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The Representations of Mathematics Education Students In Solving Algebra Problems

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Abstract. The purpose of this paper is to explore and describe the representations of mathematics education students in solving algebra problems. The data were collected through written answers of all subjects, 44 first year students of mathematics education in University of Muhammadiyah Malang, East Java, Indonesia to the assigned three tasks, before being descriptively and qualitatively analyzed. The results of the analysis showed that the students' representations in solving algebra problems are visual (figure/graph), verbal, and symbolic representations. Furthermore, the students have done more than one representation in solving algebra problems, three types of representation.

Key words: *Representation, algebra problems, problems solving*.

INTRODUCTION

Representation constitutes a pivotal part in mathematics, in a way that it facilitates students' better comprehension. In addition, it is more effective to learn mathematics by means of representation, as students can express their other mathematical competences through representation [13]. Connecting and stating mathematical ideas are requiring various representations, to name: symbolic, graph, table, or chart. In an effort to communicate those heaps of abstract ideas, it is crucial to comprehend what are to present. Representation is one of constructions that can be expressed in various forms and ways. For instance, one word can represent single entity; number can represent measurement units of an object namely length, weight, etc [7, 24].

Mathematical representation constitutes a mathematical ability to express mathematical ideas in the forms of problems, statements, definitions, etc. in a number of varying ways. It is than necessary that students' mathematical representation be continuously developed. Representation falls within two categories: internal and external representations. Internal representation, by its nature, can only be stated to one particular individual, as it is abstract. Behaving differently, external representation covers up written, spoken, symbolic, figure, as well as graph [7, 8, 20]. Friedlander and Tabach [2] assert that there are four typologies of representation, namely: verbal, numerical, graphic, and algebra representations.

Departing from previous studies, students' representation remains in low level. Most of students fail to write or do verbal representation. Accordingly, it is of urgency that students' representation ability be improved. It will enable students to solve mathematical problems [19]. The ability to solve mathematical problems plays an important role, as problem solving is inseparable from mathematics. It is the heart of mathematics [5, 12]. Problem solving ability will be beneficial for every individual, including any individuals studying other fields/disciplines, to solve mathematical problems systemically [16].

Students are required to have proper ways to solve any mathematical problems. Provided that students are used to solving mathematical problems, they can then find it easy to solve any problems from any other fields of studies as well as in daily lives. Therefore, the importance of mathematical problem solving is to be emphasized. In learning mathematics, students are to be equipped with representation and problem solving competences. However, the fact shows contradictory concern. It is far from the expectation. By that it means, teachers have crucial role in sharpening students' representation and problem solving skill [6, 19].

Students' mathematical competences will be more optimum if they are capable of presenting mathematical concepts by means of various representations. According to NCTM [13] representation is a

sort of idea expressed by students to solve problems. Students differ in their ways of thinking, and thus there comes up a number of representations with different reasons. Giving students freedom to solve any problem can stimulate their way of thinking to represent their own ideas. In other words, representation facilitates students to solve problems.

Santulli [17] and Syafri [20] further elaborate that solving mathematical problem requires an ability to design a model and interpret solution by means of representation indicators. Representation is also enabling students to communicate their ideas to their teachers or peers. The success of students in solving mathematical problems depends on students' ability to express various representations to solve similar problems.

Algebra is one of topics in mathematics offered from elementary school to university/higher education institution. It is common to use algebra in daily bases, especially for things related to input, process, and output. It is not surprising that teachers and policy makers declare that algebra is the entrance gate to success in academic and professional lives.

It is necessary to master algebra. Mathematics requires representation. Due to an abstract nature of mathematics, users can only be accessing mathematical ideas merely by means of the representations of those ideas [25]. It is then crucial to investigate students' representation in solving mathematical problems, particularly for algebra materials.

METHOD

Subjects

The subjects of this current study were 44 students of first semester majoring in mathematics education, University of Muhammadiyah Malang, East Java, Indonesia academic year 2017/2018, especially those taking elementary algebra. There were 6 male and 38 female students under investigation.

