5th ICRIEMS Proceedings
Published by Faculty Of Mathematics And Natural Sciences
Yogyakarta State University, ISBN 978-602-74529-3-0

Intertwining Characteristic In Realistic Mathematics Education (RME) In Learning of Linear and Quadratic Equations System

Florensius Widodo Yulianto

Department of Master of Mathematics Education, Sanata Dharma University
Maguwoharjo, Depok, Sleman, Yogyakarta, Indonesia

florensiuswidodo@yahoo.com

Abstract. The aim of this study were to describe intertwining characteristic or knowledge relations in realistic mathematics approach and what are teachers need to do in the mathematics learning process until the intertwining characteristic arise. The type of research used was a design research developed by Gravemeijer and Cobb. The subjects of this research were 33 students of class X of MIPA 1 in one of high schools in Yogyakarta.The data collection was done by observation, student's worksheet and video recording during the learning process and then data analysis technique used is descriptive qualitative. The results of this study showed that the intertwining characteristic in realistic mathematics approach emphasizes a series of learning as a combination of learning series that must be used in problem solving. Things that teachers should do is to create a series of problems that are explored and solved by students so mathematics formal language is well achieved.

INTRODUCTION

A linear and quadratic equation system is a system containing one linear equation and one quadratic equation which may be one straight line and one parabola. Systems of linear and quadratic equations can be solved by graphical method or algebraic method and both methods have intertwining. The linear and quadratic equation system is one of the compulsory materials that should be taught to the students of class X in this 2013 curriculum [1]. Before doing the research, the researcher gave the problem about solving system of linear equation in two variables with two methods namely graphic and algebraic method. As a result, most students have not found a relationship between the graph and algebra methods. This shows the students' understanding is not intact, they see it as something separate. In fact, solving with algebraic and graphical methods has intertwining.

In addition, the researcher's opinion is supported by interviews with teachers, students have difficulty in studying linear and quadratic equations system because students have not understood the prerequisite materials that isfinding graphical and algebraic solutions of two-variable linear equation system; students also do not understand well how to draw lines and parabola; students do not yet understand how to solve a system of equations.

To teach this topic, teachers should not directly use the example but rather give the students a problem. Then in order to occur a meaningful learning process [2], [3], discussion of a topic must be explored by the students because the structure and the concept of mathematics is intertwining [4]. This is to avoid the student's lack of understanding of the concept, so when given other more complex problems, students are able to solve it because the knowledge is student construction. The ideal learning model is student-centered learning rather than teacher-centered [5]So in order to achieve the purpose of learning then the teacher needs to design a meaningful learning on this material. In this case, a suitable learning approach is an approach with RME.

Realistic Mathematics Education (RME) is a way of teaching mathematics that was conceived and developed at the Freudenthal Institute in the Netherlands from about 1970. [5] RME is a teaching approach that stems from the 'real' things for students, emphasizes the skills of 'process of doing mathematics', discusses and collaborates,

argues with classmates so that they can find themselves (' student inventing 'as opposed to' teacher telling ') and in the end using the math to solve the problem both individually and in groups. In this approach the role of the teacher is nothing more than a facilitator, moderator or evaluator while students think, communicate, train the nuances of democracy by respecting the opinions of others [6].

Learning using RME should contain the characteristics of the approach. Treffers [7] mentioned five characteristics of RME, as follows:

1. **Context Use**
   Context is a bridge for students to understand mathematical concepts. Context used as a realisticproblem should be imaginable for students.
2. **The use of mathematical models for Progressive mathematization**
   The term model refers to situation models and mathematical models that are developed by the students themselves. This means that the students develop models in solving problems. At first, themodel is a model of a situation that is familiar to the students. By a process of generalizing andformalizing, the model eventually becomes an entity on its own. It becomes possible that is used as amodel for mathematical reasoning.
3. **The Utilization of Student's Construction**
   Mathematics is not a finished product. Students are given the opportunity to construct knowledgethrough exploration. The results of their findings are the basis for building a mathematical concept.
4. **Interactivity**
   Group activities, the indirect negotiations, and discussions are very important activities in theprocess of knowledge construction. It is an informal method for students in acquiring formal knowledge.
5. **Intertwining**
   Single mathematical concepts are not enough to apply the concepts and problem solving.Intertwining is needed in solving mathematical problems.

