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Mathematical Abstraction of Junior High School Students With Process CRA (Concrete Representational Abstract) Approach

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Abstract. This research is motivated by the low skill of mathematical abstraction. Efforts made in this problem is to implement a learning approach CRA (Concrete Representational Abstract).The purpose of this study to illustrate the process of abstraction of junior high school students using the CRA approach. This case sends is a part of mixed methods research, the population of all eighth grade students in one of the Junior High School in Tangerang taken only two classes. One class as an experimental class and one other class as a control class, with a total sample of 81 students. The design of the study is the design concurent triangulation design. The data sample for quantitative data was obtained using random cluster sampling while qualitative data was conducted using purposive sampling. Instruments is used in the form of test instruments and non-test. Test instruments and instruments is in description form about the non-test in the form of observation (field record), interviews, attitude scale, student performance, and documentation. It is known that there are 40 students in the experimental class and 41 students in the control class with 43,4 as the average score for the experimental class and 42,1 for the contro class , included 12,3 and 15,91 as the standard deviation for each class respectively for the pre-test. In the post-test, the average score of the experimental class (81) shows that the experimental class' abstracting skill is low, while the control class' average score (70,1) shows that the abstracting skill of that class is also low. In the table, the SD (Standard Deviation) of the experimental class and control class' post-test are 21,2 and 25,2 respectively. The research showed that the description of abstraction process through CRA approach is an empirical abstraction process which the dominant aspect is identifying the objects' characteristics through direct experience. We conclude from this research is abstraction process in students learning with CRA during the class is an empirical abstraction process and the dominant aspects are aspects that identify the characteristics of objects through direct observation.

INTRODUCTION

Quite many skills that must be sharpened in education during the school age. In order to help students to solve any kinds of problem in learning, students needs to be taught the basic leaning skills. Based on [4] statement that during the school age, students develop their basic learning skills: writing, reading, and counting on which those skills act as the basic skills to obtain knowledge. From various explanation, it can be concluded that one of the basic learning skill is counting, and counting

William (2007) also investigated the spontaneous abstraction process of junior high school students in the learning equation of a straight line. Some of the topics raised in the field of abstraction also algebra[2]. In contrast, research on the topic of geometric abstraction is still rare, while the abstraction process which is very important in learning geometry [6] .Tall and Gray stated that the process of abstraction occurs when students learn geometry, for example when students analyze the geometry object in two dimensions. Geometric objects such as points, lines, angles, and all their relations are abstract objects.

Some of the opinions of researchers and experts can be summarized that abstraction is the process that leads students perform and experience activities that eventually form abstract concepts [9]. It indicates that the notion of

abstraction in mathematics learning requires a process that can help students make sense or mathematical concepts [5].

From preliminary studies conducted by test and the questionnaire. The results of tests the skill of abstraction that has been done by the students, most are unable to resolve the problem and get a numerical value that indicates the abstraction of students is still low. Evident from the 37 students who trial the test on abstractions that only 8 students who scored above the KKM, while two people were absent, and the rest are under KKM, with the standard of the KKM in the school is 70. This is very clear, the results of matter taken from [12] and [14] by using indicators abstraction capskill is low.

Sttedly's mathematics education journals, et.al, entitled *Effective Mathematics Instructions* [14], explains that the CRA approach is instructive in learning mathematics that combines visual representation. The approach has a three-part instructional allows teachers to use *Concrete* (such as *chip-colored*, geometrical figures, pattern blocks, or cubes) to model mathematical concepts to be learned, then show the concept through *Representational* (like describing a form), and the latter is *Abstract* or symbolic (such as numbers, notations, or other mathematical symbols), usually abbreviated with the CRA *(Representational Concrete*Abstract). [13] states that the CRA approach teaches students through three stages of learning, namely: (1) concrete, (2) the representation, and (3) the abstract.

One advantage of the CRA approach mentioned by NCTM. According to the NCTM *(National Council of Teachers of Mathematics)*, the advantage of this approach lies in the intensity and concreteness that help students maintain a framework in their working memory to solve the problem [10]. This is proved by the results of research conducted by[15] that learning by using CRA approach is able to improve students' mathematical abstraction. CRA approach is particularly suitable in researching students' mathematical abstraction. This is reinforced by the statement Bruner Lestari in [14]"For children between the ages of 7 to 17 years, to obtain absorption and comprehension which includes memory, understanding, and application still requires the eyes and hands".

From some explanation about abstraction, importance of abstraction, and use of CRA approach, hence researcher have aim which must be achieved from this research that is know explanation of abstraction process of junior high school student by using approach of CRA.

