The Characteristics Of Students' Refractive Thinking about Data

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Abstract—Refraction is the process of produced the decision through reflection and critical thinking. Therefore, thinking that characterized by reflective thinking towards critical thinking until produce a decision called a refractive thinking. This shows that an important component of the refractive thinking is reflective thinking, critical thinking and a product. The reflective thinking is a process that occurs when a person encounters perplexity and conducted an investigation to find a solution. While, critical thinking is a process construct an alternative solution and evaluate various alternatives that can be considered to produce a decision. This study aimed to classifies the characteristics of students' refractive thinking about data. The approach used was a qualitative approach. The data source is the students of the 2nd semester by considering his communication skills. In this study, the students are required to complete the task and revealed with think a loud. If the student encounter reflective thinking and critical thinking in making decision, then students were included in the group of refractive thinking. The results showed that there were three characteristics of the refractive thinking performed by the students, namely: (1) the refractive thinking with a single strategy, (2) the refractive thinking with a dual strategy, and (3) the refractive thinking with multi strategies

Keywords: refractive thinking, reflective thinking, critical thinking, data

I. INTRODUCTION

Data considered as "number" [1]. In this case, data is not viewed as information on the specific situation about decision. Consequently, students always using statistical procedures to solve it, such as count the average or sum without attention of context provided. This proved with research [2] that some students in processing data by means comparing average or sum in making decisions about "most favored of cafeteria food". Some phases of completion to avoid misleading, among other: interpretation, description, conjecture, explanation and evaluation [2]. In view of [3] and [4] that descriptions and interpretation developed by [2] is the phase of reflective thinking. In the other, [5] and [6] identified that conjecture, explanation and evaluation are phase of critical thinking. Thus the phase of completion developed by [2] consists of two processes, namely reflective thinking and critical thinking. [7] and [8] defines that process produce a decision through reflection and critical thinking called refraction. Therefore, thinking that characterized by reflective thinking towards critical thinking until produce a decision called a refractive thinking. This indicates that a important component of refractive thinking is reflective thinking, critical thinking and decision.

Reflective thinking is one of the important thought process in construct knowledge and experience. The reflective thinking signed with difficulty (trouble) experienced by person so that he/she doing continuously behavior changes. Behavior changes are the process of investigated with explore information on the problem [9]. Investigations done to resolve the situation of uncertainty, instability, uniqueness, and conflict so that as provide answers the questions [9]. Reflective thinking occurs because of process connected one's knowledge with new information. This is matching with opinion of [3] and [10] that reflective thinking is process take knowledge and experience it then used to resolve problem. Reflective thinking is process occurs when person experienced perplexity and doing investigation repeatedly until finds completion [11]; [12]; [13]; and [14]. Perplexity are uncertainties or difficulties when solving problems. Inquiry is activity repeatedly searching for information that leads to settlement the problem. Reflective thinking has important role, namely as tool someone to solve the problem. With the thinking, can provide an opportunity for someone to step back and think about the best strategy to achieve goal [15] and [16]. Thus, reflective thinking is very important because it can help develop strategies and apply new knowledge the complex
situations. If reflective thinking done right, then help person to next step which called critical thinking [7] and [8].

Be related with critical thinking, [7] that critical thinking signed with process of evaluated a variety of relevant information when doing reflection in the problem solving. Implicitly "evaluation" revealed [7] is process of selected alternative solution obtained thus be taken consideration to decision making. It shows that decision must based on relevant information or attributes the problem. This is mismatch with the opinion [17]; [18] and [5] that critical thinking is process of considering and evaluating of some information obtained so that possible to decision making. Critical thinking signed with activity interpretation and evaluation of the problem solving[18]. Interpretation revealed the definition is process construct some settlement and produce an alternative solution. Moreover, the word "evaluation" is process of determining some-thing. Evaluation is a process signed with selected solution or best answer from some alternatives [5].

