ME – 04

Adaptive Reasoning and Strategic Competence in Solving Mathematical Problem: A Case Study of Male-Field Independent (FI) Student

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Abstract-Adaptive reasoning and strategic competence are two important components that can be assembled into a logical steps in solving mathematical problem so that they reflect a math proficiency. This paper describes the adaptive reasoning and strategic competence of student in solving contextual mathematical problem designed to involve settlement through some mathematical concepts and efficient strategies to formulate, represent, and solve problem situation. Cognitive styles and gender of students are assumed to make an impact on mental activity student in solving mathematical problems involve adaptive reasoning and strategic competence. Thus, in depth interviews carried out to a eleventh-grade male senior high school student and has cognitive style Field Independent (FI). Male-field independent (FI) student related the concept of mathematical to situation mathematical problem encountered clearly and completely that appropriate with the rules of the concept. In addition, male-field independent (FI) student used reading and imagining strategies in understanding the problem situation, used verbalizing strategy in formulating the problem situation, used imagining, symbolizing and verbalizing strategies in representing the problem situation and then solved the problem analytically.

Keywords: a*daptive reasoning, field Independent, gender, mathematical problem solving, strategic competence*

I. INTRODUCTION

This paper presents the investigation of adaptive reasoning and strategic competence in solving mathematical problem. Kilpatrick [1] revealed five strands that are entwined shaped mathematical proficiency. The five components are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Five strands are not independent, but rather entwined form a bond with each other and influence each others to establish mathematical proficiency. This suggests that the two strands of the adaptive reasoning and competence strategies also influence each other. Adaptive reasoning and strategic competence are two key component that reflect understanding of mathematics [2]. Both adaptive reasoning and strategic competence respective have many aspects that are assembled into a logical sequence in solving mathematical problem.

Adaptive reasoning refers to mental activities adapt mathematical concepts, facts, procedures, and methods into mathematics problem situation and than provide explaination, justifying and reflection regarding relationship between concepts and situation. Strategic competence refer to mental activities apply strategy to formulate, representate, and solve the problem situation [1]. Having an excellant adaptive reasoning and strategic competence, means that the students have been successful in learning and has established a mathematical proficiency. The realization of success in learning is not only seen on the ability of students to arrived to solution of the mathematical problem, but is also capable of logical thinking to provide an explanation and justification of the results of their thinking and strategy of solutions used in the process of mathematical problem solving. As the opinion of Kilpatrick[1] and Herbert [3] that every experience in solving problems, students should also be able to think logically to explain and justify each of solution and results of their thinking to others. According Skemp [4] that knowing something is done without a coherent reason is a form of instrumental understanding. Then

Skemp [4] identified the primacy in mathematics learning is to build understanding of "relational" than understanding "instrumental".

The tendency of students through the process of mental processing information to produce a solution of the problem situation is called cognitive style. According to Kuo [5] that cognitive style refers to how the tendency of individuals to organize and represent information. According to Leng [6] that describes how the cognitive styles of students to recognize and represent a problem, plan, produce and execute a plan, determine and evaluate the solution when students think about how to solve the problem-solving task. Cognitive styles directly related to the thought processes which form the results of his thinking in accordance with the characteristics of the student. Field-dependent (FD) and field-independent (FI) cognitive style is one dimension of cognitive style that most attention in the educational implications [7]. According to Kuo [5], FI student is internally directed and process information with their own structure, analyze problems that require all elements in the context, as well as accept the idea reinforced through prior analysis. Holmes [8] concluded FI Students set their own standards for thinking and behaving. FI students active and goal oriented. FI students have excellent logical reasoning and analytical reasoning skills. According to Johnstone and Al-Naeem, FI students can capture the essence of the problem and ignore unimportant information that is not excessive in processing information [9]. FI students demonstrated the ability in solving problems better than FD students [10].

