

Students' Metacognition in Problem Solving of Trigonometric Identity in term of Learning Styles

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Abstract. Problem solving ability lies at the heart of mathematics education. Metacognition has an important role in regulating and controlling people cognitive in solving problems. This study aims to describe student's metacognition in solving trigonometry identity problems in term of learning styles. Learning styles are classified into three types, namely visual, auditory and kinesthetic learning styles. This study involved the senior high school tenth grade students, the subjects chosen were three female students which are SV (visual subject), SA (auditory subject), and SK (kinesthetic subject). All subjects were given problem solving task related trigonometry identity, and then were interviewed. Our results showed that at stage understanding problem, SV and SK did planning, monitoring, and evaluation, while SA did planning and evaluation without monitoring in understanding the problem. The subjects SV and SK monitored what they were thinking by reviewed and enactive what they have thought was appropriate and correct. The subject SV and SA more modest in presenting proofing answer, while SK was more detail and systematic. When devising a plan of execution, all subjects did the planning, monitoring, and evaluation of the plan to be carried out. In carrying out the plan, all subjects finished the task by planning what they have done before and plan for the next, monitored what was being done and evaluated to ensure that what have finished was appropriate. When looking back, SK was more detail and systematic rechecking the final answer than SV and SA which were simpler and shorter.

INTRODUCTION

Metacognition is of particular importance in the process of mathematical problem solving [1]. Metacognition comes from the words "meta" and "cognition", meta is the Greek word meaning after or after, cognition is a process of gaining knowledge [2]. Metacognition is the process by which students monitor, assess, and modify their own learning progress, it can help students develop their knowledge to teach themselves and improve positive learning of new rules and knowledge [3]. Metacognition is another important aspect of children's learning [4]. Metacognition and problem solving have a very close relationship. Metacognition in problem solving can help problem solver identify problems to solve, help to see what the problem really is, and help better understand how to achieve goals or solutions [5]. The metacognition aspect involves the knowledge and skills students need to understand and direct control over their cognition [6].

Metacognition has an important role in the learning process, especially problem solving. With the use of metacognition, students will be aware of the process of thinking. This will minimize student errors, so students can develop appropriate strategies to solve problems [7]. Problem solving can be done well if it involves both metacognition and cognition processes [8]. The most important objectives of mathematics learning are to improve students' abilities, mathematics achievement and develop students' skills in solving math problems [9]. Associated with mathematical problem solving, Problem solving can build a person's understanding of a concept, and increase his ability to understand new knowledge [10]. Polya states that there are two types of problems that are problems to find and problems to prove [11]. Proving is a very important part of mathematics. That is because mathematics is developed through theorems that are proved true [12]. According to mathematicians and educators, evidence and proof are important mathematical activities for students [13]. The reasons for this study are: (a) the learning of a body of mathematical knowledge and gaining insight into why assertions are true [14]. A mathematical proof is a formal way of expressing the particular kinds of reasoning and justification [15]. Proof is a set of logical

arguments to show the truth of a statement [16]. Evidence communicates the mathematical knowledge that students have acquired, so that knowledge lies in an organized structure [14]. In mathematics learning, the problem of proof one of them is encountered in the material of trigonometric identity. Trigonometric identities are identities involving the trigonometric functions [17]. The low absorption of students on trigonometric identity material shows that students still have difficulty in studying the material. Therefore, indirectly indicates that students also have difficulty in solving the problem of proving trigonometric identity.

Each individual has a different style in processing an information. Learning styles are the way in which individuals concentrate on process, internalize, and retain new and difficult information [18]. Learning or thinking styles refer to the preferred way of individual processes information and also describe a person's typical mode of thinking, remembering or problem solving. There are several perspectives about learning-thinking styles. Two of which are the sensory preferences and the global analytic continuum. Sensory preferences that are visual inputers, auditory learners and tactile or kinesthetic learners [19]. A person's learning style is a combination of how he absorbs, organizes and processes information [20]. If someone knows his learning style, then he will learn faster and easier. There are three sensory modalities that individuals use in processing information, namely visual, auditorial, and kinesthetic modalities (known as the V-A-K (Visual-Auditorial-Kinesthetic) learning modality and hereafter called the V-A-K learning style [20].

