

# A Two-Tier Diagnostic Test Instrument on Calculus Material: What, Why, and How?

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**Abstract.** Students' difficulties in learning mathematics have become one of the interesting things to reveal. The difficulties can be found from some errors made by the students in the process of problem solving. This issue becomes important to note because one of its effects is the learning outcomes will not be maximal and if the errors in solving math problems are not corrected then the student will make the same errors at the same math problems. One tool for analyzing student errors is a diagnostic test. This article aims at describing the importance of a two-tier diagnostic test and how to develop it. So that it can be used as a tool to diagnose students' difficulties on calculus material based on errors made by the students when completing items on diagnostic test instruments. The description is conducted based on a review of 40 literatures related to diagnostic tests and two-tier diagnostic tests as well as relating them to students' error on calculus material. The meaning of a two-tier diagnostic test on calculus material as well as why and how to develop are also discussed, including the strength and weakness of two-tier diagnostic test.

## INTRODUCTION

Mathematics is the branch of science that's studied almost on every secondary Education, not only in Indonesia but also studied in almost all corners of the world. It's because math is the Foundation of the development of science and technology, so that math needs to be studied, understood, and mastered it well. But in fact not all students are able to understand and master the math well. This can be seen from the difficulties still experienced by students while studying or while completing a math problem.

Based on the study of literature there are some researches on mathematical learning difficulties of students. Based on the Research conducted by [1] that most students have difficulty is in the stage of transformation (the inability of transforming the problem into context-based mathematical models) and stages of comprehension (inability to understand the meaning of the problem). Based on the description before then can be obtained information that students still have difficulties in solving math so students make many mistakes in the process of mathematical problem solving.

the difficulties experienced by students in understanding mathematics is a lack of understanding of the problem posed, the lack of knowledge to do strategy in solve a problem, the inability to translate the problem into a mathematical form, and the inability of students in using the correct math [2]. Furthermore, students are required to find a stationer from point of the curve  $y = x^3 - 3x^2 + 4$ , that is the point on the graph where the gradient is zero [3]. Students managed to find the gradient of a function or its derivatives, i.e  $y' = 3x^2 - 6x$ , but in finding a value  $x$  to the point there are 24 stationer from 110 students in normal use. Of the 24 students 12 students wrong in giving factor, they divide  $3x^2 - 6x = 0$  with  $x$  so that it becomes  $3x - 6 = 0$  so they only get one value  $x = 2$ . Then 6 other students when giving the wrong factors, they give the factor  $3x^2 - 6x = 0$  into  $3x(x - 6) = 0$ . Based on these studies many students have difficulties in learning mathematics.

One of the mathematical topics that's considered hard is Calculus topic. Calculus learned at the school includes a limit of a function, derivative, and integral. Calculus is one of the important areas in mathematics, and this is the underlying argument to introduce it to the non-specialist [3]. But the students continued to experience problems and difficulties in study [3,4,5]. So that it becomes one of particular concern for teachers. Difficulties in learning mathematics can be seen from the mistakes done while completing math problems. In addition to the challenges that student teachers regarding the error it's more important or more fundamental's how to analyze and identify

the errors experienced by students[6,7,8,9,10,11]. Errors experienced by students is a tricky thing, because the cause of the error can be from a variety of factors, could be from students themselves, teachers, the curriculum, the topic, and etc. Or a combination of these different factors[12]. Hence the need for tools or instruments that can be used to analyze the errors experienced by students, the instrument can be a diagnostic test.