Instrument

The instrument used in this current study was test item. There are 3 items/problems, two items for proving and one item for finding [15] adopted from Alders [1]. The solutions to those items enable the students to utilize various representations

Problem I (P1). Investigate whether the line $y = 2x - 1$; $y = x - 1$; $y = 4x - 5$ *are moving through one point?*

Problem II (P2). Investigate whether point $(-2,0)$, $(1,2)$, *and* $(3,6)$ *are located in one line?*

Problem III (P3). Calculate the side lengths of right triangle that are consecutive integers.

Data Collection

The data were focused on the subjects' written responses and answers towards the given problem items. The data were collected after discussing linear function topic in class. To solve the three given problems, the allocated time was of maximum 45 minutes.

Data Analysis

The analysis of the data was conducted by means of coding every student's answer, as done by [3,4]. To achieve data credibility, the coding was done by two personnel. The second coder codified 80% of the data. The agreement between two coders was that 95% of each problem has the same coding result. The coding was focusing on the correctness of answer and representations used by the students. The data were further analyzed by means of descriptive statistic.

RESULTS AND DISCUSSION

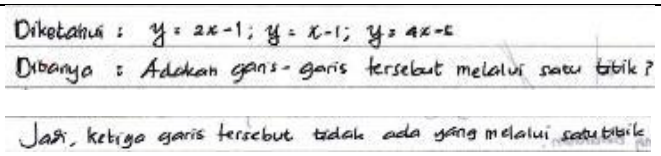
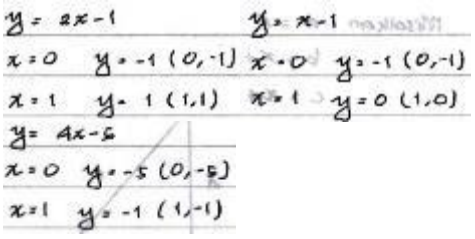
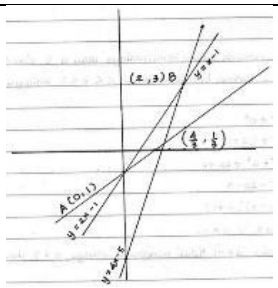
Based on the written responses of the students on each problem, the coding was mainly focused on representations used by the subjects in solving algebra problems. Correct answer was codified as R (Right) and wrong answer was codified as W (Wrong). The students' representations in solving algebra problems were classified into one of these categories: verbal representation (**VBR**), symbolic representation (**SR**), and visual representation (**VSR**). The use of those three representations is in line with the research done by

[23] following the steps proposed by [9]. The students' responses were categorized as **VBR** when the students did the representations by means of wordings or phrases to explain and describe mathematical ideas, both formally and informally. The responses were categorized as **SR** when the students focused more on the use of numbers, variables, tables, and other mathematical symbols. The students' responses were categorized as **VSR** when there were graphs, number lines, diagrams, or any other mathematical figures.

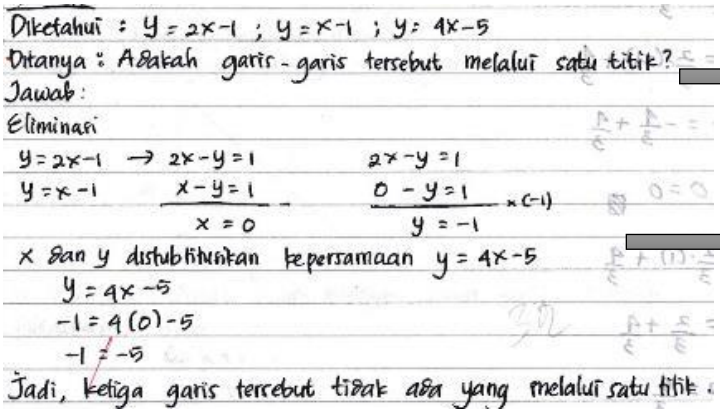
The Students' Mathematical Solution to Problem I (P1)

The students' representations to solve **P1** are shown in Table 1. Based on the analysis of the students' representations for **P1**, in Table 1, it has been shown that the subjects made use of verbal representation in the stage of understanding the problem [15] and drew a conclusion based on the solution. This is underpinning [14] in a way that very few students are making use of symbolic representation upon understanding a problem. Mostly symbolic representations are used when planning to solve a problem; however, most of students fail to execute this sort of representation [15, 21]. Visual representation has been mostly done when it comes to students to recheck their problem solution [15].