Gender in this study are the characteristics that distinguish between male and female are shaped by social and cultural factors and biologically formed. The results of research related to gender differences are very diverse [10]. In general, mathematics achievement of male better than the mathematics achievement of female [11],[12]. Hyde, Fennema, and Lamon expressed that male outperform female in terms of solving complex problems [13]. The research results of Awan [14] states that female students significantly have math self-concept more positive than male students. But contradict with the research results of Hergovich [12] that males showed higher self-concept in mathematics and females have higher self-concept in language. Therefore, in this study unique investigate the male-Field Independent (FI) student in order to present the description of adaptive reasoning and strategic competence in solving mathematical problem.

Adaptive Reasoning is mental activity to relate mathematical concepts, facts, procedures, and methods into mathematical problem situations so as produce an idea that used to solve mathematical problems. Mental activity in the reasoning adaptive can be observed through: mental activity to relate concepts into problem situations by explain logically their relationship; mental activity to select procedures and methods that appropriate with the situation problem by explaining logically the procedures and methods; mental activity to adapt between mathematical concepts, facts, procedures, and methods and the situation problem by justify logically [1],[3],[15],[16],[2].

Table 1. Adaptive Reasoning in Solving Mathematical Problem

Aspect	Sub-aspect	Sub-aspect observed
Explaining	Selecting concept	• Selecting appropriate mathematical concepts with problem situations
	Explaining the relationship	• Explaining the relationship of mathematical concept with the problem situation
	Explaining strategy	• Explaining strategy that has been selected
	Explaining procedure	• Explaining procedure of the strategy that have been selected
Justifying	Justifying strategy	• Justifying strategy that have been used

Strategic competence refer to mental activities apply strategy to formulate, representate, and solve the problem situation and than looking back. Mental activities in strategic competence can be observed through: mental activities use strategy for understanding the problem situation; mental activities use strategy for formulating known information from problem situation; mental activities use strategy for formulating unknown information, mental activities select strategy/method as a solution; mental activities use strategy for representing problem situation that appropriate with method or concept selected; mental activities use strategy for solving the problem [1],[16],[2],[17].

Table 2. Strategic Competence in Solving Mathematical Problem

Aspect	Sub-aspect	Sub-aspect observed
Formulating	Selected strategy for Understanding	• Selected strategy for understanding the problem
	Formulating known information	• How is the strategy used for formulating data / information is known from the problem situation
	Formulating unknown information	• How is the strategy used for formulating data/information is unknown from the problem situation
Representing	Selecting methods	• Selecting methods as a solution
	Representing problem situation	• How is the strategy used for representing problem situation that appropriate with method or concept selected
Solving	Solving problem	• How is the strategy for solving the problem

II. METHODOLOGY

This study employed qualitative research methods. The goal of the study was to present an accurate description of real situation regarding aspects of adaptive reasoning and strategic competence rather than simply asses mathematics expertise observed from the male-field independent (FI) student in solving mathematical problem. This subjek has a high mathematics achievement, that was known from given the test mathematics competence. This test arranged from nasional exam questions in 2013, 2014, and 2015 that have been converted into essay questions. Furthermore, subject also have been given GEFT (*Group Embedded Figures Test*) test to know that the subjek is field Independent student. This subject is eleventh grade student.

Since the aim of this study to examine the adaptive reasoning and strategic competence rather than simply asses mathematics expertise, it was necessary to supply a nonrutine problem that would challenge the student and was suitable for the study of adaptive reasoning and strategic competence. Nonrutine problem refer to a task that the student has not previously seen and done problem. The problem involve some mathematical concepts as settlement and need an efficient strategiy. The nonrutine problem given is as follows:

There is a land size $(200 \times 200)m^2$. *Within the land there is a warehouse size* $(40 \times 40)m^2$, *which is located in center of one edge of the land and overlooking the land. The whole piece of land overgrown with greengrass and dense, except on the warehouse. An lawn mower has a cable length of 80 m. On the walls of the warehouse in right front corner there is an electricity source to turn on the lawn mower. Determine the area of land that allows the grass can be mowed?*

Student were given nonrutine problem above and then interviewed in solving the mathematical problem. Subject was investigated to express his thingking regarding all aspects of adaptive reasoning and strategic competence in solving the problem.