RESEARCH METHOD

Types of research

This study used a combination approach *(mixed method*), so that the processing of data, there are two studies that mutual support between qualitative with quantitative. Opinions [1] that is a combination of research approaches in research that combines or links between quantitative research methods and qualitative.

Research subject

In accordance with the purpose of this research, the research was done in one of the junior high schools in Tangerang, batch 2013/2014 during the even semester. Since there were 8 classes that had been grouped by the school, the would be only two classes choses through cluster random sampling. One class was assigned as the experimental class, while the other was control class. After assigning the classes, the researcher utilized purposive sampling to see the qualitative data in the experimental class only since this comes in line with the third and fourth problem.

Data collection technique

There are four technics of qualitative data collections, there are observation, interview, documentation, and mixed/triangulationn [11] According to [8] in qualitative research, data collection is conducted in the natural setting, primer data source, and the data collection itself is mostly participant observation, in depth interview, and documentation.

Data analysis technique

The research design used specifically is concurrent triangulation (combination of qualittative and quantitative equally). According to [1] this model or strategy is the model that is the most familiar compared with other combined models. This model will utilize quantitative and qualitative methods altogether for both data collection and analysis

processes, for later on the researcher can determine which data can be merged and separated. The research in this model will be done in one stage but using both qunatitative and qualitative methods at the same time. The content between the quantitative method and qualitative method is supposed to be even, but the content of each method might differ from each other in the practice. The data integration is done in the data preparation, interpretation, and discussion.

RESULT AND DISCUSSION

The average results, minimum score, maximum score, standart deviation, and variants for all pre-test, post-test, and gain of the students' abstraction skillin the experimental and control class can be seen in the Table 1 below.

Table 1. Statistik Deskriptif Skor Pretes, Postes and *Gain* Tes

Statistis	***Kelompok Eksperimen***			***Kelompok Kontrol***		
	Pretes	***Postes***	***Gain***	***Pretes***	***Postes***	***Gain***
N (banyak siswa)	40	40	40	41	41	41
Skor Minimum	19	47	0,02	15	20	0,01
Skor Maksimum	72	135	0,84	70	120	0,76
Jumlah Skor	1750	3223	14,02	1752	3088	12,28
$\bar{x}$ (rata-rata)	43,4	81	0,35	42,1	70,1	0,25
Simpangan Baku	12,3	21,2	0,17	15,91	25,2	0,22
Varians	153,64	452,3	0,031	253,2	636,7	0,048
SMI				**150**		

Based on Table 1, it is known that there are 40 students in the experimental class and 41 students in the control class with 43,4 as the average score for the experimental class and 42,1 for the contro class , included 12,3 and 15,91 as the standard deviation for each class respectively for the pre-test.

The abstracting skill scores from both classes in the pre-test tend to be very low, which is not surprising, as the material used in the questions have been taught to the students. There is a difference in the average of the experimental class and control class, but not really significant, since both classes' standart deviations are considered very low. Standard deviation of the control class is higher than the experimental classs. This shows that the abstracting abilities in the control class is more varied than the experimental class.

Overview of Mathematical Abstraction of Students Using Concrete Representational Abstract Approach (CRA)

At the Concrete stage the teacher performs apperception, beginning to form a group to observe through the Discussion Sheet that has been given. Based on the results of data field note field students showed that at this stage students make observations by using concrete objects and write the results of observations into the discussion sheet.

The first aspect is identifying the characteristics of the object through direct experience is the basic component of the abstraction process. All scholars agree with this, because the process of abstraction requires the student's experience of objects or objects directly in the real world representing abstract objects, an opinion which clearly describes this indicator is [7]; Piaget ([6]; [3]) in [12] which says abstraction requires experience or direct experience.

A very visible activity for the activity mentioned the elements of prism and ??limas?? are students touching the ribs ABCDE, ABT, BCT, CDT, DET, EAT to be able to find the side of the pentagon lime, then they will trace the vertex of the pyramid lime to know which part will form the diagonal of the base side or form a diagonal plane.

Figure 1. Students working on Discussion Sheet 1

Based on Figure 1 can be explained, that the students make observations of the concrete objects first, before they find the elements associated with the concrete object. This is their first step before being faced with problems related to everyday life. Can be seen in the picture above, from student observation. Students mention elements of the corner, rib, diagonal of the base side, the diagonal plane, the diagonal of the space, and represent the observed results into the image form corresponding to the observed concrete object.

