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Categorizations of Students' Reasoning Behavior for Solving Integer Comparison Problems in Elementary School

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Abstract. Students reasoning behaviors become an important tool to investigate how students learn to think mathematically. This study investigates students' reasoning behaviors in solving integer comparisons. The sample were the 5th grade students of the Unesa's Labschool, consisting 11 girls and 9 boys. Considering the existing students' portfolio, all students completed both maths ability and integer comparisons tests, and were ranked the result based on the results. Three volunteer students, with different in maths ability (high, medium, and low), were selected as research subjects. The interview was conducted based on the integer comparisons' answers. Two categorizations of students' reasoning behavior were utilized, namely ordered-based and magnitude-based. The high ability subject applied ordered-based reasoning by writing the order of integers. In contrast, the medium and low ability subjetcs used consistently both ordered-based and magnitude-based reasoning. Moreover, the low ability subject implemented an out of context's illustrations in magnitude-based reasoning by drawing pencils and erasers, and the medium ability subject described the context using properly words. This result suggest that teachers should exercises the students to more explore their reasoning behavior in any problem.

INTRODUCTION

Integers are the first numbers that are introduced to elementary students [1]. Learning integers, students have often difficulties, as well as some misconceptions, in modeling integers. When students are asked to compare two integers with negative signs, they are often answered incorrectly, for example $-3 > 2$ [2]. In addition, more than 40 students aged 9–13 years cannot compare two informations which had presented in the table or bar chart [3]. In comparing integers, 3 students of the 5th grade gave differently ways of comparing between five pencils and the number of pencils. While another student made a sequence of numbers 1 through 5 to show that the number five is greater than the number one. There are various ways of students in comparing integers [4]. The way of students' reasoning is very rich and interesting, especially the solving of integers problems [5]. When the first grade students had to order integers and compare their values, there were 71% of students removed automatically negative signs, 23% knew the concept of negative numbers so that they can compare negative numbers with positive numbers, and only 11% can sort and compare numbers consistently [6]. The students' reasoning of the 2nd and 4th grade tend to utilize the developmental reasoning in non-negative integers. Meanwhile when involving negative integers, students are more likely to use reasoning based on order [7]. These results are reinforced the condition that students are often difficult in reasoning especially in solving comparison integers. Hence, it can be concluded that learning performed in schools should not require only the ability in the application of concepts, but also the application of concepts can be applied in various situations, and the ability of students' reasoning.

Students are reasonable if they are able to present oral, written, drawing, or diagrammatic statements, be able to pose assumption, manipulate, construct evidence, give reasons or evidence to the truth of solutions, draw conclusions, examine validity and arguments, and find patterns or traits to make generalizations. From that above description, reasoning is an important element not only used to solve problems but also used during classroom learning activities. It has long been agreed that reasoning is important in mathematics, that is in learning and its use[8]. So this study focuses on investigating the behavior of primary school students' reasoning in solving integer comparison problem.

Based on the explanation above, we may answer the following questions: (1) How do elementary students reason in integer comparison? (2) What is the student's reasoning behavior of elementary students in solving the given problem of integer comparisons?

METHOD

Participant

This study applied a qualitatively descriptive approach that aimed to investigate the behaviors of elementary school students' reasoning in completing the integer comparisons' problems. One class that consists of 20 students (11 girls and 9 boys) was purposively chosen from the existing three classes, then all students had to perform the mathematical ability and integer comparisons tests. Combining the results and the existing students' protfolio, all students were ranked into three groups which were high, medium and low mathematics abilities respectively. To asses students reasoning behaviors, three volunteer students, with different mathematics abilities, were selected as research subjects. Furthermore, all subjects were then interviewed based on the their answer sheets. The data obtained were analyzed and categorized based on students' reasoning behaviors.