Table 1. The Students' Representation Types to Solve Problem I (**P1**)

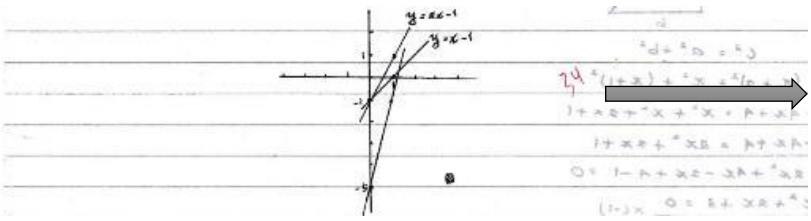
Student's Representations	Answer	Frequency	Right (R)	Wrong (W)
VBR		57%	57%	0%
SR		100%	80%	20%
VSR		73%	63%	10%

There have been some subjects who have done three representations at once in solving the algebra problems. There has been 45% of the subjects who have made use of the three representations to solve **P1**. Out of 45%, 37% of the subjects did the representations correctly and 8% of whom did it wrongly. The following illustrates the ways how the subjects did the three representations to solve **P1**.



Verbal Representation

Symbolic Representation

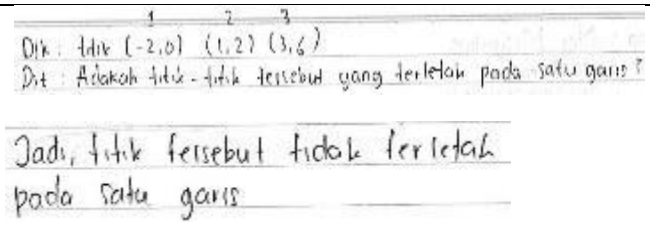
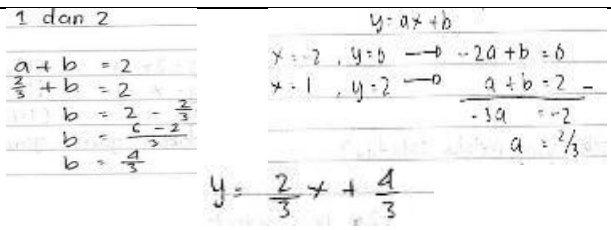


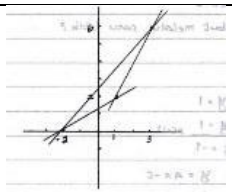
Visual Representation

The Students' Mathematical Solution to Problem II (P2)

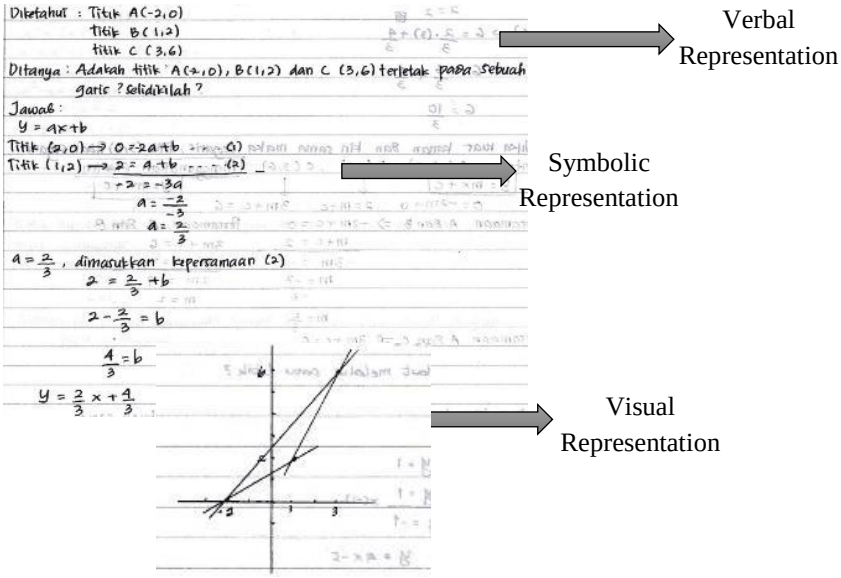
The students' representations to solve **P2** are shown in Table 2. Based on the analysis of the students' representations for **P2**, in Table 2, it has been shown that the subjects made use verbal representation in the stage of understanding the problem [15] and drew a conclusion based on the solution. This is underpinning [14] in a way that very few students are making use of symbolic representation upon understanding a problem. Mostly symbolic representations are used when planning to solve a problem; however, most of students fail to execute this sort of representation [15, 21]. Visual representation has been mostly done when it comes to students to recheck their problem solution [15]. It is necessary that students' visual representation be trained continuously, as its use requires specific skill [18]. This has shown that each representation owns its regularity [11].

