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Analyzing Student's Ways of Thinking on Fraction Estimation: A Case of Student from Rural Area

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Abstract. The objective of this study is to know the types of student's way of thinking when they deal with fraction estimation problems. This study used qualitative with the descriptive method and involved one of the 6th-grade students at one of the primary schools in the rural area of West Java, Indonesia. A student from the rural area was selected for consideration of the equity principle in mathematics education. The results are some types of student's way of thinking when dealing with fraction estimation, that are common denominator approach for addition and subtraction of fractions, a direct multiplied method for multiplication of fractions, inverse multiplied algorithm approach for the division of fractions, and change to decimal when dealing with ordering fractions problem. The other characteristics of the student's way of thinking are failed to make an estimation and cannot represent word problems regarding fractions. All these types of student's ways of thinking and student's characteristic are a manifestation of the procedural ways of thinking and lack of the conceptual understanding. The findings are then elaborated using some related theories to justify the results.

INTRODUCTION

Fractions are one of the topics in mathematics that is difficult to understand; this happens because understanding fractions require a high level and complex thinking [1]. On the other hand, fraction competency is essential for students. Research finding stated that competence with fractions in Grade 5 predicted next gains in mathematics knowledge five years later [2]. Moreover, the importance of fractions extends beyond math classes and the school years. Fractions are essential for a wide range of work related to the physical, biological, and social sciences, also for various other jobs such as nursing, carpentry, and auto mechanics ([3], [4]).

Let us consider the definition of fractions. Fractions is is a symbol that represents the results of two numbers $\frac{a}{b}$ (with b not equal to zero). Thus, all rational numbers written in form $\frac{a}{b}$ are fractions, but rational numbers 1.45 are not fractions; however, if 1.45 is written $\frac{145}{100}$ then it is a fraction [5].

Based on the definition above, the fraction is a symbolic expression. Although fraction is just a symbol in a number, understanding fractions are crucial for students in learning mathematics. Fractional topics are the foundation for learning algebra and math at higher levels ([6],[7],[8]). The importance of fractions makes it an attractive topic for research. Therefore, the research problem of this study is how student think when she deals with the fraction estimation problems. In this study, we aim to examine student's ways of thinking on fraction estimation. The student was chosen from a rural area because we consider the issue of equity in mathematics education [9]. Furthermore, we want to know how a student that far away from urban areas understand fractions.

METHOD

This research used a qualitative approach with descriptive method [10]. The respondent is one student in the Public Elementary School in the rural area of West Java, Indonesia; after this, we called the student as Juju (pseudo name). The student was selected from a school located in the rural area. Data were collected by providing tests and interviews. There are ten questions on the test with the following details:

1. Put in order from smallest to largest:
   $\frac{99}{100}$ $\frac{3}{4}$ $\frac{1}{10}$ $\frac{4}{5}$ $\frac{6}{12}$ $\frac{3}{8}$
2. $\frac{15}{16}:\frac{19}{20}=\cdots$. Is the answer more or less than one?
3. $\frac{3}{4}+\frac{1}{5}=\cdots$. Is it more or less than $\frac{1}{2}$? More or less than one whole?
4. $\frac{2}{3}+\frac{1}{4}=\frac{3}{7}$. Do you agree? Why or why not?
5. $16\times\frac{41}{83}=\cdots$. About what the whole number is the answer closest to?
6. $\frac{2}{3}:\frac{4}{6}=\cdots$. What is the exact answer? Why?
7. Mona brings 3 pints of ice cream. Each serving is $\frac{2}{3}$ of a pint. About how many meals?
8. $1\frac{3}{4}:\frac{2}{3}=\cdots$. Estimate, explain why it is reasonable?
9. Estimate the distance traveled by a person who runs $2\frac{4}{5}$ blocks. Is each block $\frac{1}{3}$ of a mile long?
   Is it more or less than 1 km?
   Is it more or less than 5 km?
10. Estimate $3\,\frac{1}{2}-\,2\,\frac{5}{7}=\cdots$.
    (adapted from [11])

RESULT AND DISCUSSION

Before the interview, Juju was given 30 minutes to work through the problem first. The following is a complete description of the interviews with the students.

Question number 1 is as follows:

Put in order from smallest to largest:

$\frac{99}{100}$ $\frac{3}{4}$ $\frac{1}{10}$ $\frac{4}{5}$ $\frac{6}{12}$ $\frac{3}{8}$

The transcript of the interview is as follows:

The researcher: Put the fraction from smallest to largest?
Juju: Fraction from the smallest to largest are $\frac{1}{10},\frac{4}{5},\frac{3}{4},\frac{3}{8},\frac{6}{12},\frac{99}{100}$
The researcher: How do you get that answer?
Juju: By changing to decimal and ordering the decimal number
The researcher: Can you think $\frac{1}{2}$ *as a benchmark to solve that problem?*
Juju: No, I cannot do that.