Analysis

The data was obtained from the reasoning test about comparison and a semi-structured interview based on reasoning's test. In this paper, we focus on three students' reasoning behavior to the integer comparison problem. The problem was printed on a sheet and posed to students one at time. First, the students were asked to give justification then give the reason for their justification. After the data were identified their justification, the justification within each way of reasoning were combined to determine how the behavior of reasoning were used. The interviewer posed some questions to know the way of students' reasoning, such as "Why do you agree with that statement? How did you think about that?. Then posed additional question if necessary to know deeply about the behavior of students' reasoning in comparison integer. Each student way was based on their justification. In particular, we present the way of reasoning [6] below.

Ordering-based reasoning

This way of reasoning is based on the sequence of integers in the count or its position on the number line. This method focuses on numbers that lie before or after other numbers when counting, or that are located more to the right, more to the left, or near zero on the number line.

Magitude-based reasoning

Students who use reason-based reasoning tend to compare the magnitude or the cardinality of a number. In accordance with this reasoning, students argue that zero and negative numbers are meaningless, as well as some integers associated with a specific context.

Developmental-based reasoning

In this reasoning, students are not familiar with negative numbers. This indicates that the students are more knowledgeable about the number of counts and their operations. As a result of that, students prefer to remove the minus sign, such as replacing –7 as 7. There is also a sign indicating minus as a reduction and there are also assume that there is no negative number.

TABLE 1. The reasoning behavior

The reasoning behavior categories	Students' activities
Ordering-based reasoning	• Comparing two integers, a negative integer is always less than a positive integer or zero, or a positive integer is always greater than a negative or zero integer. • Comparing two integers, integers close to zero or close to positive integers will be larger. • Comparing two integers, more to right integers are greater than, or more to left integers are less than. • Comparing two integers, the more left integer is greater (assuming a negative integer of the positive integer)
Magitude-based reasoning	• Comparing two integers, negative integers will be more or less negative than other negative numbers. • Compare non-zero integers to zeros, on the ground that zero has no meaning. • Determine that all positive integers are larger than negative integers, since negative integers do not have any magnitudes. • Describes the magnitude of an integer in a context.
Developmental-based reasoning	• Removes the minus sign so that it becomes a natural number. • Treats a minus sign as an operation.

RESULT AND DISCUSSION

The data scores obtained can be categorized as follows: 4 students are in high mathematics ability (score ≥ 80), 7 students are in medium mathematics ability (60 ≤ score < 80), and 9 students are in low mathematics ability (score < 60). Based on the above categorization, one student was volunteerly chosen from each group to assess their reasoning behaviors in solving integers comparison problem.

Reasoning Behaviors of High Mathematics Ability Student (HMAs)

In the first question, the subject HAS was agreed with the given statement, since Arman did many mistakes in reading the words of the debate texts. HAS expressed a sequence of numbers from zero to 15 and included also minus signs in the sequence of numbers. It did mean that HAS was able to locate negative integers in a well-ordered sequence. In the interview, HAS could explain that the sequence of indicated numbers and Arman had the lowest score. This can be seen from the number –15 which was farthest to the right comparing to all participants who also obtained negative scores. HAS also stated that participants who had positive scores were not the lowest because no minus sign on it.

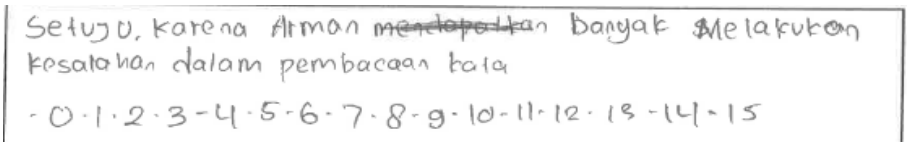

FIGURE 1. High ability subject's answer of the first question

In the second question, HAS responded that the statement was false since the different's score between Bella and Lisa was 8. HAS had arranged the sequence of integers as the previous. However, HAS focused only on a sequence of positive integers. Obviously, HAS appended a mark on the numbers 12 and 20 which were indicated the scores of Bella and Lisa. From the interview, HAS explained that the distance between 12 and 20 based on the integers line which can be observed. In fact, 20 was located more to the right of 8. This indicated that the Lisa score, 20, was greater than Bella, 8.

