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Analysis of Students' Error on Quadratic Factoring

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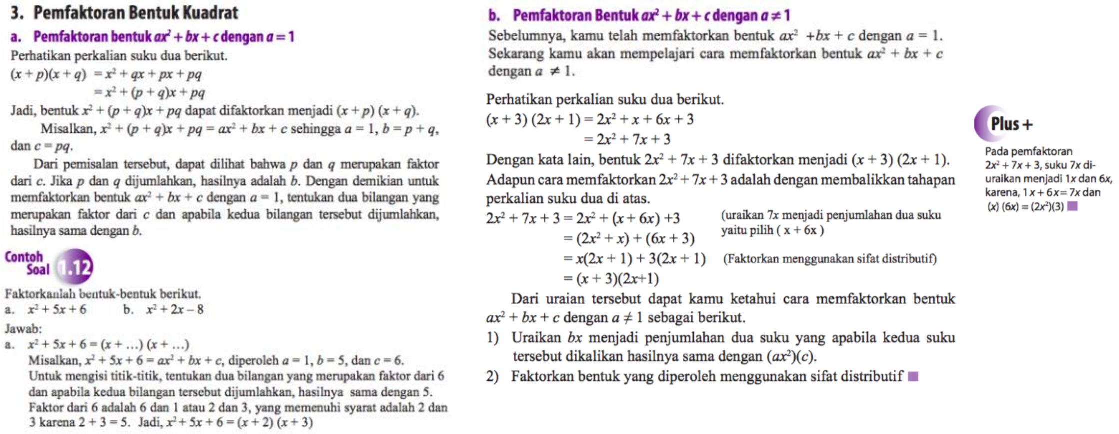
Abstract. This study aims to understand students' mistakes in factoring the quadratic form. Eight questions of quadratic formwork are given to students to identify the students' thinking processes in factoring quadratic forms. After the answers were analyzed based on the structure of the factored method used. Mistakes that occur in students caused misunderstanding of the term that exist in the quadratic form. Students who forgot the rules in factoring actually make their own rules in factoring.

INTRODUCTION

The error of junior high school students in factoring quadratic has been reported by some previous studies [1-3]. The cause of the error is that students cannot understand two types quadratic form based on x^2-term coefficients [1-3]. Thus, the student incorrectly determines the appropriate factoring procedure for each type of quadratic form [1-3]. However, the results of previous studies have not yet explained how students make mistakes and what the answer of their problem. This understanding is important, because the way of thinking and students' factoring procedures can be used as information to develop curriculum design and textbooks of mathematics subjects and teacher guidance in teaching factoring.

METHOD

This study was conducted on 8th grade students of SMP with 24 subjects who have studied factoring. The factoring method used is the method presented in the textbook of Indonesia 2006 Curriculum (Figure **1**). Eight problems in quadratic factoring was tested for 40 minutes (Table 1). After the student completes the problem, students' answers are assessed by giving a score of 0 for wrong answer and 1 for correct answer. Some students who head wrong answers were interviewed to get the rules used in answering the question.



3. Pemfaktoran Bentuk Kuadrat

a. Pemfaktoran bentuk $ax^2 + bx + c$ dengan $a = 1$

Perhatikan perkalian suku dua berikut.

$$(x + p)(x + q) = x^2 + qx + px + pq$$
$$= x^2 + (p + q)x + pq$$

Jadi, bentuk $x^2 + (p + q)x + pq$ dapat difaktorkan menjadi $(x + p)(x + q)$.

Misalkan, $x^2 + (p + q)x + pq = ax^2 + bx + c$ sehingga $a = 1$, $b = p + q$, dan $c = pq$.

Dari pemisalan tersebut, dapat dilihat bahwa p dan q merupakan faktor dari c. Jika p dan q dijumlahkan, hasilnya adalah b. Dengan demikian untuk memfaktorkan bentuk $ax^2 + bx + c$ dengan $a = 1$, tentukan dua bilangan yang merupakan faktor dari c dan apabila kedua bilangan tersebut dijumlahkan, hasilnya sama dengan b.

