ME – 03

A Study of Late Formal-Junior School Student's Geometric Thought in Understanding the Relationship Between Quadrilateral

Agustan S.
Fakultas Keguruan dan Ilmu Pendidikan (FKIP)
Universitas Muhammadiyah Makassar
Indonesia
agus.sahabat@gmail.com

***Abstract*—**Currently, geometric thought is one of the important things which become a concern in learning mathematics, especially in learning of geometry. Learning of geometry must be allowed to the students' geometric thought will enhance the intellectual engagement of students, because it can help the student in directing his thoughts so that the solution of the problem being solved tends to be true and correct. The purpose of this research paper is to describe how geometric thought is used in understanding the relationship between quadrilaterals. The participant of this research is a student who is at the early formal operational stage based on Piaget's development of cognitive enrolled in class VIII SMP, Kabupaten Bone, South Sulawesi, Indonesia. This research is a descriptive explorative study with data analysis using qualitative approach. Qualitative approach is chosen to describe in depth related to student's geometric thought in understanding the relationship between quadrilateral that can be seen from the subject's behavior in completing a given task and semi-structured interviews are administrated to the subject. During the interview, participant is questioned by researcher to investigate subject's geometric thought. To test the credibility of the data, the researcher used triangulation time. The results of this research showed that the student who is at the late formal operational stage based on Piaget's stages of cognitive development stringing up or composing 7 relationships from 15 relationships between quadrilaterals and the student or subject drew various kinds quadrilateral by using 4 attributes i.e. position, the size, shape of quadrilateral and rotational symmetry.

Keywords: *Geometric Thought, Late Formal Stage, Quadrilateral*

I. INTRODUCTION

Geometry is one subject that addresses school math objects associated with spaces of varying dimensions. The concept of geometry itself occupies a special position in the secondary mathematics curriculum because it is closely related to other forms of objects that are often encountered by students in everyday life. Various opinions appeared that discusses geometry both definitions and chances to be taught in SMP [1]. One of the shapes is taught in junior class VIII is a quadrilateral. Topics include the quadrilateral parallelogram, rhombus, rectangle, square, trapezoid, and kite. Muser [2] defines that the types of quadrilateral as follows: A square is a quadrilateral with four sides the same length and four right angles. A rectangle is a quadrilateral with four right angles. A parallelogram is a quadrilateral with two pairs of parallel sides. A kite is quadrilateral with two no overlapping pairs of adjacent sides that are the same length. A rhombus is a quadrilateral with four sides the same length. A trapezoid is a quadrilateral with exactly one pair of parallel sides.

One of the relationships between the various types of quadrilateral described by Soedjadi [3] based on the intention can be seen in Figure 1. below:

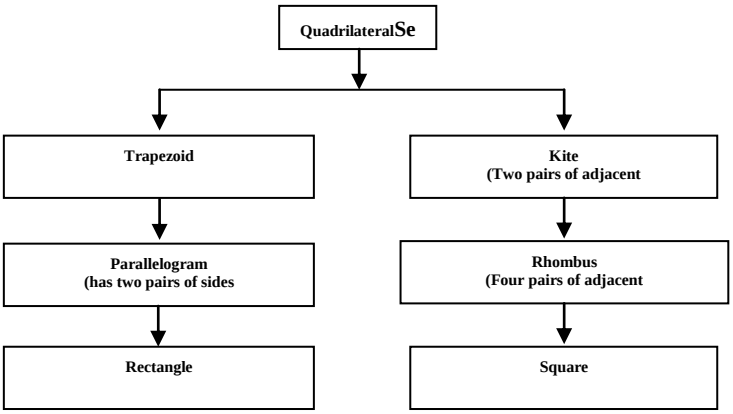

FIGURE 1. THE RELATIONSHIP BETWEEN VARIOUS TYPES OF QUADRILATERALS

Based on the picture above, trapezoid is is defined using the genus proksimum (closest family) "quadrilateral" by adding the terms "has a pair of parallel sides". Thus the trapezoid is is a quadrilateral which has a pair of parallel sides. In a similar manner, a trapezoid is parallelogram which has two pairs of parallel sides and a rectangle is a parallelogram that has a right-angle. Likewise for kite defined as a quadrilateral having two pairs of adjacent sides equal in length. Rhombus is a kite that four adjacent sides of equal length. Square is a rhombus is has a right-angle.