The diagnostic test is a test used to find out students' weaknesses so that based on these weaknesses may be made granting the right treatment. Diagnostic tests have two main functions, i.e., identify problems or difficulties experienced by students and planned a follow-up in the form of appropriate resolution efforts problems or difficulties that have been identified [13]. One form of the diagnostic test is a two-tiered multiple choice or two-tier diagnostic test. Two-tier multiple choice is a multiple choice test that has two stages (tier) option. The first stage is the multiple choice questions with five choices of answers while in the second phase contains questions regarding the reason the selection of an answer in the first stage with a few choice reasons[14]. The advantages of two-tier multiple choice if compared to multiple choice tests in the form of conventional or ordinary one is reducing the errors in measurement. Use the multiple choice tests conventional or plain consisting of five students then answer choices have a chance by 20% to respond right at random, but if you use a two-tier test multiple choice then the opportunity the student to answer true random or guess is only 4% [15].

Based on the description above, this research aims to describe the important of a two-tier diagnostic test and how to develop it that can be used to analyze the difficulties students through mistake that he did in finishing mathematical problems. In addition it will be also described about the strength and weakness of two tier diagnostic test instruments.

## METHOD

This article is a literature review article. In this article we will examine the ideas, knowledge, and findings contained in the literature so that it will provide information about the two tier diagnostic test instrument. The literature used in this article contains 30 literatures consisting of literature in the form of books, journals, and research reports.

Data analysis techniques in this article there are three stages, namely organizing, synthesizing, and identifying. The first stage is organizing the literature that will be used. At this stage the authors review first the literature that will be used so that relevant to the topic discussed in the article. Furthermore, the second stage is synthesizing that is looking for linkage between the literature used and then synthesize it into a writing. Then the third step is identifying important information from the literature related two-tier diagnostic test so that it can provide new information in this article.

## RESULTS AND DISCUSSION

### Diagnostic Test

The test was a spate of questions or exercises as well as other tools that are used to measure skills, knowledge of intelligence, ability or talent possessed by individuals or groups [16]. The purpose of performance of tests, among others, is to: (1) find out the level of students' ability, (2) measure the growth and development of students, (3) the difficulty of diagnosing student learning outcomes, (4) knowing the results of instruction, (5) knowing the results of the study, (6) knowing the achievement curriculum, (7) encourage the students to learn, and (8) encourages teachers to teach better. [17] based on the purpose of the test is differentiated into several types, one of which is the diagnostic tests aim to diagnose learning difficulties of students.

The diagnostic test is used to determine the elements of a subject that has special weaknesses and provide guidance to find the cause of the shortage [18]. Diagnostic tests can be used to find out the strengths and weaknesses of students in learning[19]. One of the goals of the use of this test is to determine the teaching need to be held in the next. Diagnostic tests essentially looking back to back about the difficulties that arise and flourish[20].

A good diagnostic test can give an accurate picture of student misconceptions based on error information[21]. A good diagnostic test is a test which can indicate whether a person has mastered the skills or not[22]. the results of the diagnostic tests can provide information to teachers about the ability of the beginning and the misconception of their students before the start of the learning activity. Diagnostic tests also provide information about the lowest limit to start learning activities[23]. The diagnostic tests used to assess student understanding towards the key concepts of a particular topic that tends to be understood wrongly[24].

A diagnostic tests can be considered as a valid test if: (1) parts of the ability test components should emphasize on just one type of error, and (2) differences part test should be trustworthy[21]. This can be achieved if and only if the test has a high reliability and the correlation between low test. Based on the understanding of the definitions

in this article is the diagnostic test is a test used to diagnose students' learning difficulties in the layout of a given topic as well as to find out the cause of the trouble with how to handle difficulties.