Some researchers have review the reflective thinking as process towards critical thinking, among others: reflective thinking is the one tool to develop higher-level thinking [19]; critical thinking is the result of one's reflection in learning [20]; reflective thinking to support critical thinking skills in solving social and political problems [21]; reflective thinking increases one's critical thinking and understanding which learned [22]; reflective thinking the beginning of the process of critical thinking specifically refers to the process of analyzing and making judgments [23] and [24]; Reflective thinking is the key of critical thinking [23]. In the study, [2] showed the students experienced when using phase shift in thinking reflective thinking and critical thinking so as to produce variations of the model answers. However[2] did not review how the thinking of students in produce the answer. Whereas [7] write a study of refraction theoretically and not review in the mathematics education. In the research have not provided description about how process of reflective thinking continued to critical thinking till produce decisions. Therefore this article review the process of reflective thinking towards critical thinking till produce decisions called as refractive thinking

II. LITERATUR REVIEW

A. Definition of Refractive Thinking

Using a metaphor light to describe the refraction [25]. Refraction is process light hit a medium thus result reaction which triggered the refraction of light towards a certain point. Based on the metaphor, [7] and [8] states that the refraction occurs because of the reflection continued critical thinking and produce new knowledge. Therefore thinking is signed with reflective thinking continued critical thinking till produce decision called refractive thinking. This indicates that an important component of refractive thinking is reflective thinking, critical thinking and decision. The process of refractive thinking can be illustrated in Fig 1.

![Figure 1. The Process of Refractive Thinking](image.png)

Based on the illustration, process of refractive thinking occurs because of the process reflective thinking continued critical thinking until produce decision. This indicates that important component refractive thinking is reflective thinking, critical thinking and decision. The refraction is the transformative knowledge that occurs the which validates the use of critical analysis and problem solving providing interpretation and Conclusions of important issues and situations considering the course content and context [7]. Knowledge transformative in this case is ability of person resolve problems through some alternative solution. The purpose of refraction is process decision-making by considering some possible alternative solution. This shows that the refraction is focusing of information since there are some alternative solution obtained when reflection and do critically analysis as consideration to establish a
decision. Be related with refraction, [8] defined that refraction is new knowledge acquisition from critical thinking of reflection. This shows that the refraction is the process of acquiring new knowledge (decision) resulting from reflection and critical thinking. Therefore refractive thinking this study is process of decision making through reflective thinking continued with critical thinking

B. Reflective Thinking

Basically, component of reflective thinking implicitly contained in the notion of reflective thinking. To make component of reflective thinking determined beforehand some notion of reflective thinking of some of the views. The reflective thinking is a process occurs when someone experiencing perplexity and then conducting an inquiry repeatedly until find the solution [11]; [12]; [13] and [14]. Based on some notion reflective thinking, implicitly important component of reflective thinking namely perplexity and investigation.

Reflective thinking is initiated by the perception of something troubling or promising, and it is determined by the production of changes one finds on the whole satisfactory or by the discovery of new features which give the situation new meaning and change the nature of questions to be explored [9]. This shows that, reflective thinking signed with difficulty (trouble) experienced hyperson so that he doing continuously behavior changes. Behavior changes are the process of investigated with explore information on the problem. Investigations done to resolve the situation of uncertainty, instability, uniqueness, and conflict so that as provide answers the questions. Based on the above definition reflective thinking, implicitly there are some components of reflective thinking. Components of reflective thinking according to [11]is perplexity and inquiry. According to [9] is the trouble and experiment. Two opinions can be compared. The equality of reflective thinking [11] and [9] presented in Table 1 below. Implicitly, Based on the similarities in the nature of each component, the obtained result of the development of reflective thinking in this article.

<table>
<thead>
<tr>
<th>Perplexity</th>
<th>Trouble</th>
<th>Perplexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty about something that is difficult to understand.</td>
<td>Difficulties experienced by someone</td>
<td>Difficulties experienced person to continue the next process; doubts about the answer or solution is found or confusion when someone obtained unexpected results</td>
</tr>
<tr>
<td>Inquiry</td>
<td>Experiment</td>
<td>Investigation</td>
</tr>
<tr>
<td>The process of repeatedly information that directs the mind to a certain direction.</td>
<td>Investigations conducted by exploring information to obtain an idea to solve the problem.</td>
<td>An investigation by exploiting existing knowledge to look back information or settlement process because of a uncertainty or doubts in obtaining answers</td>
</tr>
</tbody>
</table>