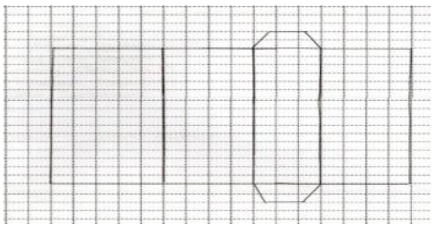
From the way the students to complete the discussion sheet, it can be seen how the students identify in the direct experience, then will be supported by data that students complete the student worksheet 1 in accordance with the aspects of identifying the properties of objects with direct experience.

Bades on Figure 2 can be seen that student worksheet 1 related to the surrounding environment, this aspect also appears in the students, through the image of students can identify the properties of objects in direct experience in accordance with the problem of student worksheet 1. Seen from the students mention the elements or properties of the requested space wake, such as mentioning the side of the pyramid or prism, the corner of the pyramid or prism, the rib of pyramid or prism, the diagonal of the base side, the diagonal of space and the diagonal plane.

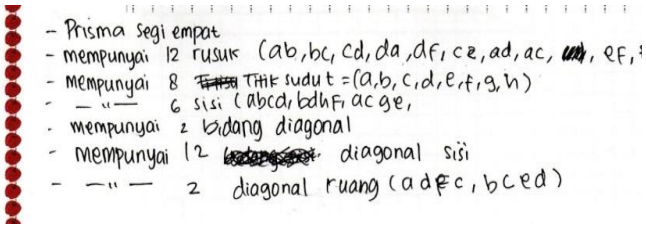
Student Abstraction Process at Representational Stage

At the Representational stage the teacher provides an opportunity for students to plan for problem solving in the discussion process. By giving directions to students to understand the contents of the student worksheet presented. The activity observed at this stage is to identify objects by manipulation.

This aspect begins to appear when students complete student worksheet 2, there are some questions that require students to manipulate by imagining an object wake up space so that students can form a web-nets. Before students do student worksheet 2, students are lured by observing concrete objects, and writing them down into discussion sheets. Here's a picture of how students make observations and observations in the discussion sheet:



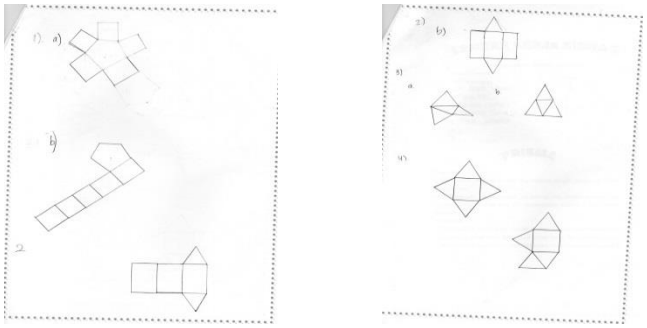
(a)



(b)

Figure 2(a) and (b). Student Work Results on Discussion Sheet 2

Can be explained from Figure 2a ??dan?? 2b that the students make observations, by cutting out concrete objects in order to know the shape of the webs of the concrete objects, then write the observations into the discussion sheet 2. In the observation results can be seen that students draw up the space that has been observe and draw the form of webs that have been observed and mention the various information from the space such as space, how many sides, how many vertices, how many ribs, and other info on the build space that has been observed.



(a) (b)

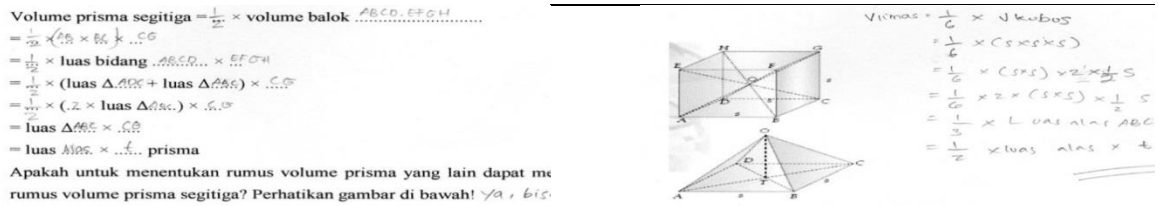
Figure 3 (a) and (b). Student Work Results on Student worksheet 2

Based on Figure 3a and 3b can be seen that students mention various nets. Where students have to manipulate and imagine a wake up space contained in the matter of student worksheet 2 in order to form the requested webs. The webs in the drawings are like triangular prisms, triangular pyramids, quadrilateral quadrants, and a pentilous prism without cap.

Student Abstraction Process in Abstract Stage

In the abstract stage, students are required to change the problem from concrete observation into the form of an image and then solve any matter related to the material into abstract mathematical form. Students conduct discussions, writing, drawing, asking, reasoning, and finding solutions to problems. this can stimulate students to do good activities during the learning takes place. The result of the group disc is written on the answer sheet of the student worksheet. The problems in the student worksheet relate to the indicators of mathematical abstraction ability of the students studied. The role of the teacher as a facilitator and guide the students when experiencing difficulties.