Related to the topic of the quadrilateral as one of the subjects in the field of geometry taught in junior high, Sunardi [4] found many students were wrong in solving the problem of parallel lines. Based on this, the geometry is looked as part of math given to students classified as difficult. Students' s learning difficulties can not be separated from the practice of learning that has been in progress [5]. Idris [6] suggested that learning of geometry is not easy and some students fail to develop an understanding of the concept of geometry, geometric reasoning and skill to solve the problems of geometry. Furthermore, Idris stated that a number of factors that lead learning of geometry is difficult which they are language of geometry, visualization and learning abilities are less effective for the low mastery of facts, concepts and principles of geometry.

According Soerjono [7] one of among the causative factor is the intellectual ability of students. The results of Burger and Shaughnessy's research [8] demonstrated that the intellectual ability of students plays an important role in the mastery of facts and concepts of geometry. Intellectual abilities are spatial ability and auditory ability which are very close relationship with the cognitive aspects of students in general. Research shows that the understanding of spatial knowledge can affect the performance related to academic tasks especially math, reading and science [9]. According to Piaget and Inhelder [9] states that spatial ability which is an aspect of cognition that develops in line with the cognitive development.

Piaget [10] describes the sequence into four stages of cognitive development which is qualitatively different, namely: (1) the stage of sensory motor (2) pre-operational stage, (3) the concrete operational stage, and (4) the formal operational stage. He claimed that all children pass through four stages with different speeds, but none of the children who passed through one of the four stages [10, 11]. Piaget suggested that students should use logical operations to get structure of knowledge and their changes [12]. The more often children move and find new things, the children will increasingly have new schemes that are used to develop their logical operation [11].

Formal operational by Sawyer et al, Dickinson and Lee in Philip Adey split into 2 of the formal operation of early formal operations and late formal operations [12]. Students at the early formal have a good effort to solve the problems by using all its logical operations. The resulting solution is correct but there are little mistakes in using a type of logical operation. They can predict the final answers so that any data and information geared towards achieving that goal. Students at the late formal is able to answer correctly. They show the use of logical operations well. They relate the data and information to resolve pemasalahan. If there is an impasse in resolving a problem they can find another alternative. Children demonstrate a broad understanding of the problem, using a cognitive schema to build understanding of the structure of the problem, as well as choosing the right strategy. By using the scores as a basis and pay attention to the level of understanding of the students showed in solving problems, level of cognitive development of students based on tests of logical operations Piaget classified as shown in Table 1 below.

TABLE 1. LEVEL OF STUDENTS'S COGNITIVE DEVELOPMENT BASED ON TEST OF LOGICAL OPERATION PIAGET

Interval Score	Level Of Students's Cognitive Development
0-35	Early Concret
36-70	Late Concret
71-105	Early Formal
106-140	Late Formal

(Adopt from Loengson dan Limjap, 2003: 13-14)

II. METHOD

This research is exploratory descriptive study with data analysis by qualitative approach which main data in the form of words that are linked into sentences. Qualitative methods is chosen for profile students' understanding of the natural background and the main instrument is the researcher's own research. It means that the data which is analyzed in form descriptive and not in the form of figures as well as in quantitative research.

This research was conducted in Watampone, Bone district, Makassar, South Sulawesi. The participant in this study is student of class VIII SMP. The reason for choosing a class VIII student junior high school student is at the stage of formal operations, so that students are able to think more abstractly, capable of inductive and deductive thinking and be able to think logically.

To determine the stage of cognitive development of students who are at the late formal stage, then used the test instrument Logical Operations Piaget (Piaget TOL) developed by Leongson, JA & Limjap, AA (2003). The instrument is a matter of form description which consists of 35 questions. With reference to the opinion of Schoenfeld and Avalanche & Limjap, researchers determined the students' level of cognitive development by Piaget logical operation test. If the scores obtained by students 106-140, the level of cognitive development of students at the level of late formal.

To obtain valid data in this study, then do the validation data. One of the qualitative research validation procedures that can be performed is by means of triangulation. Validation of the data in this way is done by repeatedly checking with different time. Sugiyono (2005) called the data validation process by triangulation of time.