The table 1 shows a comparison of reflective thinking [11] and [9], namely: (1) Trouble partial indicator illustrated also in perplexity, such as someone difficulty in problem solving. Perplexity developed by Dewey is not just in trouble, but rather confirms the existence of doubt or lack of confidence their completion. If students are having trouble, doubt or confusion in solving the problem then it is said the students experienced perplexity; (2) Inquiry can be compared with the experiment, because the inquiry has the same properties as the problem that is causing the effort provide a solution. In the process of looking at the problem, a person can remember what you learned and utilized to solve the problem. The process is known as behavioral changes. In other words, students conduct an investigation by leveraging existing knowledge to look back the settlement process due to a lack of confidence or doubt in obtaining answers. Students who experience the process said investigation. Therefore reflective thinking in this article is the thinking process that signed the perplexity and then conducted an investigation till find a solution to the problem [26].

C. Critical Thinking

Critical thinking component is implicitly contained in the definition of critical thinking. To create a critical thinking component is determined in advance some notion of critical thinking of some of the views. [7] argued about critical thinking, Critical thinking demonstrates the ability to evaluate relevant information and opinions gathered in the reflection stage in a systematic, purposeful, efficient manner developing problem solving skills. This shows that evolution occurs because of the reflection of someone
in solving problems. Thus the "Evaluate relevant information and opinions gathered in the reflection stage" which is revealed in the definition explicitly states that the evaluation and collection of some of the information is a component of critical thinking.

Be related critical thinking, [18] states that critical thinking is skilled and active interpretation and evaluation of observation and communication, information and argumentation. Critical thinking by [18] signed with interpretation and evaluation of information and statements. Usually Interpretation is construct and produce some solutions alternative. this is case, beginning to conclusions from problem. In addition, evaluation is process of determined something. Evaluation signed with selecting the most excellent of some alternatives [5].Based above definition critical thinking, implicitly there are some components of critical thinking. Components of critical thinking according to [18]interpretation and evaluation. According to [7] is Opinions gathered and evaluation.implicitly, Based on the similarities in the nature of each component, the obtained result of the development of critical thinking in this article.

![Table 2. Development of Critical Thinking](image)

The table 2 shows a comparison of critical thinking [18] and [7], namely: (1) Gathered opinions can be compared with the interpretation, because it has the same properties that produce alternative possibilities completion. The possibility of constructing an alternative solution requires a variety of information that has been collected in the process of reflection. The situation is known as construction (construct); (2) Evaluation of critical thinking [7] and [18] can be compared as in selecting or evaluating an alternative solution or answer. This component signed by evaluated alternative solution or answer based considerations. This component is known evaluation.Therefore critical thinking in this article is thinking process that signed the construct and evaluation alternative settlement and the best answer based on various considerations [26].

III. RESEARCH METHODS

The purpose of this study was to explored and classified the processes of students refractive thinking in solving mathematics problems. Refractive thinking indicated from the process of students construction against instrument task "decision-making". This study used a qualitative approach, since according to characteristics owned. The research was carried on students in 2nd semester. For this purpose, the research took the data on student at Universitas Wisnuwardhana Malang and Universitas Negeri Malang. Research subjects not randomly selected, However taken with considered his communication skills so disclosure of the thinking process can be done well.

In this study, students were asked to complete task and expresses out loud what he was thinking (Think Out loud) when solving problem. After students obtain settlement, research check the students process settlement correct to obtain answers. If student experience reflective thinking and critical thinking in produced decision, then student will be a subject and included in the group refractive thinking. Each group is filled by two research subjects. If not obtain the desired subject, then the given the task again to students. The process of selecting subjects performed until a saturation of the data, its meaning that appears the same or remain characteristics of some subjects for each category. The many research subjects for each reflective thinking is 2 subject. Determined 2 subjects, with consideration that the method analysis used the constant comparative method. The task sheet instrument "decision making" used in this research is the development of a decision-making instrument from [2]. Problems in this article are influenced by quantity and quality that is large and increase of numbers each object. The problem given to the students as follows.