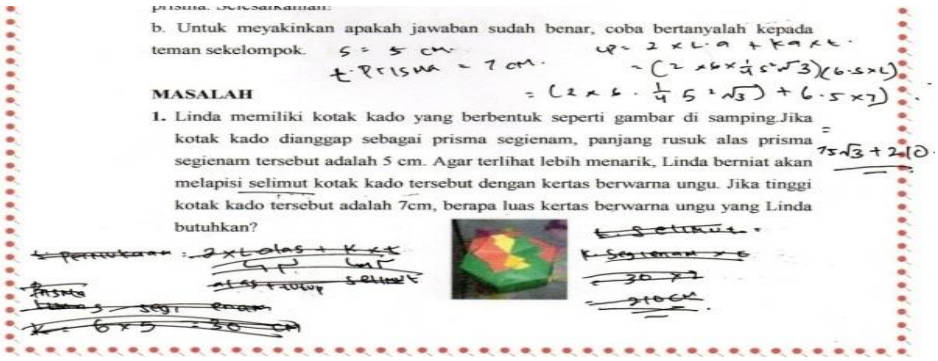
This aspect occurs during the fifth and sixth encounters, about the volume material of prism and pyramid. While the material will be studied then the students will be lured with a discussion sheet 4 at the fifth meeting, the following is the result of the fourth discussion sheet to lure students to work on student worksheet 5 at the fifth meeting and student worksheet 6 at the sixth meeting: Furthermore, this aspect can also be seen from student worksheet 5 and student worksheet 6, where students can deduce the volume of prism and pyramid. Here is the workmanship of students VIII-C to student worksheet 5 and student worksheet 6:



(a) (b)

Figure 4.
(a) Student Work Results on Student worksheet 5
(b) Student Work Results on Student worksheet 6

It can be seen from Figure 4a and 4b that the work of student worksheet 5 on the volume of prism and student worksheet 6 on volume 6. Viewed from the students' work, that they concluded to find the volume of prism from the volume of the beam, while the students also look for the volume of pyramid from the volume of the cube. In the first question the student must answer part a and b on a problem, then reply to the section looking for the volume of the prism from half the volume of the beam. Continuing student worksheet 6 at the next meeting on pyramid volume, the student has to answer first the height of a pyramid lime is in the cube and there are 6 congruent pyramids in the cube. So that students can continue the work of finishing volume of pyramid with one-sixth volume of cube.


b. Untuk meyakinkan apakah jawaban sudah benar, coba bertanyalah kepada teman sekelompok.

MASALAH

1. Linda memiliki kotak kado yang berbentuk seperti gambar di samping.Jika kotak kado dianggap sebagai prisma segienam, panjang rusuk alas prisma segienam tersebut adalah 5 cm. Agar terlihat lebih menarik, Linda berniat akan melapisi selimut kotak kado tersebut dengan kertas berwarna ungu. Jika tinggi kotak kado tersebut adalah 7cm, berapa luas kertas berwarna ungu yang Linda butuhkan?

Figure 5. Student Work Results on Student worksheet 3 and 4

Based on Figure 5 that the work of students with student worksheet 3 and student worksheet 4, it can be seen that students perform various representatives in the form of symbols to answer the questions asked about the surface area of prism and pyramid. From issues related to the application of everyday life, students must seek information in order to find the required surface area, as well as complete the surface area formula. From the work of the student worksheet students have done the mathematical ideas in language and mathematical symbols.

CONCLUSION

Based on the analysis result of research conducted in one of the State Junior High School in Tangerang to grade VIII batch 2013/2014 even semester, some conclusions can be formed: abstraction process in students learning with CRA during the class is an empirical abstraction process and the dominant aspects are aspects that identify the characteristics of objects through direct observation.

Based on the conclusions explained above, the researcher would like to give some suggestions which are: (1) CRA approach help students more in identifying characteristics of objects through direct experience (2) Further research in bigger fields such as material, population or other mathematic competences using CRA approach is suggested (3) CRA approach is one of the variations in teaching mathematics. Based on the enalysis, this can be applied in 2013 curriculum (4) To the interested researchers, it is suggested to conduct researches about abstraction skill that is related with another skill, such as critical thinking, creative thinking, or other competencies, such as learning results, learning achievements, etc (5). It is also suggested to the researchers who are interested in this matter to examine the impact of the abstraction from various learning methods, which can be an alternative to improve one of the weak aspect of abstraction.

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