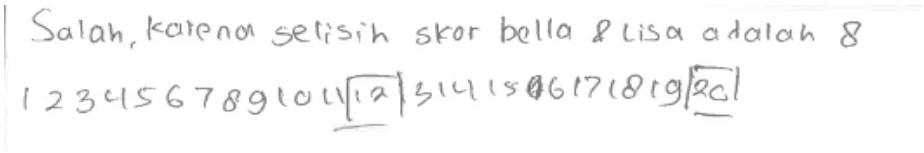

FIGURE 2. High ability subject's answer of the second question.

For the third question, HAS concluded that the given statement was false, since Arman had the lowest score. However, HAS did not write a sequence of numbers as the previous answer. Based on the interview, HAS explained that the third question was related to the first one. Hence, HAS stated that if Arman is the participant with the lowest score, then automatically will preserve the lowest score comparing to the Doni's score.

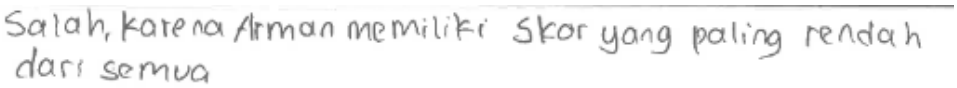

FIGURE 3. High ability subject's answer of the third question.

In the fourth question, the subject stated that the statement was incorrect because the addition was also incorrect. HAS justified that "–7 plus 5 cannot be equal to 12". HAS was unsure of the summation and concluded that the statement was false. In addition, HAS assumed that if one of the participants plus 5 scores then the score of the other students had also be added five score. This showed that HAS did clearly not understand the given question appropriately.

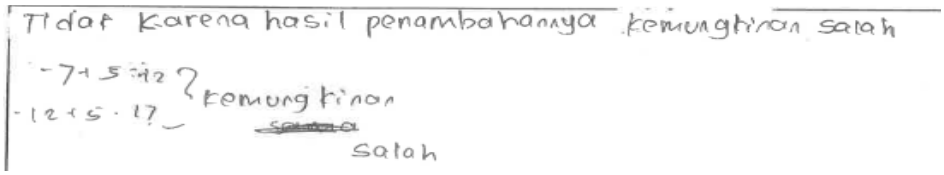

Figure 4. High ability subject's answer of the fourth question.

In the fifth question, HAS stated that the statement was false because the lowest one was the Arman's score. To justify this, HAS repeated to wrote a sequence of integers with a positive integer to the left and a negative integer to the right. Furthermore, HAS said that from the number line, the score obtained by Arman was farther to the right than others. Hence, HAS concluded that only Arman cannot proceed to the next round. In addition, HAS also reinforced the given reason by linking to the previous question that Arman was only the participant with the lowest score. Hence, HAS simply ignored the rules given to the question that a participant with a score below zero cannot continue the next round.

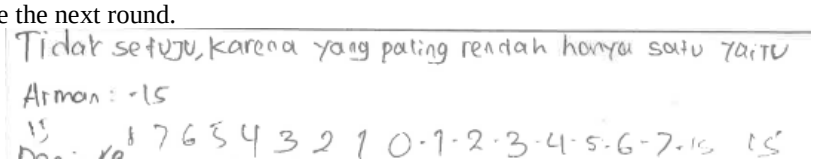

FIGURE 5. High ability student's answer of the fifth question.

Reasoning Behavior of Medium Mathematics Ability Student (MMAs)

In the first question, MAS described that the given statement was correct because Arman mispronounced the speech 15 times. However, MAS did not write any way to clarify the given answer. From the interview, MAS explained that other participants who obtained a positive score would be never mispronounced in the speech. While the participants, who collected a negative score, made a mistake in pronouncing the speech. MAS also explained that there were two participants with a negative score, but when justified the participants having more mistakes must be the participants who had the greatest minimum score. Therefore, MAS concluded that the Arman score was the lowest score.