Furthermore, as we know that mathematics consists of various materials, one of which is the material of the calculus. Calculus include three major parts, namely the differential limit, or derivative, and integral. Calculus is one of the important fields in mathematics[3]. Calculus has many benefits in life, especially in the field of engineering. But though the calculus is important in life there are many students who are experiencing difficulty in resolving the question of calculus [3,4,5]. As in other parts of the world students at the National University of Lesotho encounter problems in their learning of calculus and in most cases some of the reasons that are given for bad performance in the departmental meetings include: 'we have poor quality of students', 'students are lazy to learn' and 'students are not serious about their studies'. Whilst to some extent these may be true we cannot totally place the blame of poor performance on students alone; in one way or another teachers may also contribute. Over the years we have observed that when teachers mark students' work the focus is more on the correctness or incorrectness of an answer: where the student has committed an error that stage will be marked with a cross and where the correct answer is given a tick is put instead. Teachers seem to be less concerned about knowing the origin of errors. The question to ask is: how effective can teaching be in remedying an error that has not been diagnosed? As highlighted earlier students' errors are a function of many variables such as the student, the teacher, the curriculum, the environment and their interaction, but as a matter of focus we concentrated on just one variable, the teacher, by engaging in a study that has the potential to improve teachers' knowledge of teaching. Since error analysis is one way to contribute to effective teaching of mathematics [7,9,10,11,25]. By having the diagnostic test so teachers will be able to find out students' learning difficulties in the matter of calculus and also can handle well the difficulties experienced by students.

### **Two Tier Diagnostic Test**

As has been discussed in the previous section that the purpose of the diagnostic test is to diagnose learning difficulties of students as well as knowing the causes and how to deal with it. There are several ways to diagnose student difficulties among others through interviews and through the test, test for yourself there are a few types, namely open-ended test, essay, multiple choice, multiple tier test, and others. Based on research [26] obtained the percentage method of diagnosis that is often used to diagnose student difficulties, namely 53% for an interview, 34% for open-ended tests, 32% for multiple choice, and 13% for multiple tiered test. Each of these methods has advantages respectively according to the characteristics of the material and the difficulty that will cause errors are analyzed.

In this article will be examined further on one type of multiple tier test ie :two tier diagnostic test. Generally, two-tier diagnostic test were described as diagnostic instruments with first tier including multiple choice content question, and second tier including multiple choice set of reasons for the answer to the first tier [27,28,29,30]. Students' answer to each item were considered correct when both the correct choice and reason are given. Distracters were derived from students' misconceptions gathered from the literature, interviews, and open-ended response tests. Two-tier tests were considered a great improvement over the previous approaches in that these tests consider students' reasoning or interpretation behind their selected response and link their choices to misconceptions of the target concept [31]. Also, as stated by [27] two-tier diagnostic instruments are relatively convenient for students to respond to and more practical and valuable for teachers to use in terms of reducing guesswork, allowing for large-scale administrations and easy scoring, and offering insights into students' reasoning.

Next, mathematics material that will be discussed is calculus. Calculus topic issues demand resolution with measures of appropriate procedures. Then for two tier diagnostic test will be modified by asking students to write the procedures used with write reasons using the procedure. The example form of instrument can be seen in Figure 1. Students write down the procedure along with the reason it will be easier to find out the location of the error or cause difficulties of students.

- The two-tier diagnostic test multiple choice has two advantages compared to the usual multiple choice[15]:
1. Reduce error rate measurements. On a multiple choice with five alternatives there are 20% of the answers is selected correctly. Correct answer chosen at random will be counted also in assessment, this leads to the ability of the students cannot be known with certainty.
  2. Two-tier multiple choice allows teachers to assess two aspects in one phenomenon. On the first level the students are asked to answer problems occurred, then in the second level the students are asked to explain it. This allows teachers to assess student knowledge and understanding of students.

– **FIGURE 1.** This question is an example of initial development of this study.

(i) Diketahui dua buah bilangan positif. jika jumlah kuadrat bilangan pertama dengan empat kali hasil perkalian kedua bilangan tersebut adalah 288. dan hasil kali antara kuadrat bilangan pertama dengan bilangan kedua mencapai maksimum maka jumlah antara bilangan pertama dan bilangan kedua adalah ...

a.  $6\sqrt{6}$                       d.  $\frac{45\sqrt{6}}{6}$   
 b.  $\frac{21\sqrt{2}}{2}$                         e.  $13\sqrt{2}$   
 c.  $12\sqrt{2}$

(ii) Tuliskan cara untuk mendapatkan jawaban dari (i) dan alasan memilih cara tersebut!