Table 2. The Students' Representation Types to Solve Problem 1I (**P2**)

Student's Representations	Answer	Frequency	Right (R)	Wrong (W)
VBR		57%	57%	0%
SR		100%	90%	10%

VSR		11%	8%	3%

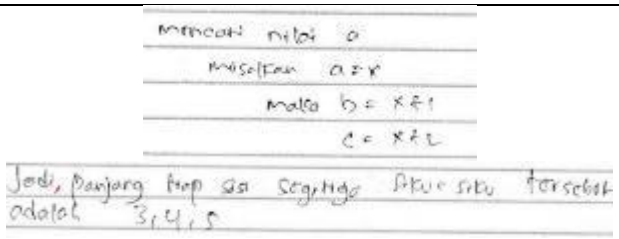
There have been some subjects who have done three representations at once in solving the algebra problems. There has been only 11% of the subjects who have made use of the three representations to solve **P2**. Out of 11%, 7% of the subjects did the representations correctly and 4% of whom did it wrongly. The following illustrates the ways how the subjects did the three representations to solve **P2**.

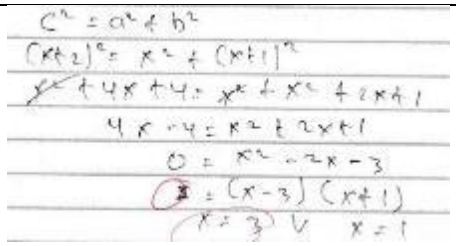
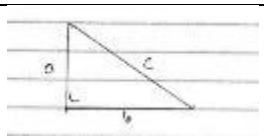


The Students' Mathematical Solution to Problem III (P3)

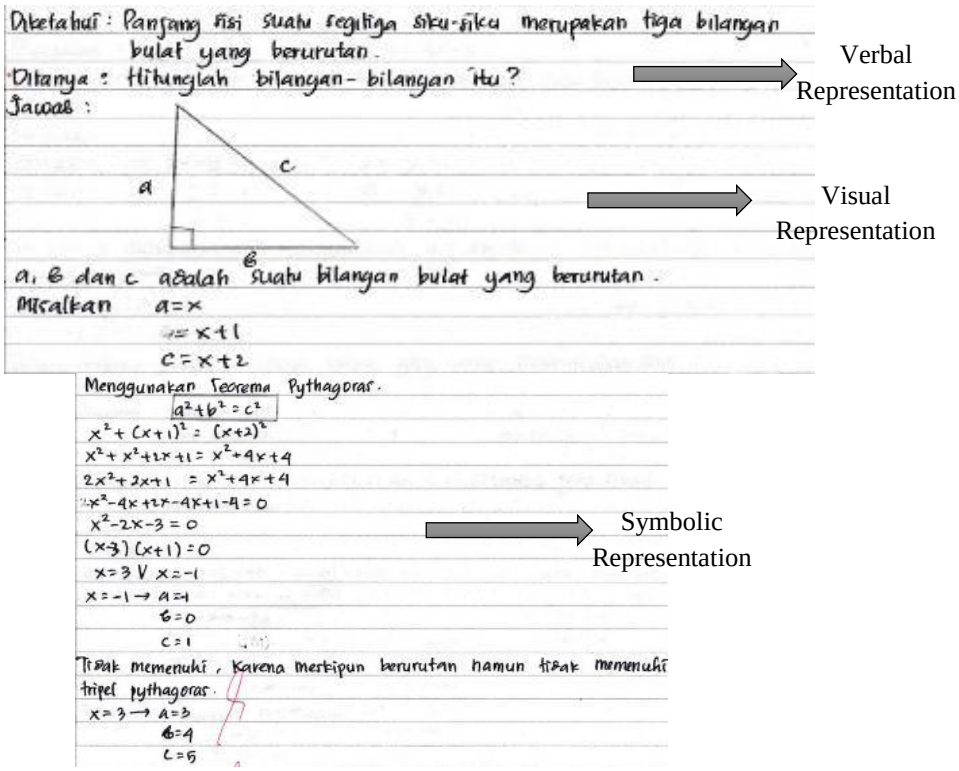
The students' representations to solve **P3** are shown in Table 3. Based on the analysis of the students' representations for **P3**, in Table 3, it has been shown that the subjects made use verbal and visual representations in understanding the problem [15]. Some subjects still found it difficult to utilize variables and very few of them used symbolic representation in solving algebra problems [10,14]. Most of the subjects made use of symbolic representation when planning to solve a problem [15, 21] The data have shown that visual representation was mostly used by the students [22].

