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A Semiotic Analysis of Pattern Generalization: A Case of Formal Operational Student

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Abstract. This study to explores the generalization process of students with logical thinking ability formal operational stage based on semiotic perspective. The semiotic materials analyzed in this study include gestures, speechs and writings. This explorative qualitative research was conducted in private junior high school in Tuban East Java Indonesia. Subjects in this study were one student of VII Grade whose logical thinking ability was at formal operational stage. The subjects were selected using GALT (*The Group of Assessment of Logical Thinking*). Data collected by think alouds and video recording, that is when completing the pattern generalization task, students expressed their thoughts orally. In addition, interviews are also conducted to complement the data that has not been collected by think alouds. The data has been collected and then analyzed using qualitative data analysis techniques. The results show that formal operational students through all stages of generalization perfectly. The student changes the regularity of the known image into a number pattern. Students declare generalizations in the form of symbolic algebra.

INTRODUCTION

Mathematics is seen as the science of patterns and order [1] and identifying for a pattern or regularity is one of the actions which are performed in mathematics on the whole [2]. Patterns are at the center of mathematical thinking and mathematical inquiry [3]. The activities related to patterns are essential in terms of realizing mathematical relationships, understanding the system and logic of mathematics [4]. These activities include searching for pattern, extending patterns, making pattern generalization [5] and contribute to the skills of organizing data systematically, conjecturing and generalizing [6].

Patterns generalization improves algebraic thinking and constructs the concepts of variable and function [7]. Generalization also helps students to understand symbolic representations and interrelate among previous knowledge of arithmetic [8]. Thus it facilitates to proceed from arithmetic to formal algebra. Searching for patterns is a fundamental step in order to make generalization it is seen as a way of approaching algebra [9] [10]. In short, patterns have an important role as a bridge between generalization and algebra in primary level for providing constitution of algebraic thinking that is the base of formal algebra.

The process of generalization is an induction process, it is begins with specific examples leading to general rules. Based on these specific examples the student will analyze the relationships, similarities and structures, then determine the predictions of the general rules applicable to the examples. [11] states that based on the results of the calculations, shapes, and drawings, the combination of abduction-induction allows student's generalization. The expression of pattern generalization not only reduces and equates, but it needs abduction with various approaches that lead to hypotheses with facts that are mediated by signs.

In completing the task of pattern generalization, students use different strategies [12]. Each of the strategies used by students is accompanied by interesting signs to be examined, both mentally (thought processes) as well as physically (gestures, words, symbols, drawings and so on). A theory that learns about signs, the functioning of signs and meanings produced by signs is a semiotic theory. Symbols or signs formed by students can be studied through semiotic theories with interpretations according to the context being studied. Mathematics is a student's knowledge associated with activity-based alerts. Semiotics are most appropriate when applied in mathematics

[13]. [14] argues that understanding abstract mathematical operations and contexts is not possible without semiotic activities.

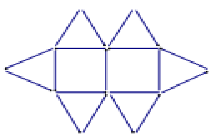
Some semiotic components that appear in the pattern generalization activity include gestures, words, and symbols [15]. The semiotic on the mathematics learning activity is a conceptual way that focuses on the symbol and its use as a way to understand the function and mental structures [13] of semiotic as an interpretation of mathematical expression [16]. The semiotic components studied in this study include gestures, spoken words and symbols. Gesture accompany students in generalizing patterns can be expression of fingers or mimic at the time of analyzing geometric images that make up the pattern. The words on the generalization process of the pattern are expressed by the student in the form of words or sentences. While the symbol can be a sign, image, and letters. Students usually use symbols or signs that have been understood meaning in the process of generalizing the pattern.

METHOD

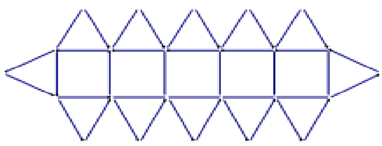
This qualitative research is conducted in Tuban Junior High School in East Java Indonesia. The subjects of the study were seventh grade students with logical thinking ability at formal operational stage. Selection of research subjects using the GALT (Group of Assessment of Logical Thinking) instruments developed by [17]. The abbreviated GALT is a paper and pencil test consisting of 12 items. The test form uses multiple choice types that present answers and possible reasons behind the answer. An image is inserted for each item to visualize the problem. Each item correctly scored 1, so the maximum score of 12 and the minimum score is 0. Next score is classified as follows; scores 0-4 are grouped in concrete operational stages, 5-7 scores are grouped in the transitional phase, and 8-12 are formal operational stages [17].