Outline of the learning process using RME approach designed by researchers is to do some learning activities as follows:

Activity 1: in group discussion, student solve a linear and quadratic equation system graphically:

$$y = x^2 - 3x + 2;$$
$$y = x - 1.$$

From the problem above, students are expected to be able to draw a parabola and a straight line on the same coordinate plane and then identify the points of intersection between the line and the parabola.

Students are expected to be able to write down the steps in drawing lines and parabola. The researcher chooses the question above as a context because the questions above can be imagined by students. The answer to the question above has two intersection points. By using context, students can be actively involved in exploring problems, and can also develop students' motivation and interest in learning mathematics. In the learning process there is an interaction between the teacher and students and there is also interaction between students and students in constructing their knowledge. They discuss each other, propose arguments in solving the above problems. If students encounter student difficulties, then they can ask the teacher so that there is interaction between students and the teacher. During the discussion the teacher went around and monitored each group. From the contextual problems given, students are given the freedom of how they solve the problems so that models, schemes they develop themselves.

Activity 2: in group, student solve a problem in activity 1 algebraically.

From the problem above, students are expected to be able to solve the system and find the solutions.

From the problem above, students are expected to be able to use the knowledge they have acquired when studying the Two-Variable Linear Equation and Three Variable Systems. Therefore, students are expected to be able to use both elimination and substitution methods. The above question has two members of the solution set.

Activity 3: student make a presentations and draw conclusions.

The goal is to let students understand and find the connection between completion with graphical and algebraic methods (intertwining). Activity 3 is designed for the presentation of several groups. During the presentation, the teacher as a presentation guide and also the discussion. One group made alternate presentations and the other students noticed. In the presentation also appeared question and answer activities. After the presentation activity is over, the teacher guides the students to draw conclusions. Students are expected to understand the solution of systems of linear and quadratic equations with algebraic methods interrelated with graphical methods.

From the experiences and interviews as above, the researcher intends to conduct research that can lead to intertwining characteristics in the learning process experienced by the students. Then this study aims to describe what teachers need to do in the learning process of mathematics until the intertwining characteristics in RME arise.

RESEARCH METHOD

The subjects of this research were 33 students of class X of MIPA 1 in one of high schools in Yogyakarta. The data collection was done by observation, student's worksheet and video recording during the learning process and then data analysis technique used is descriptive qualitative. The type of research used was a design research developed by Gravemeijer and Cobb. According Gravemeijer and Cobb [9],[10] there are three phases in the design research, namely: (1) The first phase: preparation of trial design, (2) Second phase: trial design. (3) The third phase: a retrospective analysis. In *the first phase*, the researcher determines the learning objectives and builds the instructional design.

Instructional Activities

Objective: student can solve a linear and quadratic equation system graphically and algebraically

Activities of Teacher and Students:

1. Activities to construct social norms in class
   If there are students who want to ask questions, express opinions, or answer questions both from the teacher and from other students, the student must raise his hand first and then the teacher invites the student to start speaking. Whereas other students must pay serious attention.
   When group or class discussions occur, all students have the right to express their ideas.
2. Review of Learning material
   The teacher gives apperception to students in the form of recalling the material of the Two Variable Linear Equation System and the Three Variable Linear Equation System and how to solve the system
3. Exploring and solving problems with group discussions
   The teacher gives group assignments: solve this system!
   $$\begin{cases} y = x^2 - 3x + 2 \\ y = x - 1 \end{cases}$$
4. Class discussion
   a. The teacher gives students the opportunity to present their answers in front of the class.
   b. Students can ask or respond to their friends' answers
   c. The teacher guides students in the presentation.
5. After the class discussion ends, the teacher and students make conclusions.
6. The teacher motivates students and closes learning activities

In *the second phase*, researchers tested the learning design in two meetings. In *the third phase*, the researchers conducted a retrospective analysis of the results achieved by the students in the trial design phase. In accordance with the formulation of research problems, the retrospective analysis conducted by researchers, researchers focused on the intertwining characteristics in trial design.

RESULT AND DISCUSSION

The intertwining characteristics in the RME approach emphasize that the learning sequences cannot be treated as being completely separate. Conversely, a connection of the learning sequences should be utilized in problem solving [4]. The things teachers need to do in order for students to form connections from the series of learning are as follows: *First*, the teacher creates a set of problems or phenomena that can be explored and completed by the student so that students can gradually arrive at the mathematical formal knowledge desired to be achieved by the student. *Second*, teachers develop a two-way interaction pattern both in the interaction between teachers and students as well as among students [4].