Table 3. The Students' Representation Types to Solve Problem 1II (**P3**)

Student's Representations	Answer	Frequency	Right (R)	Wrong (W)
VBR		68%	60%	8%

SR		77%	65%	12%
VSR		77%	70%	7%

There have been some subjects who have done three representations at once in solving the algebra problems. The following illustrates the ways how the subjects did the three representations to solve **P3**.



There has been 55% of the subjects who have made use of the three representations to solve **P3**. Out of 55%, 50% of the subjects did the representations correctly and 5% of whom did it wrongly when representing their answers.

CONCLUDING REMARK

Problem solving constitutes one of mathematical abilities that determines students' competence in mathematics. Algebra is a branch of mathematics with the most and broad coverages. One of the ways to detect students' ability in making representation is through assigning them to solve algebra problems. There have been three ways to solve algebra problems, namely: verbal, symbolic, and visual representations. Verbal representation is done when students want to understand a problem and draw a conclusion especially for making proof. Symbolic representation is generally to solve problem on finding out. Visual representation is done when students want to understand and recheck a problem on finding. Students are mostly using symbolic representation compared to those of verbal and visual representations.

The types of problems used in this current research were finding and proving [15] that are not contextual (words/stories). This research suggests that students' mathematical representations in solving algebra problems that are more contextual be further investigated.