Contoh Soal 1.12

Faktorkanlah bentuk-bentuk berikut.

a. $x^2 + 5x + 6$ b. $x^2 + 2x - 8$

Jawab:

a. $x^2 + 5x + 6 = (x + \ldots)(x + \ldots)$
Misalkan, $x^2 + 5x + 6 = ax^2 + bx + c$, diperoleh $a = 1$, $b = 5$, dan $c = 6$.
Untuk mengisi titik-titik, tentukan dua bilangan yang merupakan faktor dari 6 dan apabila kedua bilangan tersebut dijumlahkan, hasilnya sama dengan 5.
Faktor dari 6 adalah 6 dan 1 atau 2 dan 3, yang memenuhi syarat adalah 2 dan 3 karena $2 + 3 = 5$. Jadi, $x^2 + 5x + 6 = (x + 2)(x + 3)$

b. Pemfaktoran Bentuk $ax^2 + bx + c$ dengan $a \neq 1$

Sebelumnya, kamu telah memfaktorkan bentuk $ax^2 + bx + c$ dengan $a = 1$. Sekarang kamu akan mempelajari cara memfaktorkan bentuk $ax^2 + bx + c$ dengan $a \neq 1$.

Perhatikan perkalian suku dua berikut.

$$(x + 3)(2x + 1) = 2x^2 + x + 6x + 3$$
$$= 2x^2 + 7x + 3$$

Dengan kata lain, bentuk $2x^2 + 7x + 3$ difaktorkan menjadi $(x + 3)(2x + 1)$. Adapun cara memfaktorkan $2x^2 + 7x + 3$ adalah dengan membalikkan tahapan perkalian suku dua di atas.

$2x^2 + 7x + 3 = 2x^2 + (x + 6x) + 3$ (uraikan $7x$ menjadi penjumlahan dua suku yaitu pilih $(x + 6x)$
$= (2x^2 + x) + (6x + 3)$
$= x(2x + 1) + 3(2x + 1)$ (Faktorkan menggunakan sifat distributif)
$= (x + 3)(2x+1)$

Dari uraian tersebut dapat kamu ketahui cara memfaktorkan bentuk $ax^2 + bx + c$ dengan $a \neq 1$ sebagai berikut.

1) Uraikan bx menjadi penjumlahan dua suku yang apabila kedua suku tersebut dikalikan hasilnya sama dengan $(ax^2)(c)$.
2) Faktorkan bentuk yang diperoleh menggunakan sifat distributif

Plus +

Pada pemfaktoran $2x^2 + 7x + 3$, suku $7x$ diuraikan menjadi $1x$ dan $6x$, karena, $1x + 6x = 7x$ dan $(x)(6x) = (2x^2)(3)$

FIGURE 1. Factoring method in certain text book [4]

TABLE 1. Instrument

Question number	Quadratic form and its factors
1	$x^2 + 7x + 12 = (x + 4)(x + 3)$
2	$x^2 - x - 12 = (x - 4)(x + 3)$
3	$6x^2 + 5x - 6 = (3x - 2)(2x + 3)$
4	$2x^2 + 8x + 6 = (2x + 2)(x + 3) = 2(x + 1)(x + 3)$ $= (x + 1)(2x + 6)$
5	$x^2 - 9x + 14 = (x - 7)(x - 2)$
6	$x^2 + 8x - 9 = (x - 1)(x + 9)$
7	$9x^2 - 12x + 4 = (3x - 2)(3x - 2) = (3x - 2)^2$
8	$3 - x - 2x^2 = (3 + 2x)(1 - x)$

RESULT AND DISCUSSION

The results of the tabulation of student answers are available in Figure 2. From the eight questions worked out, question number 1 is the question most correctly answered by the students. This is because the quadratic coefficient on the problem is equal to one. But unlike the problem of number 2, the negative sign is in the x-term and the constant term in the quadratic form. This negative sign causes the students difficulty in determining the two numbers whose product and sum is negative number. The more terms that have a negative sign, the more difficult it is for students to complete. This case can be seen in questions 2, 5, and 6. This difficulty has also been expressed by other researchers [1-2].