Although Juju's answer was wrong, we can still analyze his strategy in solving the problem. Juju uses an approach of changing fractions to decimals, then ordering the decimal from smallest to largest. The researcher elaborated whether there was another way to solve the problem, for example, by using $\frac{1}{2}$ as a benchmark, but Juju replied cannot. The researcher wants to encourage Juju's opinion about another way to sort the fractions, for example, by an informal method of using a benchmark, a residual or common numerator.

Question number 2 is as follows:

$\frac{15}{16}:\frac{19}{20}=\cdots$. Is the answer more or less than one?

The transcript of the interview is as follows:

The researcher: Is the answer more or less than one?
Juju: More than one
The researcher: Why?
Juju: Because I calculate by changing numerator become denominator and denominator become the numerator in the second fraction, and then I multiply the numerator by numerator and denominator by the denominator.

The researcher: Can you use another strategy to solve that problem?
Juju: No, I cannot.

Juju gave a wrong answer. Juju used a strategy of changing numerator become denominator and denominator become the numerator in the second fraction, and then she multiplied the numerator by numerator and denominator with the denominator. The researcher elaborated by asking Juju about another strategy to solve that problem, but Juju replied cannot. Indeed, the researcher tried to tell Juju to think about other approaches like using the residual plan to judge the relative size of fractions and to estimate quotients of two numbers close to one.

Question number 3 is as follows:
$\frac{3}{4}+\frac{1}{5}=\cdots$. Is it more or less than $\frac{1}{2}$? More or less than one whole?
The transcript of the interview is as follows:

The researcher: Problem number 3, is it less than $\frac{1}{2}$? Or is it less than one? Or is it more than one?
Juju: Less than $\frac{1}{2}$, and less than 1
The researcher: Why? That you know, $\frac{3}{4}$ is more than $\frac{1}{2}$
Juju: Oh yes, I mean less than 1.
The researcher: How do you get the answer?
Juju: I make the same denominator and add the numerator of both fractions.
The researcher: Can you use $\frac{1}{2}$ as a benchmark for $\frac{3}{4}$ and $\frac{1}{4}$ for $\frac{1}{5}$?
Juju: No, I cannot.

On the first occasion, Juju was wrong in answering the question, but after being reminded, that $\frac{3}{4}$ is more than $\frac{1}{2}$, she mentioned the correct answer. She used a strategy of making common denominator and added the numerator of both fractions. The researcher asked her about another approach to solving that problem like using $\frac{1}{2}$ as a benchmark for $\frac{3}{4}$ and $\frac{1}{4}$ for $\frac{1}{5}$, but she replied cannot. She can think that $\frac{3}{4}$ is more than $\frac{1}{2}$ and $\frac{1}{5}$ is less than $\frac{1}{4}$, so she can conclude that the sum must be less than 1.

Question number 4 is as follows:
$\frac{2}{3}+\frac{1}{4}=\frac{3}{7}$. Do you agree? Why or why not?

The transcript of the interview is as follows:

The researcher: Do you agree in the result for problem 4?
Juju: I disagree.
The researcher: Why?
Juju: Because I calculate the result is equal $\frac{to\ 11}{12}$.
The researcher: How do you get the result?
Juju: I make the common denominator.
The researcher: Can you see that $\frac{3}{7}$ are less than $\frac{1}{2}$ but $\frac{2}{3}$ more than $\frac{1}{2}$?
Juju: I do not think that

Juju gave the correct answer. Her strategy was making common denominator and added the numerator of both fractions. The researcher suggested her to use $\frac{1}{2}$ as a benchmark. She can think that is less than $\frac{1}{2}$ but $\frac{2}{3}$ more than $\frac{1}{2}$, so the sum must be more than $\frac{1}{2}$, but he replied cannot.

Question number 5 is as follows:
$16\times\frac{41}{83}=\cdots$. About what the whole number is the answer closest to?

- Is the answer higher than or less than 16?
- Is the answer a lot bigger than 16?
- Is the answer greater than or less than 8?

The transcript of the interview is as follows:

The researcher: Now, for problem 5, how about your answer?
Juju: Less than 8
The researcher: How do you get the answer?
Juju: I multiply 16 with 41, and the result is divided by 83.
The researcher: Can you use another strategy?
Juju: No, I cannot.