REFERENCES

1. Alders.C.J. (1991). *Ilmu Aljabar untuk SMTA Jilid 2 (Algebra for Senior High School, 2nd Edition)*. PT Pradnya Paramita, Jakarta
2. Friedlander, Alex & Tabach, Michal. (2001). *Promoting Multiple Representations in Algebra*, in Albert A. Cuoco and Frances R. Curcio, *The Roles of Representation in School Mathematics*. Reston, Va: National Council of Teachers of Mathematics.
3. Baiduri, (2015a). Mathematics Education Students' Understanding of Equal Sign and Equivalent Equation, *Asian Social Science*, **11(25)**, pp. 15 – 24. doi:10.5539/ass.v11n25p15.
4. Baiduri, (2015b.) Mathematics Education Students' Strategies in Solving Equations. *Proceedings of The 7th SEAMS-UGM Conference 2015*" , pp. 28 - 33.
5. Branca, N.A. (1980). "Problem Solving as a Goal, Process and Basic Skill", dalam *Problem Solving in School Mathematics*. Reston, VA: NCTM.
6. Effendi, L. A. (2012). Pembelajaran Matematika dengan Metode Penemuan Terbimbing untuk Meningkatkan Kemampuan Representasi dan Pemecahan Masalah Matematis Siswa SMP (Mathematics Instructional Activities by Means of Guided Discovery Method to Improve Junior High School Students' Mathematical Representation and Problem Solving). *Jurnal Penelitian Pendidikan, **13(2)**,* 1-10.
7. Goldin, A.G. (2002). Representation in Mathematical Learning and Problem Solving. in English, L.D (Ed) Handbook of International Research in Mathematics Education (pp. 197-21). Mahwah NJ: Laurence Erlbaum.
8. Kartini (2009). Peranan Representasi dalam Pembelajaran Matematika (The Role of Representation in Mathematics). *Proceeding in Nasional Seminar of Mathematics and Mathematics Education in FMIPA UNRI*.
9. Krulik, S. and Rudnick, J.A. (1995). *The New Sourcebook for Teaching Reasoning and Problem Solving in Elementary School*. United States of America: Allyn & Bacon.
10. Manibuy, R., et al. (2014). Analisis Kesalahan Siswa dalam Menyelesaikan Soal Persamaan Kuadrat berdasarkan Taksonomi SOLO pada Siswa Kelas X SMA Negeri 1 Plus di Kabupaten Nabire – Papua (An Analysis on Students' Errors in Solving Quadratic Equation Problems based on SOLO Taxonomy in Grade X of SMA Negeri 1 Plus di Nabire Regency – Papua. *Jurnal Elektronik Pembelajaran Matematika*. **2(9)**:933-945.
11. Miller, F. K. (2000). Representational tools and conceptual change: The young scientist's tool kit. *Journal of Applied Developmental Psychology*, **21(1)**, 21-25.
12. National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
13. National Council of Teachers of Mathematics. (2014). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
14. Neria, D. & Amit, M. (2004). Students Preference of Non-Algebraic Representations in Mathematical Communication. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematical Education*, 2004. Vol. 3 pp 409 – 416.
15. Polya, G. (1973). *How to Solve it.* 2nd Ed. Princeton University Press.
16. Ruseffendi, E.T. (2006). Pengantar kepada Membantu Guru Mengembangkan Kompetensinya dalam Pengajaran Matematika Untuk Meningkatkan CBSA, edisi revisi (Introductory for Teachers in Improving their Competence in Teaching Mathematics to Enhance Active Learning, revised edition). Bandung: Tarsito.
17. Santulli, Tom. (2009). Representation from The Real Word. *Mathematics Teaching in the Middle School*, Vol.14, No.8. www.jstor.org.
18. Stylianou, D. (2001). *On the reluctance to visualize in mathematics: Is the picture changing?* In M. van den Heuvel-Panhuizen (Ed.), Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education (Vol. 4, pp.225-232). Utrecht, The Netherlands: Freudenthal Institute.
19. Sulastri, Marwan, & Duskri, M. (2013). Kemampuan Representasi Matematis Siswa SMP Melalui Pendekatan Pendidikan Matematika Realistik (Mathematical Representation Ability of Junior High School Students through Realistic Instructional Activities). *Beta*, **10(1)**, 51-69.

20. Syafri, S. (2017). Kemampuan Representasi Matematis dan Kemampuan Pembuktian Matematika (Mathematical Representation and Proving Abilities). *Jurnal Edumath*, ***3(1)***, *49-55*.
21. Team of Consortium for Policy Research in Education. (2012). *Symbolic Representation: Student Misconceptions and Strategies for Teaching*. Online. http://www.epcae.org/uploads/documents/Symbolic%20Representation_Sept%202.pdf. Retrieved on January 12, 2018.
22. Van Garderen, D. (2006). Spatial Visualization, Visual Imagery, and Mathematical Problem Solving of Students with Varying Abilities. *Journal of Learning Disabilities*, **39(6)**, 96-506.
23. Tyas, H.W., Sujadi. I, & Riyadi. (2016). Representasi Matematis Siswa dalam Menyelesaikan Masalah Matematika Pada Materi Aritmatika Sosial dan Perbandingan Ditinjau dari Gaya Kognitif Siswa Kelas VII SMP Negeri 15 Surakarta Tahun Ajaran 2014/2015 (Students' Mathematical Representation in Solving Mathematical Problems on Social Arithmetic and Comparison from the Perspective of Cognitive Style of 7th Graders in SMP Negeri 15 Surakarta Academic Year 2014/2015). *Jurnal Elektronik Pembelajaran Matematika*, ***4*(8)**, 781–792. Retrieved from http://jurnal.fkip.uns.ac.id
24. Walle, John A Van de (2010). *Elementary and Middle School Mathematics: Teaching Developmentally Seventh Edition*. Boston: Pearson.
25. Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.