To obtain credible data that what is observed in accordance with the fact that the credibility examination technique is done using triangulation of time. Examine the results of the interview data from a subject at different times. In addition, this research data partially obtained by using handycam and voice recorder and field note.

III. RESULTS

Next section discuss the contribution of this study on how adaptive reasoning and strategic competence in solving mathematical problem for male-field independent (FI) student. The following all

aspects from adaptive reasoning and strategic competence are assembled into a logical steps in solving mathematical problem.

A. Selected strategy for understanding the problem

Subject understand the problem situation using a reading strategy then imagine it. Subjek spent approximately two minutes to understand the problem situation. Subject read the problem calmly without touching the sheet of problem stored on the table. Before the two minutes was up, the subject then smiled as he continued to read about it. This indicates that the subject read the questions and then imagine easily. After reading, the subject received the information and then formed a mental image. Subject looked sure have understood the problem situation.

B. Explaining and justifying strategy that had been selected

Subject explained and justified strategies that was used to understand the problem situation with confidence that the strategies was appropriate. Subject used reading strategy then imagined the problem situation to be able to know and see all information of the problem. The information consist the core and aim of the problems encountered. The core problem is something which is the base to arrive at a final solution. The aim of the problem is the direction on what to do. Thus, the subject could understand problem situation after going through the process of receiving information from a problem situation and then through the process of forming mental images so that they can see and know the essence of the problem and the aim. Subject justified reading and imagining strategies for understanding the problem situations because they were a common and the right way to direct what should be done.

C. How was the strategies used for formulating data/information is known and unknown from the problem situation

Subject formulated data was known and unknown from problem situation verbally. Subject disclosed information that was in a problem situation with recounted all the information of the situation using his own words without looking at the text of the problem. Each sentences were recounted by his own words to make image clearly. The situation of problem were understood than subject recounted by his own understanding. The subject also illustrates the situation with using paper as a land and his fingers as a warehouse.

D. Selecting appropriate mathematical concepts with problem situations and explaining the relationship of mathematical concept with the problem situation in understanding the problem situation

Initially, subject select concepts of square area and circle as concepts that appropriate with problem situations. Subject explained that land and warehouses has form square. The land was said to be square because the land has the size $(200 \times 200)m^2$. Which means that land has the same length is 200 m. Writing the size of square is “side multipled side”. As well as the warehouse also was said to be square because the warehouse has the size $(40 \times 40)m^2$that means has the same length is 40 m. The subject also explained that the concept of a circle has relationship with the problem because based from the problem situation that there was a lawn mower has a cable length of 80 m. Then will be found land area that allowed the grass cut by the lawnmower. Subjects used a pen then rotated one end and the other end have support on one point. Subject explained that if for example this pen is the cable length then land area that can be passed by the lawn mower is like this pen that is rotating in a circle with the electricity source as the center and the cable length as radius.

Subject explained that the electricity source as center point because the electricity source attached to the wall that can not be moved (fixed point). Then subject also explained that the cable length as radius because radius are connecting between the center to the edge or side of the circle. While the cable length is connecting between electricity source dan lawn mower, so that the lawn mower is limited mobility and can only in a circle.

E. Selecting methods as a solution

After understanding the problem situation, furthermore subject was asked to demonstrate how to solve the problem. Subject later imagine before drawing all the information received from the problem situation. At this stage of the representation of the problem situation, the subject has not immediately found the right solution. Initially, the first subject expressed will calculate the area of the warehouse first and then calculate the area of land overgrown with grass. After that, the land area is reduced to the warehouse area. Then, to calculate the area of land that grass can be cut by the lawn mower,and then

subject drew a circle with radius of 80 meters with its center at the front corner of the warehouse. Subject sure that the grass inside a circle with a radius of 80 can be mowed by a lawn mower. The subject then make a new draw regarding all of the information contained in the problem situation. Subject drew only used a pen without using a drawing tool. Subject considered the size of the current drawing although not using the right scale.