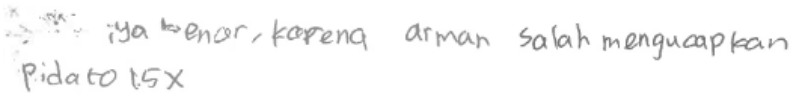

FIGURE 6. Medium ability student's answer of the first question.

In the second question, MAS explained that the given statement was incorrect one because Bella had score 12 and Lisa was 20. In the given answer, MAS did not clarify his reasoning. In the interview, MAS said the Lisa's score was greater than of that Bella. MAS showed that his reasons were as the previous question. The Lisa score is bigger than of that Lisa, because Lisa spoke correctly speechs were 20 times, while Bella spoke the correctly speechs were only 12 times. This emphasized that it was not true that the Bella's score was greater than the Lisa's score.

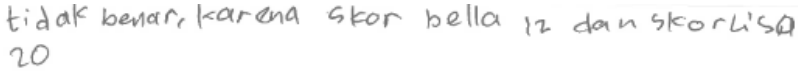

FIGURE 7. Medium ability student's answer of the second question.

In the third question, MAS stated that the given statement was incorrect because Doni mispronounced 7 times and Arman did mistake 15 times. Take into account the first question, MAS explained that the participant who utters with the highest errors was the one with the lowest score. In clarifying the reasoning behaviors, MAS performed the sequence of numbers (by circling the numbers 7, 15, –7 , –15) and stated that "the numbers indicate the most mistakes made by Doni and Arman, and Arman score is bigger than Doni's score, hence Doni performes less incorrect one."

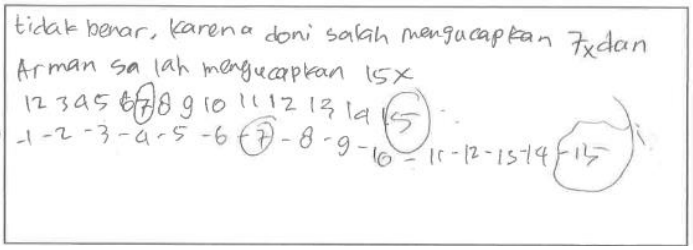

FIGURE 8. Medium ability student's answer of the third question.

In the fourth question, MAS stated that the given statement was incorrect, because Doni mispronounced 7 times but if added 5 then it was unchanged because –7 and –1 were equal to zero, then 5 = 5". Here, MAS assumed that the minus score of Doni was equal to –1, and it refers to zero. So if the score 0 plus 5 then the result was exactly 5. This stressed that the negative number was meaningless and was automatically equal to zero. MAS also said that "*zero is like a strait that separates numbers by minus marks and not. So a negative number can not be added to the positive because it must pass through the strait.* "

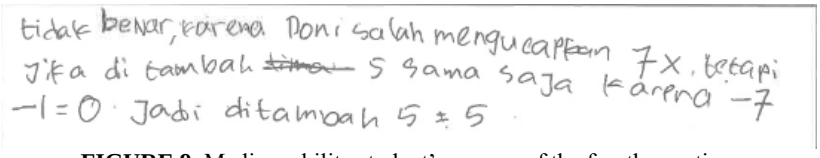

FIGURE 9. Medium ability student's answer of the fourth question.

In the fifth question, MAS stated that the given statement was true because it was below zero. MAS explained that the given requirement was a score below zero which was not able to continue the next round. This was reconnected with the participants' mistake in the speech. MAS performed that there were two participants who mispronounced the speech and obtained a negative value and the negative number were lie below zero.

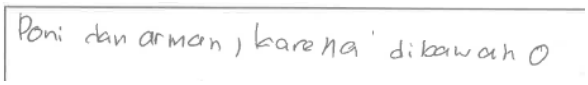

FIGURE 10. Medium ability subject's answer of the fifth question.

Reasoning Behavior of Low Mathematics Ability Student (LMAs)

In the first question, LAS stated that the given statement was not true because there was other participant with lower score, i.e. Doni. LAS described clearly that Arman and Doni were both making mistakes in performing speeches. The minus sign obtained showed that they made a mistake and did not affect the nominal. Then participants who achieved the lowest score was Doni. Doni score was only 7 and Arman 15. It was clear that Doni score was smaller than Arman one. This indicated that Doni obtained the lowest score.