There are three relationships between learning styles and metacognition. First, knowledge involves information processing; This implies active monitoring and arrangement in information processing activities to achieve certain goals, such as learning. Second, awareness of one's own habits for a particular process and a particular learning modality is the initial condition for metacognition. Third, teachers 'awareness of students' learning styles has a significant impact on teaching learning [21].

METHOD

This study aims to describe the process of metacognition of students when solving the problem of proving trigonometric identity in terms of learning style. This study involved the senior high school tenth grade students This research is a descriptive research that is qualitative. To obtain an in-depth description of the data, students are given problems about proving trigonometric identities and being asked to do so. The result data of the problem solving and the interview result are analyzed in depth then described.

Table 1. Student Metacognition Indicators in Solving Trigonometric Identity Verification Problems

Problem Solving Stages	Metacognition Activities	Indicator
Understanding the problem	<i>Planning:</i> Plan thinking activities in developing planning when understanding the problem	Think about how to understand the problem by reading, drawing or other representations Determine how to identify what is known and asked
	<i>Monitoring:</i> See progress / progress of planned activities when understanding the problem	Convince himself that the way to understand the problem is correct Estimating the time needed to solve the problem
	<i>Evaluation:</i> Perform an assessment of the process and the outcome of his thinking while understanding the problem	Think of a more effective and efficient alternative way of understanding the problem
Creating a trigonometric identification problem verification plan	<i>Planning:</i> Plan thinking activities in developing planning when planning	Think about the strategic plan to be used Think of usable knowledge
	<i>Monitoring:</i> View progress / progress of planned activities during planning	Convince himself that the strategy plan to be used is in accordance with the purpose of the problem
	<i>Evaluation:</i> Perform an assessment of the process and the outcome of his	Re-check whether the knowledge to be used can solve the problem

Problem Solving Stages	Metacognition Activities	Indicator
	thinking at the time of making the plan	Think of a more effective and efficient alternative strategy plan from the chosen strategy plan
Implement the trigonometric identification problem verification plan	<i>Planning:</i> Plan thinking activities in developing planning when implementing the plan	Thinking about the implementation procedure of the strategic plan
	<i>Monitoring:</i> View progress / progress of planned activities during implementation of the plan	Convince himself that the implementation procedure of the strategy plan is done right Ask yourself if the use of the knowledge you have to solve the problem is correct
	<i>Evaluation:</i> Perform an assessment of the process and the results of its thinking while executing the plan	Re-examine the correctness of the execution procedure of the strategy plan in solving the problem
Looking back the result of trigonometric identification problem verification	<i>Planning:</i> Plan thinking activities in developing planning at the time of checking again	Think about what needs to be checked again Think about what needs to be checked again
	<i>Monitoring:</i> View progress / progress of planned activities during checking back	Convince himself that the way back is done is correct Ask yourself whether to check back more thoroughly
	<i>Evaluation:</i> Perform an assessment of the process and the results of his thinking at the time of checking again	Think of a more effective and efficient alternative way of checking again

Resources: Adapted of Cohors & Kaune [22] & Shetty [23].

RESULTS AND DISCUSSION

This study involved the senior high school tenth grade students, the subjects chosen were three female students which are SV (visual subject), SA (auditory subject), and SK (kinesthetic subject). The three study subjects were selected based on the learning style questionnaire and the mathematical ability tests given. Based on the results of mathematical ability tests, the three subjects have high mathematical abilities, and they have obtained the material of trigonometric identities.

SV (Visual Subject)

SV can solve the problem of proving trigonometric identity well, although SV has not overall understood the correct way of examination when viewed from the results of his work. At the stage of understanding the problem, SV performs the three components of metacognition, ie planning, monitoring and evaluating. This is shown in the answer given by SV when interviewed

- R : What do you think of when you read the questions?
 SV : I am looking for a way to identify what is known and asked from the problem
 R : How do you identify it?
 SV : To find out, I read about three times
 R : Are you sure your way is right?
 SV : Yes sure
 R : How long can you solve this problem?
 SV : my estimate is about 30 minutes
 R : Do you think there is another way to identify the problem?
 SV : After I think about it, nothing.