F. How is the strategy used for representing problem situation that appropriate with method or concept selected and explaining strategy that has been used and explaining procedure

Figure 1. The subject's representation of problem situation

Subject was asked: *In what way you further solve this problem?* Subject make and produce a picture above (see figure 1). Subject representated the problem situation with make two square area, one are as a land with side length is 200 m and the other square as a warehouse with side length is 40 m . Subject also representated two circle, one circle with radius 80 m and the other circle with radius 40 m. Subject explained that circle with radius 80 m formed from the cable length. Circle with radius 40 m formed from remain of the cable length beyond the front wall of the warehouse with it's length 40 m, so that the length of remain is eighty minus forty equal forty meter and form a quarter circle with radius of 40 m. All part of the quarter circle exist in land area. Subject explained that the area of the land that it's grass allows mowed by lawn mower is composed of a half circle with radius 80 m, a quarter circle with radius 40 m, and an area that placed at right of the warehouse.

Subject were asked: *How your procedure to complete the area to look for*! Subject explained the procedure, first, calculate the area of a half circle with a radius of 80 m. Second, calculate the area of a quarter circle with radius of 40 m. Third, calculate the area of rectangular that it's sides 40 m multiplied by 80 m.

Furthermore, subject were asked*: How do you calculate the area of rectangular that it's sides 40 m multiplied by 80 m?* Subject think about 5 minutes. After that, subjek make a line that divides the that area so as form a right-angle triangle and a sector. Then make another high line in the area of sector as shown in figure 2.

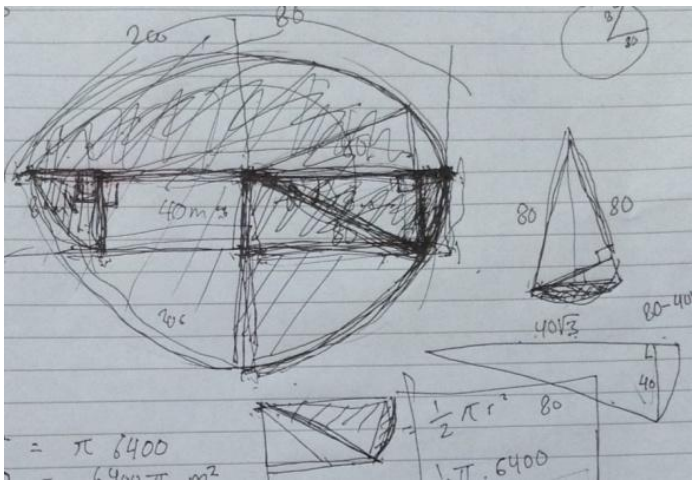

Figure 2. Student's Representation by Picture

G. Justifying strategy that have been used

Subject justify strategies used verbally and used of picture aids. His justification in accordance with the rules and properties concepts related . Subject divided the land area which allows the grass mowed by the lawn mower into four parts, namely a half circle with radius of 80 m, a quarter circle with radius 40 m, a one over twelve of the circle with radius 80 m and a right-angled triangle. Subjects were asked: *why do you say that this is a half circle*? Subject justified a half circle picture with the aid of picture that has been made. His justification was that "Due its center point is here (pointing to the center point of a circle with radius 80 m). Whereas here (pointing straight line to the left from the center to the circle line) 80 m and same here (pointing straight line to the right from the center to the circle line) is also 80 m and if we draw the line from here (center) to here (one point on the circle line so that the line is perpendicular to two line that have been mentioned earlier) in length is also 80 m. If we also use the tool for draw a circle with the center here (pointing to the center) will inevitably result in a half circle ". The subject asked again: Is There another explanation to justify it? Subject to re-explain that “if we draw a full circle, then this section (pointing a half-circle section with radius 80 m) is a half circle because, it's cleavage, forming diameter.”