Instruments used in this study are researchers as the main instrument, sheet generalization task patterns, and audiovisual recording device. The researcher as the main instrument acts as a planner, data collector, data analyzer, data interpreter, and research reporter. During completing the task of pattern generalization, students are required to express verbally all that is being thought of. Therefore, data collection in this study is said to think aloud. This activity was recorded with audiovisual. Interviews on students are based on results. The generalization task of the given pattern is as follows the task of generalizing the pattern.

Look at the following diagrams containing squares and triangles built from matchsticks.



For 2 squares you need 19 matches
(a)



For 5 squares you need 40 matches.
(b)

Give an algebraic "rule" or "formula" to work out the ***total*** *number of matchsticks you will need if there are "n" squares.*

The result data of the task of generalization of the pattern in writing, the result of think aloud, the result of the observation, and the result of the interview, both in the form of verbal data and behavior analyzed by stages: 1) transcribing all verbal data, 2) reducing data by making abstraction, 3) into units of units categorized by coding, 5) analyzing the process of generalizing patterns based on semiotic perspectives, and (5) drawing conclusions. The TGP answer sheet and interview results were analyzed based on the generalization process indicator according to [18].

RESULTS AND DISCUSSION

Based on the subject selection process, after experiencing the saturation obtained two students with logical thinking ability is at formal operational stage. Of the two students, one student meets the consistency criteria. Based on the problem solving generalization of the patterns that have been done by the subject, then obtained data derived from think aloud, interviews, field notes, and the work of the subject.

The student who was the subject of the study was a male student with logical thinking ability at the formal operational stage, SF. Data analysis in this study focused on the generalization process based on the stages of generalization [18], which is (1) discovering of regularity, (2) confirming of regularity, (3) producing of general formula (generalization) and (4) proving of general formula.

SF finds the regularity of configuration (a) and configuration (b). SF looks at the arrangement of rectangles and triangles. SF finds the regularity of every square arrangement surrounded by triangles. There is an upper triangle, lower triangle, right triangle and left triangle surrounding the square.

SF found the number of matchsticks in two known configurations. SF finds the number of matchsticks in configuration (a) and (b). Here is a passage of think aloud from SF.

SF: *"If the two squares have 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19. There are 19 matchsticks. If the five squares have 1,,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38 ,39,40. There are 40 matchsticks ".*

SF does not read the information under configuration (a) and (b). SF recalculates the number of matchsticks in configuration (a) and (b). In calculating the number of matchsticks, SF is mediated by the gesture pointing. SF points to matchsticks that make up both configurations using a pencil, as shown in Figure 1 below

Figure 1. Gestures SF in calculating the number of matchsticks of configuration (a) and (b)

Figure 1 shows the gesture pointing SF in performing quantitative analysis, ie calculating the number of matchsticks. Gestur pointed to SF also followed by a speech, which is to say number of matchsticks.

SF understands the regularity of configurations (a) and (b), but SF does not understand the rules of the pattern. Therefore, SF determines the number of matchsticks that make up a square and three squares. At first SF, painted a matchstick array consisting of a square and three square like a matchstick configuration (a) and (b) then calculated the number of matchsticks that composed it. Here's a painting of SF

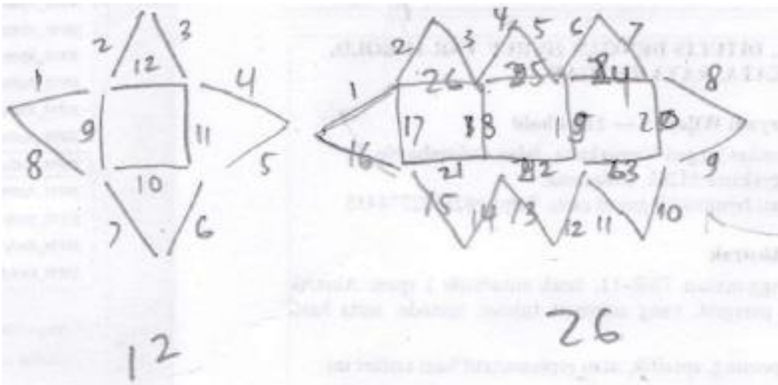


Figure 2. SF paintings for matchstick configurations consisting of 1 square and 3 square

SF understands that the configurations of one square, two square, and three squares form a pattern, that is, each sequence increases by seven matchsticks. SF is able to analyze quantitatively, which is to look for links between many matchsticks between sequences with simple. Here think aloud from SF.

SF: *"If 1 square there are 12 matchsticks. If 2 square, there are 19. 3 square there are 26 stems. If 5 square there are 40 matchsticks. The pattern increases 7. If 6 square means there are 47 matchsticks. Since 40 plus 7 equals 47 "*

SF is also able to find many matchsticks on a larger configuration, the fifty square configuration. SF is able to determine the quantity in larger configurations and is able to show the relationships that occur in the pattern. The relationship analysis capability occurring in the SF pattern appears in the work presented in Figure 3 below.

