

Literatur Study: The Relationship of Mathematics Problem Solving and Students' Higher Order Thinking Skills

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Abstract— The literature review is focusing on the use of mathematics problem solving in assessing students' higher order thinking. There is evidence that mathematics problem solving is related to critical, reflective, Metacognitive and creative as parts of higher order thinking. Theorizing from these findings, the study describes the relationship of mathematics problem solving and higher order thinking, how mathematics problem solving assesses students' higher order thinking and how mathematics problem solving might develop students higher order thinking. Findings suggest that more research is needed to gain new insight in categorizing mathematics problems that can be used in assessing and developing students' higher order thinking.

Keywords: *higher order thinkin skills, mathematics problem solving*

I. INTRODUCTION

This paper review about the use of mathematics problem solving in assessing and developing higher order thinking. The objective that gaided and motivated this review is our advocacy the use of problem solving in developing students' higher order thinking (HOT) since it is a necessary skills in the future [1]. Moreover, the opportunity to develop students' HOT skills has been shown by researches ([2], [3], [4]). Realizing the important of HOT skills, Indonesia government has conducted several efforts. The efforts are changing curriculum, teacher training and supporting research in education. Developing a learning instructions that can develop student competencies, HOT skills, becomes necessary. Pre-research that needed to be conducted is literature study. Doing a thorough analysis of literature study will provide information about theory and practices that related to development of HOT skills. This analysis can change researcher paradigm, point of view and theoretical perspectives about research area. As a result, the plan research can be guided to a new findings in HOT skills. Therefore, this literature review is conducted in order to find the relationship between HOT skills and mathematics problem solving.

The aim of this study is to review literature that related to HOT skills and mathematics problem solving in order to evaluate what is currently knowledge about how mathematics problem solving assess and develop HOT skills. The evidences are looked specifically in types of problems and reasons why problem solving is used in assessing and developing HOT skills.

II. HIGHER ORDER THINKING SKILLS

Higher order thinking has been defined by many researchers. Byrnes[5] defined higher-order thinking as a way of handling a situation that have not encountered before and is generally recognized as some combination of non-algorithmic, complex, effortful, nuanced judgments, application of multiple criteria, uncertainty about what is known and regulation. The characteristics that encounter of individuals are may vary in many ways, numbers and difficulties.

Regarding Bloom taxonomy, thinking processes that remarked as higher order thinking are synthesize, evaluate, and create. Based on this theory, many researchers defined HOT in a wide variety of definitions. The definition is summarizing as bellows:

1. King, Goodson, and Rohani [6] viewed than HOT skills are critical, logical, reflective, metacognitive and creative thinking. All of this aspects are activated when an individual encounter un familiar problems, uncertainties, questions and dilemma.
2. National Council of Teachers of Mathematics (NCTM) [7] proposed that HOT skills are required in solving non routine problem.
3. Anderson and Krathwohl [8] proposed the same definition as in bloom's Taxonomi which are a process that involve analyze, evaluate, and create.

4. Lopez and Wittington [9] believed that HOT skills Appeared when an individual receives new information and the information mixed such that it caused a new arrangement and extent the individual knowledge.
5. Weiss [10] suggested that HOT will appear when an individual faces a Collaborative, authentic, ill-structured, and challenging problems
6. Miri [11] showed that HOT skills are the strategy-where critical, systematic, and creative thinking are activated as a tactics. These activities are needed in order to get the objectives.
7. Rajendran [12] viewed HOT skills as the way to expand the use of mind in facing new challenges
8. Thomas and Thorne [13] defined that HOT skills require to think to higher levels. It requires that individual should deal with the facts, understand them, connect them each other until achieving new or novel knowledge.
9. Kruger [4] believed that HOT skills required the involvement of concept formation, critical thinking, creativity, problem solving, mental representation, rule use, reasoning and logical thinking.

Based on the definition of HOT skills that have been discussed, the further discussion will focus on several properties of HOT skills which are critical, reflective, metacognitive, and creative thinking.