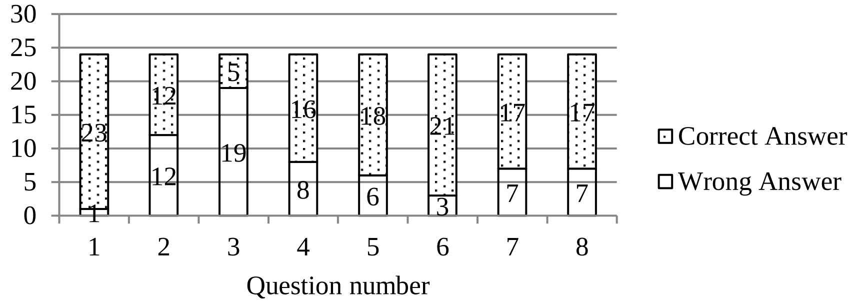


FIGURE 2. Frequency of error of 24 students

From interviews with students who make errors in factoring, authors get information that students who forget or do not understand the factoring procedure, they tend to make their own rules in factoring. The answer they use has the same pattern. Student answer patterns are illustrated in Table 1 for $ax^2 + bx + c$ with $a = 1$, and Table 2 for $ax^2 + bx + c$ with $a > 1$.

TABLE 2. Illustration of students' thinking step about question number 5

Step Order	Illustration
1. Making two parentheses	$(\quad)(\quad)$
2. Putting the variable $\boldsymbol{x}$ on each parenthesis	$(x \quad)(x \quad)$
3. Factoring the constants associated with the degree coefficient of degree one, which then the result of the factor is placed on both parentheses.	$x^2 - 9x + 14$ $2 + 7$ & 2×7 $(x \quad 2)(x \quad 7)$
4. Putting a negative operating sign on the first factor and positive sign of operation on the second factor.	$x^2 - 9x + 14$ $(x - 2)(x + 7)$

In Table 2, students make an error in step 3. The students consider the coefficient of the x-term is 9. This means that the students understand that the negative sign does not involve in the x-term. In this case, students do not understand that there is a term that has a negative coefficient. Therefore, this error continues in step three. The students understand that the $+$ and $-$ sign separated from its terms and was drag down to factoring form. Besides the error, students also divided coefficient on x-term and the constant with a number, whose results are written as the value of both factors, and the sign of operation on both factors is obtained by adjusting the sign of operation on the constant.

TABLE 3. Illustration of students' thinking step about question number 3

Step Order		Illustration
1.	Making two parentheses	$(\quad)(\quad)$
2.	Factoring the constants and put the result of the factor into the second parenthesis.	$6x^2 + 5x - 6$ 3×2 $(\quad)(3 \quad 2)$
3.	Factoring the coefficients of the two-degree terms associated with the coefficients of the first-degree terms, and place them in the second brackets and place the variables x first-degree terms on the first factor.	$6x^2 + 5x - 6$ $2 \times 3 \;\&\; 2 + 3$ $(2x \quad 3)(3 \quad 2)$
4.	Putting a positive sign of operation on the first factor and a negative sign of operation on the second factor.	$6x^2 + 5x - 6$ $(2x + 3)(3 - 2)$

In Table 3, students' errors begin in the second step. The students place two factors from the constants into the second parenthesis. Then, the students seek the factors of the x^2-term coefficient that the sum are equal to the x-term coefficient. The factor is placed on the first parenthesis. The variable x is also only placed on the first factor. Then, the sign of operation on the quadratic form is directly was drag down to each factor.