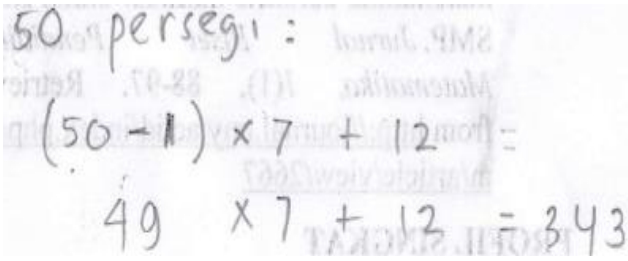


Figure 3 The results of SF’s quantitative analysis of matchstick

SF was able to find the relationship between the number of matchsticks on the matchstick array consisting of one square, two square and three square. SF is mediated by a representational gesture to determine the quantitative relationship with hand gestures, making the arrows start from the first to the second and from second to third, as shown in Figure 4 below.

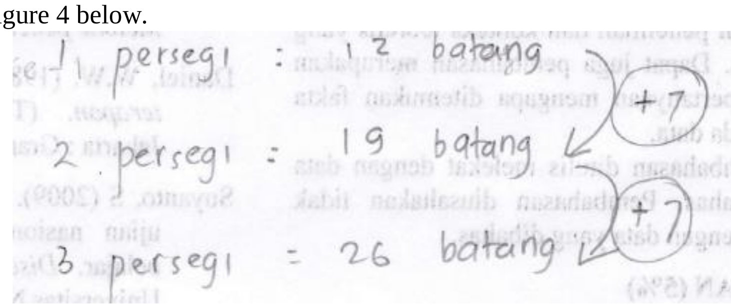


Figure 4. Gestures of SF’s Writing that indicate the relationship of the pattern quantity

Figure 4 shows the writing movement performed by SF in determining quantitative analysis. SF creates a curved arrow to illustrate the relationship between the number of patterns.

In generating the general formula for nth configuration, SF subjects use symbols such as alphabet to generalize the patterns. The use of the matches in the Figure 5 below. To determine the -n sequence of the given pattern, the gestures accompanying SF are shown in Figure 5 below.

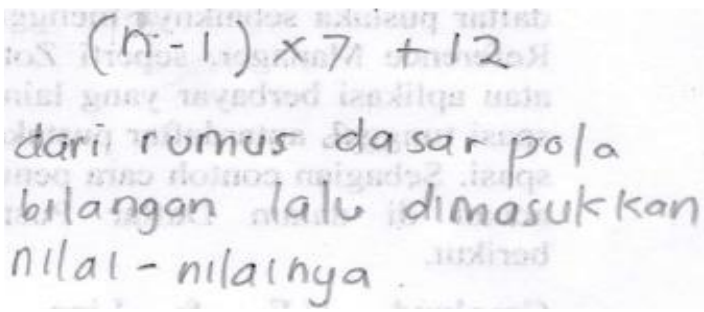


Figure 5. The results of SF’s use of alphabet symbols

When determining the general form of the pattern, namely the general formula of the number of batang korek api on the –n sequence, SF’s gestures were the fingers pointing to a pattern in the form of pictures and the eyes looking at the general formula of the determined pattern. This showed that SF performed a pattern quantity relationship known in the first, second, and third sequences to the –n sequence. SF conducted a variable cognition to determine the –n sequence.

In validating the general formula, SF validates by replacing the value of n by 3. SF also draws a matchstick configuration consisting of 3 squares. Then, SF matches the number of matchsticks obtained from the formulas and drawings. SF gets the same result. In validating the general formula formula, SF is mediated by symbols. SF makes a special symbol, which is a curve that connects the results of the calculation of the formula and the results of the calculation of the image. The arcing line made by SF signifies a similarity of values. Figure 6 below is the result of SF's written work in validating the general formula