For the next stage, the stage of making a problem-solving plan and its implementation, SV also perform planning, monitoring, and evaluation. Even if viewed from the results of his work, SV does not look to write the complete proof of action, but the result is true work. This means that SV has understood the concept and properties of trigonometric identities well.

Penyelesaian:

$$1. 2 \csc x - \cot x \cdot \cos x = \csc x + \sin x$$

$$2 \frac{1}{\sin x} - \frac{\cos x}{\sin x} \cdot \cos x$$

$$= \frac{2 - \cos^2 x}{\sin x} \rightarrow \sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$= \frac{2 - (1 - \sin^2 x)}{\sin x}$$

$$\frac{2 - 1 + \sin^2 x}{\sin x}$$

$$= \frac{1 + \sin^2 x}{\sin x}$$

$$= \frac{1}{\sin x} + \frac{\sin^2 x}{\sin x}$$

$$= \frac{1}{\sin x} + \sin x$$

$$= \csc x + \sin x$$

Figure 1. Result of SV in Trigonometric Identity Test

R : After you finish doing this, what are you doing?

S1V : I checked it again.

R : How do you check it out?

S1V : I checked my work from start to finish, but only briefly.

At the stage of looking back the results of proof, SV perform planning, monitoring, without and evaluation is good. In the re-examination phase, SV is not very careful in checking the results of his work, because he is sure his job is correct. Visual subjects carry out all elements of metacognition, ie planning, monitoring, and evaluation. To facilitate SV in working on the problem, SV wrote the trigonometric identity formula that will be used in solving the problem. The visual student absorbs the information better when viewing [24].

SA (Auditory Subject)

SA can solve the problem of proving trigonometric identity well. When viewed from the results of his work, SA know concepts, understanding of the nature of trigonometric identities, and she has knowledge of algebraic operations, so that SA can solve the given problem.

Penyelesaian:

$$1. 2 \csc x - \cot x \cdot \cos x = \csc x + \sin x$$

misal kiri = $2 \csc x - \cot x \cdot \cos x$

$$= 2 \frac{1}{\sin x} - \frac{\cos x}{\sin x} \cdot \cos x$$

$$= \frac{2 - \cos^2 x}{\sin x}$$

$$= \frac{2 - (1 - \sin^2 x)}{\sin x}$$

$$= \frac{2 - 1 + \sin^2 x}{\sin x}$$

$$= \frac{1 + \sin^2 x}{\sin x} = \frac{1}{\sin x} + \frac{\sin^2 x}{\sin x}$$

$$= \csc x + \sin x \text{ (misal kanan terbukti)}$$

Figure 2. Result of SA in Trigonometric Identity Test

At the stage of understanding the problem, SA did planning and evaluation without monitoring. This is seen in the answers given by SA when interviewed.

- R : When reading what you think about?
SA : I understand what is known from the problem
R : How do you understand it?
SA : I read the questions carefully and slowly
R : Are you sure your way is right?
SA : Not sure yet, but I keep using it that way
R : Do you think there is another way to identify the problem?
SA : Nothing.

At the stage of making the completion plan, SA performs planning, monitoring, and evaluation, successful when creating problem-solving plans, this is because SA understanding of the algebraic operations that will be used in the proof.

- R : Once you know what is known from the problem, what do you do?
SA : I remember the formula of trigonometric identities that can be used to prove this?
R : Do you remember?
SA : Just a few that I remember.
R : Are you sure the identity formula you will use can help you?
SA : Yeah sure
R : Is there any other knowledge you have, and can be used to prove?
SA : Yes, algebraic manipulation, but it's difficult

SA performs planning, monitoring and evaluation at the stage of implementing a problem-solving plan. However SA has difficulty in solving this problem. But SA can solve this verification problem correctly.