Local Revenue Offices survey 6 district to find out the level of district dependence on the central government. The dependence of regional on the central government can be measured based contribution the Own-Source Revenue (OSR) to income of province. If the contribution of OSR greater and increased then the district dependence to central government is getting low. The value in table below
shows the percentage contribution of OSR to income of province based Natural Resources (NR) for three years.

<table>
<thead>
<tr>
<th>NR</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investock</td>
<td>19</td>
<td>9</td>
<td>12</td>
<td>24</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>Maritime</td>
<td>18</td>
<td>20</td>
<td>13</td>
<td>9</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>Forestry</td>
<td>20</td>
<td>15</td>
<td>19</td>
<td>13</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>Plantation</td>
<td>9</td>
<td>11</td>
<td>26</td>
<td>23</td>
<td>17</td>
<td>14</td>
</tr>
<tr>
<td>Agriculture</td>
<td>25</td>
<td>14</td>
<td>20</td>
<td>14</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>Fishery</td>
<td>12</td>
<td>23</td>
<td>8</td>
<td>19</td>
<td>14</td>
<td>24</td>
</tr>
</tbody>
</table>

The brother task is determine the order of district from the lowest to the highest dependence on the central government! Give an explanation for your answer!

IV. RESULTS AND DISCUSSION

In this article the author only describes refractive thinking 3 subject. Third of subject were grouped into three groups: group one is subject 1 (S1) called single strategy. Subjects were included in the group 2 is subject 2 (S2) called dual strategy. Subject were included in the group 3 is subject 3 (S3) called multi strategy.

A. Characteristics of Refractive Thinking With Single Strategy by Subject 1 (S1)

The process of refractive thinking, begins with perplexity S1 to settlement. S1 determine average of each district. Settlement by average used as strategy to determine order of district. S1 think that the settlement with average can be used solved to problem. In the process look for average, S1 describes problem into some parts. S1 completed the first district A, B until the F. Thus overall average of district is the same, i.e. 16,7%. S1 questioned again the average obtained is the same “evidently of average the same!”. S1 suspect that the strategy has not been to solved the problem. S1 reading again problems and determining relations of the increase and decrease. To determine the order of district, the first S1 determine the great contribution and increase then low dependence, how do it?”. This show that S1 experience perplexity again when obtained average of the same. S1 think long time again and suspect of criteria “if the great contribution and increase then low dependence, how do it?”. This shows that S1 experience perplexity again when obtained average of the same. S1 think long time again and suspect of criteria “if the greater and increase contribution of Natural Resources then the district dependence to centre of lower”.

Based on these criteria, then S1 used another settlement with summing the percentage of contribution per year. The process is due to determine the amount of contributions per year. This shows that, when S1 suspect that strategy cannot be used to solve problems, he tried another strategy to solve it. This shows, S1 experience a process of critical thinking [7] and [18]. S1 summing percentage contribution per year. To explore the thinking process of S1 when solved with sum the percentage contributions per year, the research performed interviews. S1 claimed that to determine order of the district, the first of summing contributions per year. The settlement is base on the criteria of “substantial revenue contribution and increased”. To determine the order of the district, the first S1 determine the great of contribution then compared with other district to determine the decrease and increase of contribution every year. The following settlement by summed the percentage contribution per year done by S1.