In order for the problem or phenomenon to be explored and resolved by the student, the teacher needs to (a) know the knowledge already possessed by the student, (b) provide an opportunity for the student to solve the problem in the way it is understood, (c) provide assistance, be able to use mathematics teaching aids, as well as with questions, so that students can develop the knowledge they have to solve the problem, and (d) to know the

variations in the learning path that students can make through the set of problems or phenomena given by the teacher [4].

Based on the results of the development of learning, the explanation for each phase as follows:

Phase 1: Define learning objectives and build learning design.

Students can determine the solution of linear and quadratic equation system by graphical method; students can determine the solution of linear and quadratic equation system by algebraic methods; students can build a knowledge between solving systems of linear and quadratic equations by graphical methods and algebraic methods.

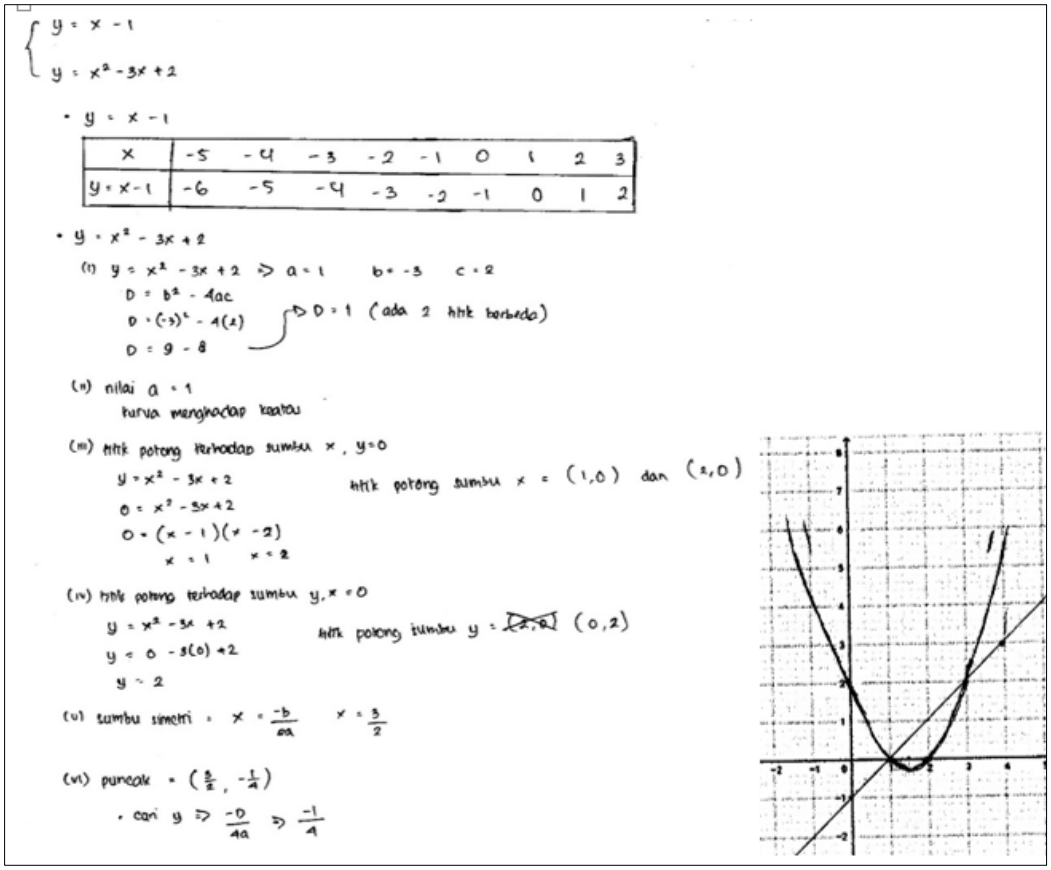
Phase 2: Test the instructional design

The learning design is built by researchers who will try the design on 26, 30 October 2017. After the learning process ends, the researcher reflects on the learning process that just happened in the classroom, and refines the design to be tested in the next meeting based on the reflection result. The role of teachers in learning activities is as a facilitator and motivator so that students can construct their knowledge.

Phase 3: Retrospective analysis / research results and discussion

The following shows the results obtained from trial phase showing the process of intertwining characteristic appearance in RME.