A. Critical Thinking

Wood [15] Defined critical thinking as process of using reasoning in judging the facts and differentiate what is true and what is false. Critical thinking enables individual to assess and calculate situations and create reasonable conclusion. Lipmann [16] viewed critical thinking as skillful, responsible thinking that facilitates good judgment because it relies upon criteria, is self-correcting, and is sensitive to context. It is believed that in order to accomplish a good judgment of situations or facts an individual utilized reasonable thinking and responsible thinking. Another viewed about Critical Thinking Skills are skills that enable one to analyze and synthesize information to solve problems in broad range of areas [17]. It can be concluded that critical thinking is an ability to calculate and assess facts, situations and problems to formulate a good judgment/solution by using reasonable and responsible thinking.

Critical thinking occurs when students construct meaning by interpreting, analyzing, and manipulating information in response to a problem or question that requires more than a direct, one-right-answer application of previously learned knowledge [18]. This can be characterized by specific core thinking skills, which can be developed in the classroom through instruction and guided practice. The list of applicable skills includes, but is not limited to: focusing, information gathering, referencing, organizing, analyzing, integrating, and evaluation. Butterworth and Thaites [19] the core activities of critical thinking are analysis (identifying the key parts of the problem dan reconstructing it in a way that fully and fairly captures its meaning), evaluation (judging how successful a solution is) and further argument (self-explanatory) how the student's opportunity to give their own response.

B. Reflective thinking

Reflective thinking – as an active, persistent, and careful consideration of any belief or supposed form of knowledge [20]. Taggard and Wilson [21] viewed reflective thinking as process of making informed and logical decisions. Meanwhile, Campbell-Jones & Campbell-Jones [22] recalling one's own experiences, beliefs, and perceptions. The basic stages of reflective thinking

1. Determination of the problem
2. Reflecting the solution to the problem
3. Design of planning for problem solving process
4. Implementation of the designed plan
5. Performing evaluation by using reflection
6. And continuing by restarting the process in [23]

This give us the big picture that problem solving activity activates students or individuals reflective thinking. In solving a problem, individual is required to understand the problem, planning a way to solve it by utilizing the known and unknown information which is need a reflective thought, implementing the idea, and evaluate the result. In evaluating the result stage, an individual uses reflective thinking to looking back to the information, looking back to blue print of the plan, looking back in implementing the plan and the result. Reflective thinking activity will be well developed when the problem that is used is a complex problems. It will trigger an trial and error activities.

C. Metacognitive thinking

Research in metacognition has been done by many researchers. There are several definitions of Metacognition. Metacognition is the mind's ability to understand, reflect, monitor and control itself or, in other words, metacognition is the ability to know about what we know. [55], defined metacognition as individual's ability to know what he/she knows such as strengths and weaknesses, and using it to next level of knowing. Therefore, metacognitive thinking can be viewed as a person ability to understand their thinking, reflect on the knowledge that they have and using it to develop new knowledge.

D. Creative Thinking

Discussing about creative thinking cannot be separated from creativity. James C Kaufman, Plucker, and Baer [25] provide the definition of creativity as interaction among aptitude, process, and environmental. They believe that an individual or a group is called to be creative if they can produce a perceptible product. The product should be both novel and useful as defined within a social context. Moreover, they categorize the definition of creativity into three categories namely creativity as a person, creativity as a process, and creativity as product. Creativity as person, an individual characteristic is viewed as creator. According to Sternberg [26], there are three psychological attributes of a person that interrelated with creativity. The attributes are intelligence, cognitive styles, and personality. The intelligence includes verbal knowledge, thinking flow, planning, formulation of the problem, preparation of the strategy, mental representation, decision-making, and globally intellectual integration. The cognitive styles include an individual act in doing things with his/her ways. Finally, the personality is related to personal factors such as flexibility, tolerance for ambiguity, self-discipline, and willingness to take appropriate risks.

Creativity as process is defined as steps in scientific method which are sensing the difficulties or problems, formulating the hypotheses, evaluating and testing the hypotheses, possibly revising and pretesting, and communicating the results [26]. It means that creativity is defined as an actual experience of being creative in encountering reality problems. An optimal experience occurs when an individual is intensely engaged in an activity.