III. RESULT

The results of task-based interview of the subject on data collection that illustrate understanding of the concept of quadrilateral of student through drawing, identifying and making diagram of relationships between quadrilaterals as follows.

Junior high school students with stage of cognitive development at the stage of **late formal operations** when drawing rectangles, students can draw variety of different rectangular shapes are infinite by paying attention to the attributes of the shape, the size and position of quadrilateral

More clearly on task-based interviews, **the activities of subject when drawing a quadrilateral**, the subject drew quadrilateral by paying attention to characteristics or attributes of quadrilateral drawn as in the following figure 2.

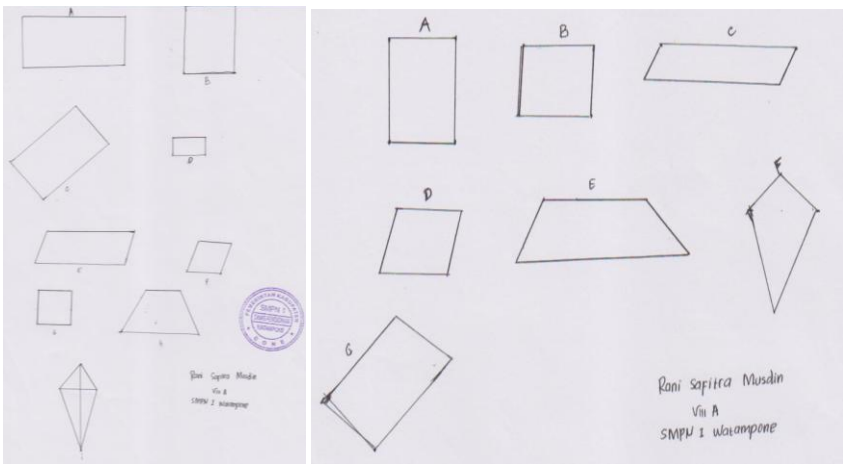

FIGURE 2. VARIOUS QUADRILATERAL DRAWN SUBJECT AT THE INTERVIEW

The following is a transcript of the interview excerpt based on the task of drawing quadrilateral

AG085	P	"Dari gambar bangun segiempat yang berbeda-beda itu, sekarang saya mau tanya. Berapa banyak bangun segiempat berbeda yang dapat kamu gambar?
RS086	R	"Enam"
AG089	P	"Ada enam? Yakin?"
RS090	R	"Yakin"
AG091	P	"Tapi tadi kamu menggambar segiempat lebih dari enam"
RS092	R	"Iya, tapi A,B,C dan D itu sama-sama persegi panjang cuma yang membedakan hadapannya, posisinya dan ukurannya"
AG093	P	Jadi dengan memperhatikan perbedaan tersebut, kembali saya mau tanya Rani, kira-kira Rani bisa menggambar bangun segiempat berapa banyak yang berbeda?"
RS094	R	"Enam"
AG095	P	"Enam, mengapa mereka berbeda satu sama lain?
RS096	R	"Ada yang berbeda posisinya, ada yang berbeda bentuknya, hadapannya dan ukurannya.
AG097	P	"Jadi Rani bisa menyimpulkan dapat menggambar bangun segiempat berapa banyak?"
RS098	R	"Banyak"
AG099	P	"Banyak, maksudnya?"
RS100	R	"Tak terhingga"
AG101	P	"Kok bisa?"
RS102	R	"Karena kita bisa menggambar segiempat beda posisi, beda ukuran, dan beda bentuk dan putaran"

From the transcript of interviews and the results of drawing quadrilateral above, data showed that the subject can draw various kinds quadrilateral by taking into differences in the difference of position, size, shape and rotational symmetry.

The activities of subject when identifying the differential quadrilateral, subjects pays attention three attributes of quadrilateral based on size of the side, kind of quadrilateral (trapezoid) and position While at identifying the same quadrilateral, subject attentioned attribute number of pairs of opposite sides and parallel, the angle of sight, the existence of a right angle which is owned by the rectangle and the size of both the adjacent sides or opposite sides and parallel. Definition of quadrilateral which is made by the subject accurate and inaccurate depending on the definition of quadrilateral used as a reference in defining a quadrilateral.