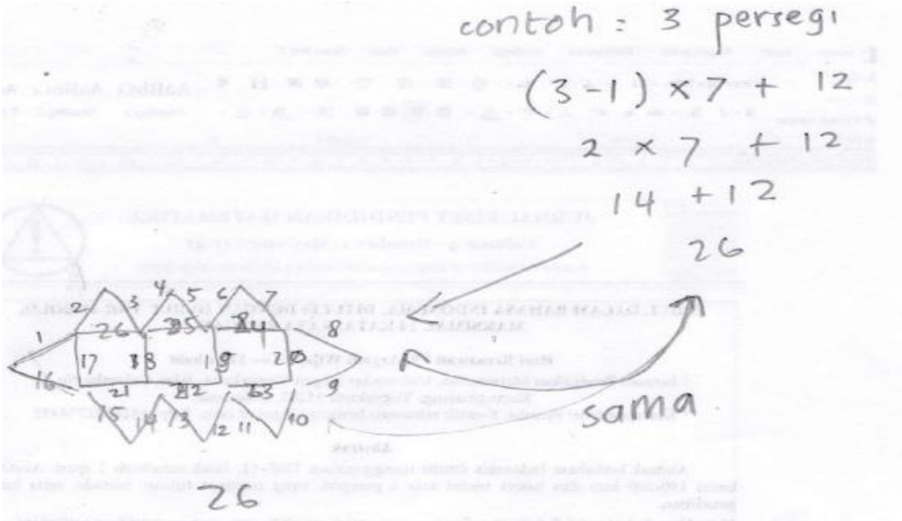


Figure 6.The result of SF's written work in validating the general formula

The results of this study indicated that in the process of pattern generalization, subject finds the regularity or similarity from the example forming the patterns. Subject's gestures with pointing the quantity of pattern when conducting the process of determining regularities or similarities in the pattern are included into the type of pointing gestures. Furthermore, the representasional gestures, gesture representing an abstract idea without physical form, also appeared. This has supported the findings of [19] which showed that children generalized pattern through gesture, observation, and sound. In addition, it was also in accordance with the findings of [20] which stated that one of the students' mathematical communications through gestures was the freedom of students' deictic attitude.

The process to determine the similarities and differences of a pattern could be observed from the expression of fingers or expressions shown by the subject towards pictorial patterns. The expressions of the subjects' finger movements followed the activities in calculating the quantity of pictorial patterns. This has showed that gestures helped the students' thinking process in generalizing the patterns. The findings of this research supported the findings of [21] which showed that gestures helped mathematical thinking and learning.

The expression of fingers in the pattern generalizing process accompanied the words expressed by the subject. The gestures expressed by the subject in pattern generalizing activity were in accordance with his words. This was consistent with the findings of [22] which stated that there was compatibility in gestures, observation, and sound when a child generalized a pattern. In addition, the findings of [23] stated that gesture was integrated with words expressed in learning mathematics.

Based on the regularities of the pattern that had been found, the subject predicted the next sequence. The students identified the similarities occurring in one pattern to be applied to the next pattern. This supported the findings of the study by [24] [25] which stated that in the process of generalization, the subject identified some specific cases and applied them to the next cases or referring to the possibility of something to determine the next possibility. Then the general similarity found was given a general prediction expressed by using variables. This was consistent with the statement from [26] that the general prediction played an important role in pattern formation and generalization.

In proving the general formula that has been obtained, the subject proves by giving example, that is, replace the value of n with 3 on the formula 12 + (n-1) x 7 then draw a matchstick configuration consisting of three squares. Subject matching the calculation results by using the formula with the calculation through the picture. The subject finds no common outcome. Proof by example in accordance with opinion [27] stating that justification using examples can be used to convince oneself or others about certain statements. [28] says that proof using examples includes empirical evidence schemes.

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CONCLUSION

Based on the result of the research, it can be concluded that the subject of SF (VII Grade Junior High School students at the formal operational stage) performs all stages of the generalization process perfectly, discovering of regularity, (2) confirming of regularity, (3) producing of general formula (generalization) and (4) proving of general formula. In the stage of discovering of regularity, SF often uses gesture pointing, a gesture that points the existing object. The students pointed the quantity of objects forming the pattern one by one. Furthermore, there was also representational gesture, a gesture that represents an abstract idea without physical form. SF confirms of regularity by applying the principles of regularity that have been discovered. SF generates a general formula in the form of symbolic algebra. SF understands the meaning of variable n. SF proves the general formula by using empirical examples.

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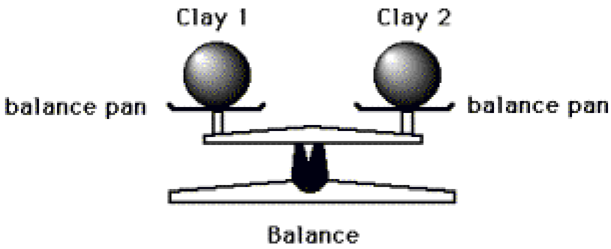
APPENDIX

The Group of Assessment of Logical Thinking
Conservational Reasoning

ITEM 1: PIECE OF CLAY

Tom has two balls of clay. They are the same size and shape. When he places them on the balance, they weigh the same.



The balls of clay are removed from the balance pans. Clay 2 is flattened like a pancake.

WHICH OF THESE STATEMENTS IS TRUE?

A. The pancake-shaped clay weighs more.

B. The two pieces weigh the same.

C. The ball weighs more.

SELECT THE REASON FOR YOUR ANSWER:

1. You did not add or take away any clay.

2. When clay 2 was flattened like a pancake, it had greater area.

3. When something is flattened, it loses weight.

4. Because of its density, the round ball had more clay in it.