Creativity as product is related to things people make, ideas, and responses. According to Baron (in Munandar [26]), creativity is an individual ability to produce/create new product. The product must be unique and novel to the creator. In this research, the definition of creativity is focused on creativity as product, especially creativity in mathematics problem solving.

In mathematics field, creativity is defined as an ability to combine logical and divergent thinking which is based on intuition but has a conscious aims [27]. The combination of logical and divergent thinking can be varied in different activities. In problem solving, divergent thinking will produce various ideas that might be possible to solve problems and logical thinking will provide considerations in choosing appropriate idea and in making decision. James C Kaufman et al. [25] summarized that there are four aspects of divergent thinking. First, fluency refers to numbers of responses to given stimuli. Second, Originality refers to uniqueness of responses to given stimuli. Third, flexibility refers to number and/or uniqueness of categories of responses to given stimuli. Fourth, elaboration refers to extension of ideas within a specific category of responses to given stimuli.

III. MATHEMATICS PROBLEM SOLVING

Problem solving is a human activity. Individuals encounter problem in their daily life. Therefore, problem solving become common activity by people. The definition of problem solving should be look carefully before the use of problem solving in assessing and developing HOT skills are discussed.

Many scientists have defined problem solving in many perspectives. [28] defined problem solving as a way to find a solution of a problem, so that a clear answer is established. A problem solver is encouraged to find a way or a structure such that the way or the structure can be used to find goals or solutions of the problem. [29] viewed "problem solving as an individual capacity to use cognitive processes to confront and to resolve cross-disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science or reading". Meanwhile [7] considers problem solving as a process of applying previously acquired knowledge to new and unfamiliar (or unforeseen) situations. Despite of the definition that have been proposed, [30] viewed in the way when a problem solving is needed. It is needed when an individual encounter a question or a problem to be solved cannot be done through routine application/procedure of previously acquired knowledge. But it can be solved only when expanded use of mind occurred that a person must interpret, analyze or manipulate information. Based on the definitions,

problem solving can be defined as an individual capacity to use cognitive processes to solve question or problem that cannot be solved by routine procedure of previously acquired knowledge.

IV. HIGHER ORDER THINKING SKILLS AND MATHEMATICS PROBLEM SOLVING

Higher order thinking (HOT) skills have strong connection with problem solving activity. It is because HOT Skills are activated when individual encounter unfamiliar problems, uncertainties, questions, or dilemmas [5]. In other words, all of the situations that can develop HOT skills are the characteristics of problem solving activity. Research found that both non-routine and open-ended problems give students more opportunities to demonstrate their problem solving skill. This activity can enhance students' Hot skills. However, if the same types of non-routine problems are given to student repeatedly, the essence to foster HOT skills will be lost (Gerald [31]).

Research that conducting by King, Goodson and Rohani [5] utilized problem solving activities both as a learning strategic to develop HOT skills and as assessment instruments. They believed that questions about dilemma, novel problems, and novel approaches that elicit answers are not learned by the students yet can be effective way to build many skills such as verbal analogies logical thinking, and inductive/deductive reasoning. In assessment of HOT skills, there are three tasks that can be used. The tasks are selection, generation, and explanation. Selection includes multiple-choices, matching, and rank order items. Generations includes short answer, essay, and performance item or tasks. Explanation involves giving reasons for selection or generation purposes.

Moreover, there are many resources (books and journal) discussed about HOT skills with problem solving activities that conducted separately to each skill such as critical thinking ([19], [32], [33]), Reflective thinking ([23], [34]), metacognitive ([35], Mayer [36], [37], [38], [39], [40]) and critical thinking ([41], [42], [43]). The problem that is used in these researches are real life problem, open problem, open-ended problem, non-routine problem, and complex problem. These kinds of problems require problem solvers to think harder in unusual ways since there is no exact formula that can be used directly. Placing problem solvers in a unclear and miserable condition that challenge their way of thinking, various ideas and unknown variable will trigger the activating of HOT skills.

Looking to the opportunity that problem solving activities created to develop students HOT skills, further research in designing problem based instruction in class in developing HOT skills become necessary. This is a vast research area since there only few of researches about that is conducted by utilizing problem solving in developing HOT skills. Moreover, every component of HOT skills also needs a further research.

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