H. How is the strategy for solving the problem

Subject was asked, "How do you solve this situation?", Then subject watched carefully the picture that has been made. Subject focused on each part of the picture that would be solved, even by separating the part of picture from the overall picture to be able solved the situation properly. Although drew that section separately repeatedly and took into account the size of the existing ones. After found the right solution, the subject later used a formula in accordance with the method used. Subjects completed the used of the formula to arrived at a final solution properly.

IV. DISCUSSION

Subject used strategies reading and imagining in understanding the problem. Subjects used strategies verbalize in formulating the problem situation. Subjects used the strategies imagining, symbolize and verbalize in representing the problem situation. Subject understood problem quickly because the subject managed to form mental images properly and appropriately. After imagined all the information captured from the problem situation, the subject admitted that he had understood the situation of the problem without have to draw these situations advance. Because obtain a good mental image of the problem situation, the subject feels confident that he has understood how the direction of what would has to be looked for and counted on the problem. The belief that the subject has really understood the problem, because the subject was able to retell a problem situation using their own language without seeing the text of the problem. Furthermore, subjects also immediately recognize quickly the concept that has the appropriate relationship with the problem, namely the concept of the circle. The subject was also familiar with other concepts quickly and appropriately during the process of solving problems. Subject provides an explanation of the relationship between the concept of a circle with a problem situation clearly. He explained all the parts of the circle associated with problem situations, such as radius, wide circle, center point, diameter, appropriately and in accordance with its nature. Subjects also justified the use of concepts and methods selected in conceptualization. From all that has been described above, it can be concluded that the subject has been linking "concept image" that has been owned by the understanding of the concepts that have been constructed since imange concept consists of "all of the mental pictures and associated properties and processes" [18]. For example, the subject explained that the cable length is appropriate with the nature of the radius because the radius connects the center point to the circle line while the cable length connects the electricity source with a maximum range of lawn mower. In addition, the subject explained that the electricity source is appropriate with the nature of the center point where the position settled at one point.

Overall, subjects correctly solve the problem until the final solution. The tendency of subjects complete the settlement procedures are analytically. For example, there is a part of the land area which also allows the grass mowed by the mower, as in figure 2. Subject analyze d this part first. And then the subject selected strategies to successfully find the wide of sector area. Based on that goal, the subject used drawing strategy three times with the size of each side in three different places. This shows that the subjects solve problems analytically. In accordance with the theory of cognitive style FI that the tendency

of individual to use an analytical approach when processing information and goal oriented [5],[8] and capture the important parts of the information it received [9].

Subject provide explanations and justifications verbally and drawing on the concepts and methods used. Subject logically explain the relationship between concepts and problem situations. In accordance with the results of Holmes [8] that students FI has excellent logical reasoning. The subject also provides an explanation and justification clearly and always feel confident with his explanations and justifications. Students are confident because in every process through in solving the problem always use mathematical concepts, facts, procedures and method that appropriate. In accordance with the expression Kilpatrick [1] that the process of adaptive reasoning holds all the facts, concepts, procedures, and methods of mathematics to steer arrive at a right final solution. In addition, the subject was always sure and confident in solving the problem because according to the research results Hergovich [12] that boys have high math self concept.

V. CONCLUSIONS

This study focus to the aspects of adaptive reasoning and strategic competence from male-field independent (FI) student in solving the mathematical problem. Male-field independent (FI) student formulate the problem by revealing the data of known and unknown verbally. Understanding the problem situation by reading and imagining and without making a picture of the problem situation. Recognized the concepts that appropriate to problem situations quickly and precisely. Selected drawing method and then represented a problem situation by drawing all information that received and then create a symbol of mathematical formula. Explaining the relationship between concept and problem situations logically, complete and correct. Justifying the used of concepts and methods logically with full confidence. Male-FI student also solved problems right up until to the final solution of the problem completely.