The errors illustrated in Table 2 and Table 3 occur because the learning process applied by teacher follow the strategy presented in the book as presented in Figure 1. The teacher begins to teach the quadratic form of $ax^2 + bx + c$ where $a = 1$ by giving an example of a simple quadratic form like $x^2 + 3x + 2$. The teacher makes two parentheses, then places two x on each parenthesis. Teacher asks students to pay attention to the coefficients on the x-term and constants to obtain two factors that if both factors are multiplied, then the result is 2 and if both factors are summed up, then the result is 3. The two factors are then placed on the second sign of parentheses, so that $(x + 2)(x + 1)$, but when the teacher teaches the form of $ax^2 + bx + c$ with $a \neq 1$, the teacher tends to introduced the way presented in Figure 3. In fact, the method requires three prerequisites to be fulfilled before: (1) knowing how to devide the x-term into two terms in sum form using the $p + q = b$ and $pq = ac$ rules, (2) using the distributive properties of the two-terms in sum form, and (3) using the distributive properties in the factoring of an algebraic expression involving addition and multiplication. The three prerequisites are not understandable yet by the students The findings of [3] which reported that only 3 of 32 students can complete the factoring $2x(x + 3) - 5(x + 3)$, so students will find difficulties in factoring of $ax^2 + bx + c$ with $a = 1$. Although the teacher has explained that there is a relationship between the factoring of $ax^2 + bx + c$ with $a = 1$ and $a \neq 1$, still the students perceive the two ways are different. This happens because, in the first way, the student can easily change the quadratic form to the multiplication of two linear factors or vice versa, while in the second way, the student must go through the factoring procedure by fulfilling the above three prerequisites to change the quadratic form into two-factor of linear multiplication. The second reason is that the textbook does not clearly show that $p.q = a.c$ in the explanation of factoring of $ax^2 + bx + c$ with $a = 1$. This gives the student's perception that a is not involved in the equation $pq = c$. The type of learning obstacle described above is a didactical obstacle [5].

A solution to anticipate the emergence of similar problems is that the teacher explains the quadratic form of $ax^2 + bx + c$ in a way like section b in the student textbook (Figure 1), since it is generally applicable to any quadratic form. Therefore, students' perceptions do not perceive that there are two different ways to factoring the quadratic form. In addition, the teacher can explain from the factoring with $a = 1$ or begins from the quadratic form of $a \neq 1$. The teacher can also explain the short procedure for quadratic form of $a = 1$. Students should always be reminded to re-examine the results of factoring by multiplying two linear factors obtained. The usual method used to multiply two linear factors is the method (FOIL) First Outside Inside and Last which means multiplying the first term with the first term, the outer side term with the outer term, the inner side term with the inner term, the last term with the last term [6]. In addition to the factoring methods presented in the textbooks in Figure 1, some other factoring methods may be used: AlgeCard, Ractangle Diagram & Box Technique [7-8]. These methods have the advantage of providing visualization of the factoring process. As an enrichment on grouping methods, teachers can give some ideas to facilitate the factoring. One of the strategies of factoring of quadratic form, that is (1) multiplying $ax^2 + bx + c$ with a, (2) let $z = ax$, so the quadratic form becomes $z^2 + bz + ca$, (3) factoring of $z^2 + bz + ca$ to obtain $(z + m)(z + n)$ where $mn = ac$ and $m + n = b$, (4) substituting $z = ax$ to the form $(z + m)(z + n)$ to obtain $(xa + m)(xa + n)$ which is a factor of $ax^2 + bx + c$ [9]. Another strategy of factoring of quadratic form, that is (1) find the factors m and n of ac such that $mn = ac$ and $m + n = b$, (2) simplify the expression $\frac{(ax+m)(ax+n)}{a}$ by factoring the common factors of a and m and of a and n [10].

The methods presented in section b of the student textbook (Figure 1) are commonly referred to as "Grouping Methods", "A-C Methods", "X-Box Methods" and "Diamond Methods" in teaching materials in the United States. This method is good for use because students can use the previous understanding that is to understand the relationship between multiplication expansion of two linear forms with quadratic foctorisation, and students can develop a relational understanding and logic understanding [11]. Looking at the example of problem 1.12 in Figure 1, it is understood that the method presented in part a of Figure 1 shows the factoring method presented only prioritizing operational thinking. Operational thinking is a low-level thinking that is only related to arithmetic processes [12]. However, the method presented in part b in Figure 1 is more likely to promote structural thinking. Structural thinking is the highest-level thinking involving formal algebraic operations [12].

CONCLUSION

The students' misconception in factoring are assuming that the negative sign on a quadratic form apart from the coefficients. This misconception affecting the students to create their own factoring rules. This error occurs because the teacher's instruction only follows the textbook. In fact, textbooks used can cause difficulties for students. Solution to anticipate the problem, teacher only need to explain one factoring method from $ax^2 + bx + c$ for every $a, b, c \in \mathbb{Z}$ and $a \neq 0$. The results of this study are expected to be used to develop the curriculum of preservice mathematics teacher and their lecturing, refreshing the concept of factoring for inservice teachers, as well as designing textbooks and curriculum on junior mathematics subjects.

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