Clearly explained that the subjects make inferences when:

a. Identifying Parallelogram

1. Student identifies several different models by taking into the attributes of size of parallelogram and the position of parallelogram.
2. Student identifies some of the same characteristics of parallelogram by regarding the attribute size of side and having two pairs of opposite sides parallel and opposite sides of the same length.
3. Referring to the definition of parallelogram is a quadrilateral of which two pairs of parallel opposite sides, or two pairs of opposite sides of equal length, or a pair of parallel opposite sides of the same length, then the attributes given by subject to construct a definition parallelogram is accurate.

b. Identifying Rectangle

1. Student identifies several different rectangular models by taking the atribute of size and the position of rectangle.
2. Student identifies a common characteristic of some rectangles that four corners is right angle, opposite and parallel sides equal in length.
3. Referring to the definition of a rectangle is a parallelogram whose one of the corners is right-angled, the attributes given by subject to construct a definition of the rectangle is inaccurate.

c. Identifying rhombus

1. Student identifies several rhombus in different models with regard to the attributes of size and position of the shape.
2. Student identifies a common characteristic of some rhombus that all sides of rhombus has the same length, opposite angle is equal.
3. If the definition refers to a rhombus is a quadrilateral whose four sides the same length as the attributes of a given by subject to build rhombus definition is accurate.

d. Identifying square

1. Student identifies some square in different models with regard the zise and position of the shape.

2. Student identifies the characteristics of some models of the same square i.e., all sides is equal in length and becomes right angel, opposite sides are parallel and equal in length.
3. If the definition refers to the square is rhombus which one of its angles is right angel or quadrilateral whose four sides the same length and the angle is right angel, then the definition given subject is inaccurate.

e. Identifying Trapezoid

1. Student identifies several trapezoids in different models with regard to side length and size attributes, position and the kinds of trapezoid.
2. Student identifies the characteristics of some models of trapezoids which have one pair of parallel opposite sides and the parallel sides of unequal length.
3. If the definition refers to the trapezoid is a quadrilateral having parallel opposite sides or rhombus which has only a pair of parallel opposite side, then the definition given subject is accurate.

f. Identifying Kites

1. Student identifies several kites in different models with regard to size of the kites and position of kites
2. Student identifies the characteristics of some models of kite is adjacent sides of the same length and opposite sides of unequal length, opposite angle is equal.
3. If the definition refers to a kite is quadrilateral that has two pairs of adjacent sides of the same length and the sides do not overlap, then the definition given by subject is accurate.

In the activities of making diagram of relationship between quadrilateral, subject made diagram relationships between quadrilateral as in Figure 3. The following

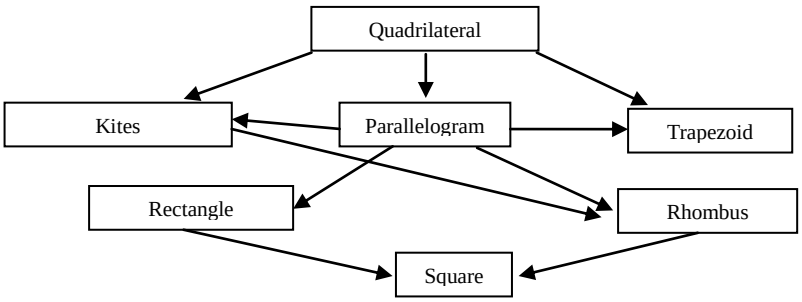

FIGURE 3. RELATIONSHIP OF VARIOUS TYPES OF QUADRILATERAL MADE BY SUBJECT

IV. CONCLUSION

From the results of this study concluded that:

1. If it is viewed from the standpoint of analytical, the definition made by subjects who is at the late formal operation presents relationship 7 relationships between quadrilateral possible. Shrinkage occurs relationship of 7 who may be 15 possible relationship. This shrinkage occurs because there is a genus that are used but not genus proksimum
2. Based on the understanding which is recognized by the subject who is at the stage of late formal operation, trapezoid is a quadrilateral which has a pair of parallel sides and a kite is a quadrilateral which is two pairs adjacent sides has the same length, then these results can be interpreted that the subject makes sense analytically.
3. There are 21 possible relationships between quadrilateral parallelogram, rectangle, rhombus, square, kite, and trapezoid. From 21 of this relationship, there are only 12 probably connections which is made by the subject, it is caused by definition trapezoid is a quadrilateral which h as opposite sides parallel and equal in length.
4. Subject recognized two accurate definition of 6 accurate definition which might and subject made 8 analytical definitions of 8 analytical definitions probably and 6 of them are accurate.