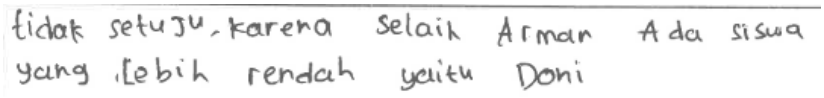

FIGURE 11. Low ability subject's answer of the first question

In the second question, LAS said that the given statement was false because the Lisa's score was higher than the Bella's score. LAS explained that a number that was more numerous must be larger than others. LAS reinforced by describing two illustrations with 20 pencils and 12 rulers. LAS compared two differently objects with different amounts. Objects that had more quantities will be larger than objects with smaler in number.

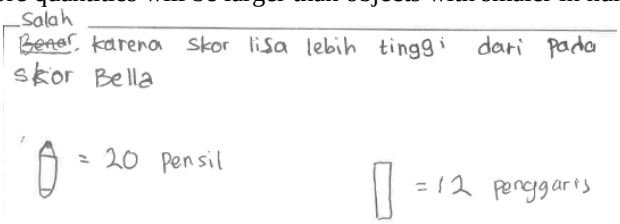

Figure 12. Low ability subject's answer of the second problem

In the third question, LAS stated that the statement was true because the Arman's score was higher while the Doni's score was smaler than the Arman. Similarly to the reason for the first question, LAS ignored the minus sign on a number. The sign only indicated that the participant had made a mistake. Hence, LAS concluded that the Arman's score was higher than the Doni's score.

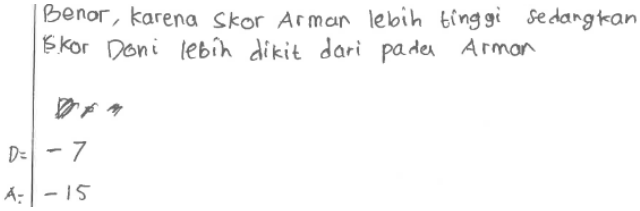

FIGURE 13. Low ability subject's answer of the third question.

In the fourth question, LAS insisted that does not agree with the statement given. The reason was because if Doni achieved the score again then his score did not equal with the Bella's score. LAS did not involve any information to clarify the proposed reason. From the interview LAS explained that Doni had made a mistake at first, and Bella did not make any mistake. This showed that if Doni was given an additional 5 scores then he would remain wrong.

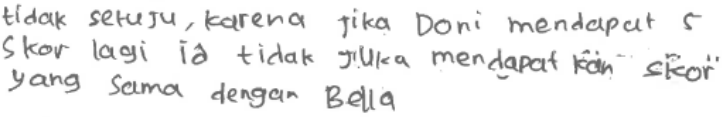

FIGURE 14. Low ability subject's answer of the fourth question.

In the fifth question, LAS was not agree with the given statement because only one participant would be out and did not continue to the next round. LAS also had no other reason to clarify the answer. But in the interview, LAS stated only Doni could not continue to the next round because Doni scored –7. While the score of other participants were greater than –7. LAS repeated to ignore the minus sign.

FIGURE 15. Low ability subject's answer of the fifth question.

Conclusion

This study had demonstrated the behavior of students reasoning in solving the integer problem, especially integer comparison. In solving the integers comparison problems, students used many ways to draw their conclusions. Students presented various ways and reasons in solving the given problem. Student with high mathematical ability often applied ordering-based reasonings to solve the problem. While student with medium and low abilities utilized magnitude-based reasonings that was associated with the given context. however, there was a difference between medium and low ability students, namely students describe pencils and rulers; in and declared the negative number was equal to zero which was as the strait that separated the positive and negative integers. We offer this result as a tool for mathematics teachers to inform students' reasoning about integer comparison more deeply and explore the behavior of students' reasoning in integer problem.

ACKNOWLEDGMENTS

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