Juju gave the correct answer. Her strategy is multiplying 16 with 41, and the result is divided by 83. The researcher asked her about another approach to solving that problem, but Juju replied cannot. The researcher tried to ask her to think about other strategies like using $\frac{1}{2}$ as a benchmark and estimating the product of $\frac{1}{2}$ of an even number, or she can adjust an initial estimation and see that multiplying by a fraction just under $\frac{1}{2}$, then she estimated the answer would be just under 8.

Question number 6 is as follows:

$\frac{2}{3}:\frac{4}{6}=\cdots$. What is the exact answer? Why?

The transcript of the interview is as follows:

The researcher: What is your answer to problem 6?
Juju: One
The researcher: How do you get one?
Juju: I multiply the first fraction with the inverse of the second fraction?
The researcher: Can you think that two fractions as equivalence?
Juju: No, I do not think that.

Juju gave a correct answer. Her strategy was multiplying the first fraction with the inverse of the second fraction. The researcher elaborated by asking her to think that two fractions as equivalence, but she replied cannot. Indeed, the researcher tried to ask her to think about other strategies like using her equivalence ideas to see that the quotient must be one.

Question number 7 is as follows:

Mona brings 3 pints of ice cream. Each serving is $\frac{2}{3}$ of a pint. About how many meals?

- Is amount more than 3?
- Is the amount more than 6?
- Is the amount less than 3?

The transcript of the interview is as follows:

The researcher: Now, for story problem. How do you think?
Juju: I do not know.
The researcher: Why?
Juju: I do not understand.
The researcher: Okay, continue to number 8.

Juju did not answer. She had no idea of this problem. The researcher tried to encourage her to answer that question, but she said that she did not understand the problem. The researcher attempted to ask her to think about other strategies like using $\frac{1}{2}$ as a benchmark to determine that $3 \div \frac{1}{2}$ would be 6; so $3 \div \frac{2}{3}$ ($\frac{2}{3}$ is a fraction larger than $\frac{1}{2}$) would be less than 3.

Question number 8 is as follows:

$1\frac{3}{4}:\frac{2}{3}=\cdots$. Estimate, explain why it is reasonable?

The transcript of the interview is as follows:

The researcher: Can you estimate the answer without make calculation first?
Juju: No, I cannot.
The researcher: How do you get the answer?

Juju: I multiply by the inverse of the second fraction, and the first fraction change becomes common fraction.
The researcher: Oh, okay.

Her strategy was multiplying the first fraction by the inverse of the second fraction, before doing that, she changed the first fraction becomes simple fraction. The researcher asking her that could she estimated the answer without make calculation first, she said that she could not do that. The researcher tried to ask her to think of the measurement interpretation to bring meaning in a fraction division task; then hopefully she can realize the answer will be higher than 2.

Question number 9 is as follows:

Estimate the distance traveled by a person who runs $\mathbf{2\frac{4}{5}}$ blocks. Is each block $\mathbf{\frac{1}{3}}$ of a mile long?
Is it more or less than 1 km?
Is it more or less than 5 km?
The transcript of the interview is as follows:

The researcher: Can you answer this problem?
Juju: No, I cannot; I do not understand.
The researcher: Okay, we continue to number 10.

Juju did not answer. He did not understand this problem. The objective of this question is the researcher wants to know about her skills in interpretation to the word problem, in this problem is multiplication.

Question number 10 is as follows:

Estimate $\mathbf{3\frac{1}{2} - 2\frac{5}{7} = \cdots}$
The transcript of the interview is as follows:

The researcher: Can you estimate the answer?
Juju: No, I cannot, I make a calculation. I change the fraction become ordinary fraction, and make the same denominator.
Juju: Okay. Thank you.

Her strategy was changing the fraction became simple fraction, and made the common denominator. The researcher asked her that could she estimated the answer without making a calculation, she said that she could not do that. The researcher tried to ask her to think about adjusting the estimation by determining that $\frac{5}{7}$ are more than $\frac{1}{2}$ so the difference will be just under 1.