V. SUGGESTIONS

Based on these results, some suggestions have to be submitted as follows:

1. From the results of this research, in general student's understanding who is at the stage of late formal operations can understand the relationship between quadrilateral well but is less able to pay attention to or identify the relationship both similarities and differences of quadrilateral. Therefore, the researchers suggested that the educators must pay attention the stage of cognitive development of students in learning, particularly in understanding the relationship between quadrilaterals.
2. In the activities drawing quadrilateral. There is student's tendency, in this study, to draw a quadrilateral by starting from images that is very familiar for subject or often encountered and recognized by subject as a rectangle and a square. Likewise, when subject made diagram relationship between quadrilaterals, the tendency was happening again. Subjects tended to start from rectangle and connect it to the square. This indicates that the learning process in schools especially for quadrilateral, teachers often taught students ranging from rectangle or square so that students are only very familiar with both of them. Therefore, researchers also suggest to educators to teach not only the quadrilateral from a square or rectangle but start another quadrilateral such as parallelogram, rhombus, kite and trapezoid.

ACKNOWLEDGMENT

I would like to express my deepest appreciation to all those who provided me the possibility to complete this report. A special gratitude I give to Lecturer, Prof. Dr. Mega Teguh Budiarto, M.Pd., whose contribution in stimulating suggestions and encouragement, helped me to coordinate my resesarch especially in writing this report.

REFERENCES

[1] Agustan, S., Analyzing Student's Understanding The Relationship Between Quadrilateral At The Early Formal Stage, Proceeding of ICERD 2015, page 501-508, December 5th 2015.
[2] Musser, Gary L., Mathematics for Elementary Teacher. Third Edition. New Jersey: Prentice Hall, Page 535, 1994.
[3] Agustan S., Profil Berpikir Geometris Siswa SMP Level Deduksi Informal Dalam Memahami Hubungan Antarbangun Segiempat Berdasarkan Gaya Belajar. Tesis, Program Studi Pendidikan Matematika Program Pascasarjana Unesa, Unpublished, 2012.
[4] Sunardi. Pengembangan model Pembelajaran Geometri Berbasis Teori van Hiele. Disertasi Program Studi Pendidikan Matematika Program Pascasarjana Unesa, Unpublished, 2005.
[5] Yeni, E.M., Pemanfaatan Benda-Benda Manipulative Untuk Meningkatkan Pemahaman Konsep Geometri Dan Kemampuan Titik Ruang Siswa Kelas V Sekolah Dasar. Jurnal Pendidikan Matematika Edisi Khusus, No.1, page 63-75. 2011
[6] Idris, Noraini. The Impact of Using Geometers' Sketchpad on Malaysia Students' Achievement and van Hiele Thinking. Journal for Mathematics Education. Vol.2, No.2 pp 94-107. University of Malaya. 2011.
[7] Suyanto. Penelusuran Tahap Berpikir Geometris van Hiele Siswa Kelas III SMP Negeri 21 Surabaya yang Mengikuti Bimbingan Belajar Primagama pada Pokok Bahasan Segiempat. Surabaya: Tesis Program Studi Pendidikan Matematika Program Pascasarjana Unesa. Unpublished, 2005.
[8] Burger, W.F. & Shaughnessy, J.M., Characterizing the Van Hiele Levels of Development in Geometry. Journal for Research in Mathematics Education, page 31-47. 1986.
[9] Tambunan, S. M.. Hubungan Antara Kemampuan Spasial dengan Prestasi Belajar Matematika. Jurnal Makara, Sosial Humaniora, Vol.10, No.1, 27-32. 2006.
[10] Hergenhahn, B.R & Olson M.H. Theories of Learning. Jakarta: Kencana. 2008
[11] Leongson, J.A & Limjap, A.A. Assessing the Mathematics Achievement of College Freshmen using Piaget's Logical Operations. Waikiki: Hawaii International Conference on Education. 2003.
[12] Adey, P., Cognitive acceleration: Science and other entrances to formal operations. London: King's College London. 1995.