Process doing by S1 is grouping district of each year. S1 again shows the relations criteria “increase” with the amount of contribution each year “the great contribution and increased ...”. The statement S1 aware that the criteria “increase” is the keyword to determine the order of the district. Implicitly, S1 can determine relations of the increase and dependency. S1 judging that the district has increased every year is district with low dependence and on the other hand, the district has decreased is an district with a high dependency.
The next process, S1 experience critical thinking with identifying and comparing each district which has increased of contribution each year. Based on the amount of contributions obtained, S1 indicates the lowest dependence is district F. The process done with compared the district F and other district. F are considered to have significant increases each year. The next process second order. In the second order, S1 connect again and compared the increase in the amount of the contribution of each district. Based on the amount of contributions obtained, S1 shows second order of lowest dependence of district is the B. If compared with other district, District B has increase in the third year despite constant. In the determine third order, S1 distrustful is district A and C. S1 compared the great of contribution and the increase in district A and C. The district A occur decrease from the first year to the second year, then increase in the third year. The district C occur increase from the first to third year and then decreased in the third year. In the selection for third order, S1 experienced perplexity. S1 think again with to give an alternative settlement to indicate the difference between A and C. In the first year of district A is 103%, while district C is 89%, this indicates that the district A was excelled in the first year. While the second year, district A is 92% and C is 114%, this indicates that the district C was excelled. In the third year, district A was excelled as 105%, while district C is 97%, this indicates that the district A has a large amount of contributions for two years, while C is only one year. The next process is fourth sequences. S1 put district C as fourth sequences. District C is the comparator A when determining third order, however the district A was excelled compared to district C. Based on these, S1 puts district C as the fourth order. In addition, S1 was connected and compared the increase amount of contribution each district. If district C was compared to other regions (areas D and E), then district C was increased. In the fifth and sixth sequences, S1 only compared the two district that have not been occupied the previous sequence, i.e. D and E. The District D has decreased but the fixed in the third year. District E has decreased the amount of contribution significantly, that is the first year until third year in a row by 114%, 96% and 90%. District D as district that occupies the fifth order because amount of contribution the same that is second and third year is 91%. This shows that in the second year and third year, district D does not decrease (constant). Based on settlement process do by S1 in making decision about district sequence begin lowest to highest of dependence i.e. district F, B, A, C, D, and E. Conclusions are based on the criteria of "amount of great contributions and increased"

With these answers, S1 believes the answer. In the process of decision making, a subject need only one settlement alternative. Subject only to clarify the criteria contained in the problem as consideration to decide, for example, to identify the contribution of each year. Based on the thinking, S1 experienced refractive thinking with single strategy. The refractive thinking process by S1 can be illustrated in Fig. 4 below.

![Diagram](image)

Figure 4. The Refractive Thinking With Single Strategy by S1

B. Characteristics of Refractive Thinking With Dual Strategy by Subject 2 (S2)

In the process of thinking, appears that S2 have difficulties to solve problem. S2 solved with summed the percentage contribution of each district based Natural Resources (NR). The process called analytical process, which is a process that describes the problem into several parts so that the parts are then
completed. The summing is done by S2 begins by calculating the percentage contribution of livestock. S2 summing the percentage contribution of the farm for three years in each area. This process is carried out from the district A to district F.

![Figure 5. The result of students' (S2) work about summing percentage contribution of the farm for three years in each area](image)

The District order first on the livestock is district D, C, B, E, A and F. This is because the amount of district contribution to livestock the greatest is district D i.e. 60, then district C is 53 and smallest amount is district F i.e. 39. While in maritime, district F is a district order first because it has the highest amount of contributions compared to other district i.e. 60. Then continue district C with contribution amount 52 and district last order is district D with contribution 40. The group process and sorted performed by S2 until fishery.

![Figure 6. The result of students' (S2) work the group process and sorted until fishery](image)

The next process, S2 combines the sequence a whole. In the process of combining sequences, S2 connects sequence with rank. It can be seen from the statement S2 "... makes the rank first, A, B, C, D, E, F, and rank 1, 2, 3, 4, 5, 6". This indicates that S2 experience process of critical thinking[7] and [18]. The statement indicates that the S2 explicitly show relationship of district with ranking. S2 connected ranking with district have greatest to smallest of contribution amount at each Natural Resources.

![Figure 7. The result of students' (S2) work about connected ranking with district](image)