Figure 1. Group IX's answer in solving linear and quadratic equation system graphically.



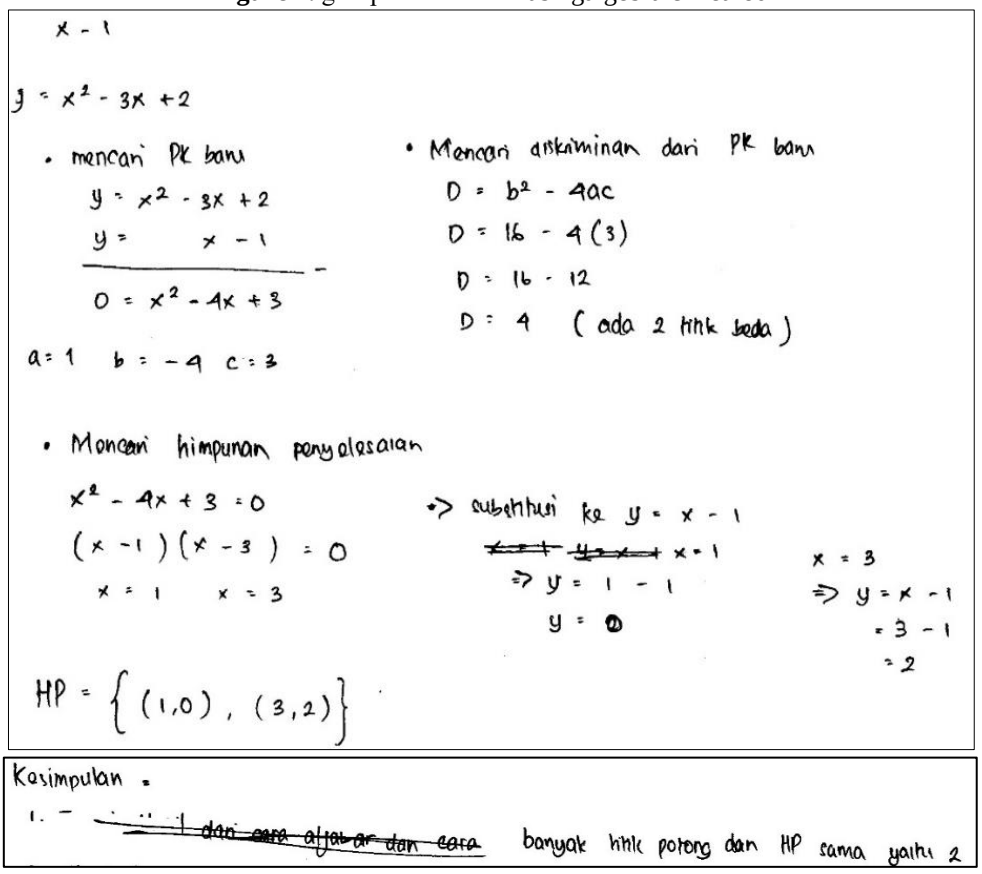
From **Figure 1**, shows that the student step in drawing the line is to determine the points through which the line $y = x - 1$, then students draw a parabola by determining the intersection of the coordinate axes, determine the symmetry axis and the peak point. Then describe it in the field of Cartesian coordinates.

Interaction by teachers and students where teachers try to stimulate students in solving the linear and quadratic equation system with the following graph method:

Teacher	:	*Please write down the completion steps. The first is how your drawing process. [students start discussing] [teacher goes around seeing student activity]*
Student	:	*can we use a variety of ways?*
Teacher	:	*Yes. Write down the process! Okay. [student just nods] [over time, teacher sees group IX and asks Alv]*
Teacher	:	*Let me see. How does your line draw?*
Alv	:	*substitute the value of x to this function, sir*
Teacher	:	*Okay. How many x points do you choose?*
Alv	:	*Nine, sir.*
Teacher	:	*Okay right. But can only 2 points be selected? please check it too. Then how do you draw a parabola?*
Alv	:	*Determine the intersection of the x and y axis. Then determine the symmetry axis and the peak. use the formula.*
Teacher	:	*Okay, if you have finished please draw. [the teacher returns to visit another group]*

At the next meeting the teacher asks students to do the same problem with the problem on activity 1, but how to solve it by algebraic method.