REFERENCES

[1] Kilpatrick, J., Swafford, J., and Findell, B., "Adding It Up," Washington: National Academy Press, 2001.
[2] Ostler, E., "Teaching Adaptive and Strategic Reasoning Through Formula Derivation: Beyond Formal Semiotics," *International Journal of Mathematics Science Education*, 4(2), pp 16-26, 2011
[3] Herbert, S., "A Framework for Teachers' Knowledge of Mathematical Reasoning," *In J. Anderson, M. Cavanagh & A. Prescott Eds, Curriculum in focus: Research guided practice. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia*, Hal 702–705. Sydney: MERGA, 2014.
[4] Skemp, R. R., "Relational understanding and instrumental understanding," *Mathematics Teaching*, 77: pp 20-26, 1976.
[5] Kuo, F.R., Hwang, G.J., Chen, S.C., Chen, S.Y., "A Cognitive Apprenticeship Approach to Facilitating Web-based Collaborative Problem Solving," *Educational Technology & Society*, *15* (4), pp 319–331, 2012.
[6] Leng, Y.L., Hoo, C. T., "Explaining the thinking, learning styles, and cognition constructs," *Association of Mathematics Educators*, *2*(1): pp 113-127, 1997.
[7] Mousavi, S., Radmehr, F., Alamolhodaei, H., "The Role of Mathematical homework and Prior Knowledge on The Relationship between Students' mathematical Performance, Cognitive Style and Working Memory Capacity," *Jurnal of Research in Educational Psychology*, 28:1223-1248. ISSN:1696-2095, 2012.
[8] Holmes, Robyn M., Liden, S., Shin, L., "Children's Thinking Styles, Play, and Academic Performance," *American Journal of Play*, 5(2): pp 219-238, 2013.
[9] Almolhodaei, "Students' Cognitive Style and Mathematical Word Problem Solving. *Journal of the Korea Society of Mathematical Education Series D:Research in Mathematical Education*, 6(2): pp 171-182, 2002.
[10] Zhu, Z. , Gender Differences in Mathematical Problem Solving Pattern: A Review of Literature," *International Education Journal*, 8(2):pp187-203,2007.
[11] Beller, M., Gafni, N., "Can Item Format (Multiple Choice vs. Open-Ended) Account for Gender Differences in Mathematics Achievement?" *A Journal of Research*, 42(1-2): pp 1-21, 2000.
[12] Hergovich, A., "Gender Differences in The Self-Concept of Preadolescent Children," *School Psycology International*, 25(2): pp 207-222, 2004. DOI: 10.1177/0143034304043688
[13] Linn, M. C., "Cross-National Patterns of Gender Differences in Mathematics: A Meta-Analysis," *American Psychological Association*,136(1): pp103-127, 2010.
[14] Awan, R. N., "A Study of Relationship between Achievement Motivation, Self Concept and Achievement in English and Mathematics at Secondary Level," *International Education Studies*, Vol. 4, No. 3; August 2011. Doi:10.5539/ies.v4n3p72
[15] Yook, E., Loong, K., "A Primary Teacher's Developing Understanding of Mathematical Reasoning," In J. Anderson, M. Cavanagh & A. Prescott Eds, Curriculum in focus: Research guided practice. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, pp 706–709, 2014. Sydney: MERGA.
[16] Suh, J. M., "Tying It All Together. Classroom practices that Promote Mathematical Proficiency for All Students," *NCTM*, 2007, http://mason.gmu.edu/~jsuh4/tenure/part4thru8/papers/tying_it_all_together.pdf.
[17] Özdemir, İ. E. Y., & Pape, S. J., "Supporting students' strategic competence: A case of a sixth-grade mathematics classroom," *Mathematics Education Research Journal*, *24*(2), pp 153-168, 2012. DOI 10.1007/s13394-012-0033-8

[18] Tall, David O. & Vinner, Shlomo, "Concept image and concept definition in mathematics with particular reference to limits and continuity," *Educational Studies in Mathematics*, 12, pp 151-169, 1981.