Based on above settlement, S2 attention district that are above (first order). The first sequence of the district there are four choices of district A, C, D, and F. S2 connected one district to other district of sequence, in this case S2 compared district A, C, D, and F. Based on this comparing, many district (mode) the first sequence is same. District mode C and D is one, while district A and F is two. S2 compared A and F with sequence thereafter (second). In sequence afterward (second), mode of district F is one, while areas A no mode. S2 identify the sequence thereafter (the second) for consideration determining the first sequence. S2 choose F as district in first sequence, while A is chosen as district in the second. This is because the district A ranks first as much two and rank second as much three. The next process is third order. To determine the sequence of third, S2 compare district B, C, D and E. Based on four district, S2 comparing many district in the first sequence. In the first sequence that appears only district C and D as many one. Furthermore, S2 consider the order after namely the order of two and three. The next process is fifth. S2 looked back at remaining district, namely B and E. S2 connects the district B and E based on the second and third sequence. In the second sequence, district B as much one, while E as much one. Shows that the district have same mode in second order. S2 choose alternative of compared that is third sequence. District B in third order as much 2 while district E only 1. Based on settlement process, S2 make decisions about sequence from lowest to highest dependence on district F, A.
C, D, B, and E. Conclusions are based on compared many district (mode) in certain sequence. The process of refractive thinking with dual strategy by S2 can be illustrated in Fig. 8 below.

![Diagram](image)

**Figure 8. The Process of Refractive Thinking With Dual Strategy by S2**

### C. Characteristics of Refractive Thinking With Multi Strategy by Subject 3 (S3)

S3 represents contribution of the most widely with summed percentage contributions for three years. S3 added the percentage of each district from first to the third year on each Natural Resources S3 using another settlement which connects with the sequence of points or score. The biggest scores indicate low dependence, whereas the lowest score showed high dependency. The first sequence district were the biggest scores, while the last order district was lowest score. S3 gives a score of 6, while final sequence given score of 1. This indicates that S3 integrates sequences with the scoring. The following settlement by S3 relating to the scoring.

![Diagram](image)

**Figure 9. The result of students’ (S3) about relating to the scoring**

The next process, S3 summed scores each district. For example, the score of district A is 6,6,4,2,2,2 if the scores are summed obtained 22. The process of scores summed in each district until district F. The following results of work by S3 related to total score of each district.

![Diagram](image)

**Figure 10. The result of students’ (S3) about total score of each district**

Based on the results work above, the lowest order first is district C. chosen C as the lowest order first by S3 because of some district, the district C has greatest amount of scores. This is consistent with previous statement by S3 "The greatest contribution is the most low dependence". S3 feel confident with the first order is district C. This appears statement by S3 "means the correct .." The next process of the second order. S3 attention again the total score of each district. Based on the results work are district have
the same amount in the second i.e. the district A and F. The total score of two is 22. This indicates that possibility of a second sequence is occupied by A and F, the following statement S3 "A and F, here the twins, second chances if not A, yes F ".

Based on the similarity score on A and F, S3 silent for moment and reading again the question. S3 experience perplexity in determine the order of second. The behavior appears with statement S3 "Then I see from where?.. we see from here highest (pointing Natural Resources) ". To determine the order of second, S3 consider many Natural Resources with a amount great contribution in A and F. District A, the Natural Resources has large amount is 4 (livestock, forestry, plantation and agriculture), while district F is 2 (maritime and fisheries). This shows that A has greater Natural Resources to contribute of district F. The following results of work by S3 related to many Natural Resources.

Based on the results work, then the second order and third is district A and F. This is because district A has more natural resources compare the district F. With these answers, S3 believes the answer. In the process of decision making, subject not only requires two strategies (addition and ranking), but he made another settlement (score) as reinforcement for answers obtained. Based on the thinking, S3 experience refractive thinking with multi strategy. The process of refractive thinking with multi strategy by S3 can be illustrated in Fig. 12 below.

**Figure 11.** The result of students (S3) about related to many Natural Resources

**Figure 12.** The Process of Refractive Thinking With Multi Strategy by S3

V. CONCLUSION

From the results study to refractive thinking of students in solving mathematics problems can be concluded that there are three type processes of refractive thinking by students, namely: (1) refractive thinking with single strategy signed the process of decision making, a subject need only one settlement alternative. Subject only to clarify the criteria contained in a problem of consideration to decide, for example, to identify the contribution of each year, (2) refractive thinking with dual strategy signed the process of decision making, the subject need two alternatives. If there are sequence of district the same, subject uses internal comparison as consideration, for example considering the mode of sequence before
and after, and (3) Refractive thinking with multi strategy with the process of decision making, subject need the settlement of three alternatives. If there are sequence of district the same, subject uses external comparison as considerations, for example identified many Natural resources with largest amount.

REFERENCES