At the stage of looking back the results of proof, SA still perform planning, monitoring, and evaluation. In this stage, SA examines the results of its work very quickly and simply. The auditory subject in proving the answer is simpler and tends to be not systematic in writing the answer on the answer sheet. SA tends to feel comfortable prove the answers directly or tell the story directly rather than writing the proofs in the answer sheet [25].

SK (Kinesthetic Subject)

SK can solve the problem of proving trigonometric identity well and complete. And SK also has understood the correct way of examination when viewed from the results of his work. The evidence that has been done shows that the SK is able to perform the steps of proof with a systematic and thorough.

At the stage of understanding the problem and preparing the problem-solving plan, SK performs planning, monitoring and evaluating. SK performs it very systematically and carefully, It is seen in the answer given by SV when interviewed.

- R : What do you think when you get this problem?
SK : I read it carefully
R : Why did you read it?
SK : In order for me to know what is known and the problem matter
R : Once you know the command of the problem and what is known from the problem, what do you do?
SK : I think of some trigonometric identities that can be used to solve problems
R : Is it enough with trigonometric identities to solve this problem?
SK : Not enough, should use algebraic manipulation as well.
R : Do you believe that knowledge can help you solve this problem?
SK : Yes sure
R : Do you think there is another way to identify the problem?
SK : There may be. But I use this way first

Penyelesaian:

$$2\csc \alpha - \cot \alpha \cos \alpha = \csc \alpha + \sin \alpha$$

$$\frac{2}{\sin \alpha} - \frac{\cos \alpha}{\sin \alpha} \cdot \cos \alpha$$

$$\frac{2}{\sin \alpha} - \frac{\cos^2 \alpha}{\sin \alpha}$$

$$\frac{2}{\sin \alpha} - \frac{(1 - \sin^2 \alpha)}{\sin \alpha}$$

$$\frac{2 - 1 + \sin^2 \alpha}{\sin \alpha}$$

$$\frac{1 + \sin^2 \alpha}{\sin \alpha}$$

$$\csc \alpha + \sin \alpha$$

Figure 3. Result of SK in Trigonometric Identity Test

In the looking back stage, SK performs planning, monitoring and evaluation well. In this stage, SK looking back the results of its work very thoroughly and systematically.

- R : After you finish doing this, what are you doing?
 SK : I checked it again.
 R : How do you check it out?
 SK : I checked my work from start to finish.
 R : Are you sure you've checked everything?
 SK : Yeah sure, I've checked it carefully and systematically.

SK looking back the results of its work from start to finish. So if there is a mistake on the job will soon be known and can be fixed again. The next stage, the stage of implementing the problem-solving plan, SK also perform planning, monitoring, and evaluation. When viewed from the results of his work, SK looks to write the steps of proof in a complete and systematic and the result is true work. This means that SK has understood the concept and nature of trigonometric identity well and mastered algebraic manipulation.

The kinesthetic subject is more systematic in solving the problem, SK tends to do everything that allows his hands to be active [20]. To check the results of evidence that has been done, SK proves from the other segment, if obtained the correct results and proven, then the verification is done is correct.

CONCLUSION

Visual subjects carry out all elements of metacognition, ie planning, monitoring, and evaluation. To facilitate SV in working on the problem, SV wrote the trigonometric identity formula that will be used in solving the problem. The visual student absorbs the information better when viewing. The auditory subject in proving the answer is simpler and tends to be not systematic in writing the answer on the answer sheet. SA tends to feel comfortable delivering verification of answers directly. The kinesthetic subject is more systematic in solving the problem, SK tends to do everything that allows his hands to be active. SV and SK did planning, monitoring, and evaluation, while SA did planning and evaluation without monitoring in understanding the problem. The subjects SV and SK monitored what they were thinking by reviewed and enactive what they have thought was appropriate and correct. The subject SV and SA more modest in presenting proofing answer, while SK was more detail and systematic. When devising a plan of execution, all subjects did the planning, monitoring, and evaluation of the plan to be carried out. In carrying out the plan, all subjects finished the task by planning what they have done before and plan for the next, monitored what was being done an evaluated to ensure that what have finished was appropriate. When looking back, SK was more detail and systematic rechecking the final answer than SV and SA which were simpler and shorter

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