Figure 2. group IX's answer usingalgebraic method



Teacher	:	*How are your steps in solving systems of linear and quadratic equations by algebraic method?*
Student	:	*We eliminate y variable and then rewrite equation in standard form. Solve the quadratic equation and then substitute x value into linear equation to get y value. We have solution that are (1,0) and (3,2).*
Teacher	:	*Oke. Good.*
Teacher	:	*what do you think if we use substitution method?*
Student	:	*it will give the same answer*
Teacher	:	*Okay. Please prove it!*

In activity 3, the teacher asked several groups for presentation in front of the class, among them group IX. Group IX describes the process of completing the linear and quadratic equation system by the graphical method **(see figure 1)** beginning with completing table points for drawing the lines. While in drawing a parabola, group IX first determines the parabolic intersection point against the x-axis where (y = 0) and the y-axis where (x = 0). Then determine the symmetry axis and the vertex of the parabola. Whereas with the algebraic method, they use elimination method.
Then the interaction of teachers and students where teachers try to explore students' ideas about finding the link between the solution of the graphical method with algebraic method.

Teacher	:	*look at your answers! What is the point of intersection between the line and the parabola?*
Student	:	*There are two, sir.*
Teacher	:	*Okay. please mention it!*
Student	:	*(3,2) and (1,0)*
Teacher	:	*What do you think about the solution of algebraic method?*
Student	:	*give the same answer (3,2) and (1,0).*
Teacher	:	*So, can you conclude about this?*
Student	:	*the intersection between the line and the parabola equals the algebraic solution*
Teacher	:	*It means?*
Student	:	*The intersection points is a solution because it gives the same answer when we solve it algebraically.*
Teacher	:	*Good. Then, is the number of intersection points on the graph equal to the number of members of the solution set?*
Student	:	*Yes. It is same.*
Teacher	:	*Okay. Do you get it?*
Student	:	*Yes...*
Teacher	:	*Okay. Please write a conlusion about our topic today.*

During the presentation, the teacher serves as a presentation guide. One group made alternate presentations and the other students noticed. In the presentation also appeared question and answer activities. But there is important questions that students do not ask, so the teacher asks:

Teacher	:	*"how to know that the solution we found is right?"*
Student	:	*From our answer (3,2) and (1,0), when substituted into the equations* $y = x - 1$ *or* $y = x^2 - 3x + 2$*, satisfies both equations at the* same *time.*
Teacher	:	*Okay. But what is the relationship with your graphical method?*
Student	:	*(3,2) and (1,0) marks the intersection of the line and parabola and for systems of equations, "solutions" are "intersections". We can confirm the solution by substitute it into the system of equations, and confirming that the solution works in each equation. It was analog with solving linear system in two variable on previous topic.*
Teacher	:	*Okay. Good job.*

To further convince the students, the teacher gives the linear and quadratic equation system problem which has one realsolution ($y = x^2 - x - 2; y = x - 3$) andhas no real solution ($y = x^2 - 4x + 3; y = -2x + 1$).

From all the results of the discussion and the learning sequence, students have understood the problem and can solve it well. One of the RME characteristics that emerges here is "the intertwining of various learning strands or units". Students have realized that the point of intersection between line and parabola is also a solution of linear and quadratic equation system. This proves that the students can find the structure of the material systematically that has intertwining andteachers have facilitated students in meaningful mathematics learning. According to Kusuma [11], the meaningful learning was the basic for making mathematical connection. Mathematic connection is the most important part needed in every level of education. The ability of a mathematic connection shown from the student in: (1) recognizing the equivalent representation of the same concept; (2) recognizing the relationship of a mathematical procedure to a representation of equivalent representation; (3) using and assessing the interrelationships between topics and interrelationships outside mathematic; and (4) Using mathematic in everyday.

CONCLUSION

Based on the results of research and discussion that has been described, then it can be concluded that the use of RME approach on topic linear and quadratic equation system has an important role. The five characteristics of RME will deliver the students to a meaningful learning process, more specifically on the intertwining characteristics that have provided benefits according to Hongki[4] namely: (1) making students' knowledge intact, (2) making the mindset of students not mechanistic, (3) making the students not dependent on procedures or formulas to solve a problem, 4) improve student creativity. Researchers saw enthusiasm arise from students. Students feel helpful because of the lessons learned through group discussion. In addition, students are also given the opportunity to learn independently.The teacher in delivering the material in the learning of mathematics is necessaryusing this approach so that students are more able to construct knowledge well.

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