The following table summarizes students' ways of thinking:

TABEL 1. The summary of student's ways of thinking

Student's Thinking	P#1	P#2	P#3	P#4	P#5	P#6	P#7	P#8	P#9	P#10
Common denominator approach	-	-	√	√	-	-	-	-	-	√
Inverse multiply	-	√	-	-	-	√	-	√	-	-
Direct Multiply	-	-	-	√	-	-	-	-	-	-
Change to a decimal	√	-	-	-	-	-	-	-	-	-
Cannot represent story problem	-	-	-	-	-	-	√	-	√	-
Fail to make an estimation	√	√	√	√	√	√	-	√	-	√

In our point of view, for question number 1, Juju cannot use a benchmark or residual strategy for ordering fraction, instead of, she changed the fraction to be decimal. However, she cannot complete the ordering fraction correctly although she can shift every fraction to be decimal. We think it because of her less careful when ordering the decimal. As a result, she had the wrong order, $\mathbf{\frac{1}{10}, \frac{4}{5}, \frac{3}{4}, \frac{3}{8}, \frac{6}{12}, \frac{99}{100}}$. About the connection between student ordered fractions ability and her skills with estimation, we can see her difficulty to make an estimation, because she always used procedural way to deal with the estimation problem during the interview.

We think Juju was very successful in fraction division problems (question number 2, 6, and 8). However, she cannot use estimation skill. She just applied a procedural way that multiplied the first fraction with the inverse of the second fraction like her answer in question number 2, 6, and 8. However, she still did a wrong calculation, maybe because of her less careful, for example, in question number 2, her answer is more than one, while the correct answer is less than one.

A procedural way to the division of fraction that multiplied the first fraction with the inverse of the second fraction is called Invert and Multiply Algorithm. According to Zembat, Invert and Multiply Algorithm (IMA) strategy provides only a little feasibility to connect with a rich understanding of the fraction. He proposes another approach called Common Denominator Algorithm (CDA) for the division of fraction. Based on Zembat, CDA strategy provides many advantages to connect with a rich understanding of the fraction [12].

In the problem involving addition and subtraction (question number 3,4, and 10), Juju always used a procedural method that is common denominator approach. She made common denominator and added the numerator of both fractions. She cannot think of another plan or representation, even though the use of various representations is highly recommended in studying fraction. Based on previous research findings, students gain a deeper understanding of a concept when they can identify and model a mathematical concept in various representation systems and have the flexibility to move from one representational system to another [13].

Question number 5 involves multiplication of fraction. Juju directly multiplied by $\frac{41}{83}$. She does not realize that $\frac{41}{83}$ are about $\frac{1}{2}$, so the answer is about 8. Students often have difficulties on how to multiply a fraction by conceptual understanding. Based on the previous research finding, to understand fraction multiplication conceptually, students should be able to adjust meaning in the numbers being operated on and have an understanding of what happens to the numbers when they are multiplied. To help students understand fraction multiplication, the previous researchers use a piece of paper and a number line [14].

In the problems that involve word problem of fraction (question number 7 and 9), Juju said that she did not understand. She was not able to change the word problem into a mathematical model. As a suggestion to the next learning, several strategies can be used to improve students' understanding of the fraction word problem, such as schema-based intervention [15]. This strategy emphasis on conceptual understanding, helped students acquire word-problem-solving skills and maintain the taught skills. Another strategy is using a number line approach, the optimal use of number line model can help students in understanding the size and fractional calculations [16].

To sum up, we think our student thinking is procedural. She used the common denominator strategy, change to decimal strategy, direct multiplied strategy, inverse multiplied approach, and cannot use the other procedures like a benchmark, residual or drawing a picture. According to Hiebert & Wearne, our student thinking is a syntax thinking or procedural. She just knows the step-by-step way to solve a fraction problem. Furthermore, she does not see any connection between properties in fractions problem. In another word, she did not have a semantic or conceptual understanding [17].

Using Herbert &Wearne opinion about the conception of three sites of understanding, in site 1, our student knows the fraction. She can interpret it, except for word problem. In site 2, execution procedure, our student cannot use conceptual knowledge. She just uses algorithmic or procedural way. In site 3, that is evaluating the solution; our student cannot assess her conceptual understanding. We think that our student always jumps to the procedural way to solve the problem. She spent a lot of time with the word problem, but she still cannot figure it out. We think it is because of lack of conceptual understanding.

CONCLUSION

Based on the result and discussion, we conclude that there are four types of students' thinking while dealing with fraction estimation. There is common denominator approach for addition and subtraction of fraction; inverse multiplied approach for the division of fraction, a direct multiplied approach for multiplication fraction, and change to decimal when dealing with the problem of ordering fraction. The other characteristics are failed to make an estimation and cannot represent the word problem regarding a fraction. All this type of students' thinking and students' characteristic is a manifestation of the procedural way of thinking and lack of conceptual understanding. Based on the results, we recommend that fraction instruction should: (1) teaches effective estimation strategies; (2) make students understand about meaning of a fraction by multiple representations; (3) Remind a student of the importance of another approach besides procedural way; and (4) teaches students how to modelling word problem about fraction.

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