exactly.

From the interview, subject MAR understood what equation was but he did not understand the equivalence equation. It influenced his difficulty to write equivalence equation. A part of his answer of the algebraic test as follows.

Bagas buys a pair of shoes Rp220.000. This price is four fifth from the initial price

Write the equation

$$\begin{aligned} & \text{Initial Price} = x \\ & 220.000 = \frac{4}{5}x \end{aligned}$$

What is the initial price?

$$\begin{aligned} & \frac{4}{5}x = 220.000 \\ & x = 220.000 \times \frac{5}{4} \\ & x = 110.000 \times 5 \\ & x = 550.000 \end{aligned}$$

FIGURE 5. MAR's Answer in Algebraic Test

Subject RGZ (High Fraction Ability)

RGZ implemented an illustration of a cake sharing for comparing two fractions. A part of interviews with RGZ as follows.

- Q** : Would you mind to describe, how did you answer the question?
S4 : I just imagined that $13/17$ was likely 13 cakes were divided for 17 children. If $9/17$ means 9 cakes are divided by 17 children. Clearly, 13 cakes were divided by 17 children was greater than 9 of that divided by 17. So $13/17$ is bigger than $9/17$.

- Q : So, did you apply the same procedure to compare all fractions?
S4 : Yes, I imagined some cakes were shared for some students.

From the interview result, RGZ exactly understood the concept of fraction very well. He explained correctly the procedure and reason for implementing the proposed procedure. In the fraction estimation task, he converted fractions into decimal. He converted all fractions, by dividing numerator by denominator, into decimal numbers. Based on these decimal numbers, he was able to estimate the given fractions on the line number which was between 0 to 1. The results of his work in estimating the given fractions as follows.

FIGURE 6. RGZ's Answer in the Estimation Fraction Test.

From interviews and fraction writing test, it showed that RGZ was able to describe a fraction as a relation between numerator and denominator which cannot be separated. He had known that fraction was a number. So he appropriately understood the magnitude of fraction, which was as a number [17].

In the algebraic test, RGZ utilized symbols and understood them. He described correctly what were given and were asked in the given problems. A part of his answer of the algebraic test as follows

Bagas buys a pair of shoes Rp220.000. This price is four fifth from the initial price
<p>Write the equation Write the equivalent equation What is the initial price?</p> <p>$X = \text{harga dasar} / \frac{4}{5}$</p> <p>$\frac{4}{5}X = 220.000$</p> <p>$4X = 220.000 \cdot 5$</p> <p>$220.000 \cdot 5 = 1.100.000$</p> <p>$\frac{1.100.000}{4} = 275.000$</p>

FIGURE 7. RGZ's Answer in the Algebraic Test

Take into account the answer of the algebraic test, a part of interviews is as follows.

- Q : What was the notation x for?
S4 : This was the symbol that I used to form the equations
Q : What did it mean?
S4 : If x = initial price. Hence x was the initial price of shoes.
Q : Could the letter x be changed with other letters?

S4 : Yes, it just the symbol to represent the unknown. Hence, it might be changed with other letters.
From the test results and interview, RGZ explained the symbol as something that represents an unknown value. Thus, he was capable in implementing any symbol. He was also able to form the equations into equivalent equations and solve them.

There was a minor difficulty faced by RGZ in forming the two variables word problem. He applied only one variable in his modeling. Therefore he was not able to form the equivalent equation. Here is the results of his test.

Mr. Toni stand near the one year old child that we don't know the height. The height of Mr. Toni is tree times the height of this child

Write the equation	Write the equivalent <u>equation</u>	What is the initial price?
$x = \text{Arik's height} = 60 \text{ cm}$	$x = \frac{1}{3} \text{ of } 180 \text{ cm}$	$x = 60 \times 3 = 180 \text{ cm}$

Figure 8. RGZ's Answer in Two Variable Word Problem

From the above figure, RGZ assumed that x as a child's of one year height and he did not assume Mr. Toni's height, so he had difficulty in forming equations that were equivalent to the previous equation. But in solving the problem when the value of x was known, he could easily determine its solution correctly. It was because he understood that Mr. Toni's height was three times the one year child's height. As the result of the interview is given below.

Q : What did you write in performing your equation?

S4 : $3x$

Q : Could you explain, what was x ?

S4 : The height of the one year child.

Q : Given the height of the one year child was 60 cm. What was the height of Mr. Toni?

S4 : $60 \times 3 = 180 \text{ cm}$

Q : If the height of child was 70cm, what was the height of Mr. Toni?

S4 : 3×70 . Because it had to be three times.

From the results of the interview, subject RGZ exactly understood the use an unkown symbols and its operations. However, he implemented his one variable knowledge to solve the two variables word problem. It was noted that a significant result for subject AKP, who is also a high fraction ability, was similarly to that of RGZ.

CONCLUSION

Students with low fraction ability indicate that the understanding of fraction magnitude are significantly low. They cannot interpret a fraction as a number. They adressed any fraction as an unrelated numerator and denominator. Hence, it directly affects to the ability in solving the algebra word problem. They apply symbol for unknown value, but do not really understand what equation is. The student with medium fraction ability understands the fraction as a number but he has difficulty in prosedural knowledge. For algebra ability, he understands the symbol used and is able to write and solve the obtained equation. On the other hand, he is not able to write more than one equation from the given word problem. In contrast, the student with high fraction ability is able to demonstrate a fraction as a number. Hence, his understanding of fraction magnitude is also high, and influences the ability for solving the algebra word problem. He is able to use any symbol correctly, and understand what an equation of algebra form is. His ability in performing some equivalent equations is significantly high, and is capable to solve the problem related to one variable word problems. His difficulties referred to the implementation one variable's knowledge into two variables equation, Our result recommend that teachers should gradually strengthen the concepts of fractions.

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