### How to Develop Two Tier Diagnostic Test

The development model that is used to develop two-tier diagnostic test consists nine steps [32] in it, i. e. : (1) to determine the purpose of the preparation of the instrument, (2) finding relevant theory or the scope of the material, (3) compile the indicator instrument grain/reserved, (4) compile the grain instrument, (5) content validation, (6) revised based on input validator, (7) perform the tests, (8) conduct analysis instrument and the grain problem, and (9) assembling the instrument.

a. Determine the purpose of the preparation of the instrument.

The purpose of the preparation of this instrument is to diagnose the trouble students in calculus based on material errors committed students in resolving item reserved instrument.

b. Finding relevant theory or the scope of the material.

The instrument developed diagnostic tests with two-tier format multiple choice then the relevant theory is a theory of two-tier multiple choice, then to the scope of the material used in this study is on the matter of calculus high school level. Calculus material referred to in this research include a limit of a function, an integral and differential, but only limited to polynomial functions only. Then for the mistake that will be analyzed in this study using the errors are categorized by Newman (Clement, 1980) which consists of six types of error i.e. (1) reading error, (2) reading comprehension difficulty, (3) transform error, (4) weakness in process skill, (5) encoding error, (6) careless error.

c. Devise indicators of the instrument.

Indicators of problem be created adapted to the scope of the existing material, i.e. material covering the limit functions, differential, integral for functions and polynomials.

**TABLE 1.** Indicator format of the instrument

No.	Basic Competencies	Material	Indicators	The Form and Numbers

d. Compose grains of the instrument.

After grating composed then the next will begin organized item instrument. The following is a sample format of two-tier diagnostic test that will be developed in this research.

(i) The Value of  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{4 - \sqrt{5x + 1}}$  is ...

- a.  $-1$                       d.  $\frac{1}{2}$   
b.  $-\frac{1}{2}$                      e.  $1$   
c.  $0$

(ii) Write how to get answer from (i) and reason for choosing the procedure to get the answer!



In this section, we will know how the student get the answer. For example, the procedure of problem solving are right but the ending answer was wrong. From the student writing we can know where the error is.

- e. Validation of content.  
Validation of content include the validity of the content and validity of invalid constructs. For the validity of the content will be done by the experts. Experts selected by the researchers as the validator is two lecturers State University of Yogyakarta and three teachers of mathematics in schools. While the analysis to prove the validity of invalid constructs using confirmatory factor analysis or factor analysis (CFA).
- f. Revision based on input validator.  
After the validity of the content or the assessment by experts when the grain problem hasn't been in accordance with the criteria it will be repaired or may be disposed of in accordance with the advice of the validator.
- g. Doing a tryout.  
In this study a test will be administered at grade XII in one High School who has studied the calculus material ranging from the limit of a function, differential, integral for functions and polynomials.
- h. Conduct analysis instruments and grain problem  
After doing a test run results from a test run is then analyzed to see reliability, power, difficulty level of distinction, and also the distractor effectivity.
- i. Assembling the instrument.  
After all the above stages are completed then the next instrument is assembled and arranged so that it will be built into an instrument of two-tier diagnostic test

## CONCLUSION

Based on the literature review that has been done is known that diagnostic test is a test used to diagnose the location of students' learning difficulties on a particular topic and to determine the causes of the difficulty and how to deal with the difficulty. One form of diagnostic test is a two tier diagnostic test that is a two-level diagnostic test, the first level of multiple choice questions and then the second level is a matter of description by asking students to write down the procedures and reasons for an answer at the first level. Diagnostic tests with two tier or two tier diagnostic test can be used to uncover students' difficulties in solving calculus mathematical problems based on mistakes made at the time of completion. This brings benefits to the teachers because they get information where the errors are made by the students, so that the teachers will be easier to identify the difficulties done by the students.

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