ME– 10

Characterization of Mathematical Connections in Calculus

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***Abstract*—**This paper aims to determine the types of mathematical connections when undergraduate students solve connections problems in calculus, and to describe the characteristics of each type of the connections. The approach of this research is qualitative research with descriptive exploratory method. Itsparticiants were undergraduate students of Mathematics Education Study Program, Faculty of Teaching and Education, University of Mataram in Academic Year 2015/2016. Data of research collected through the connections assignment sheet in calculus and followed up by interview based on the tasks. The study resulted that the type of mathematical connection can be characterized into two types, that are the proceduralconnections and the conceptual connections. Each of these type links two or more topics among some topics in calculus. In the future studies, result of this characterization will bedetailed schematically to describe its process of thinking.

Keywords:*calculus, characterization, mathematical connections.*

I. INTRODUCTION

When learners solve a problems, it was often happened that they already have learned and mastered the knowledge required to solve the problem, but they can't associate or make a connection and use it to solve the problem. Therefore, the ability of mathematical connections needs attention in the learning of mathematics. Mathematical problems solving need skill to connect mastered knowledge relating to the problems encountered.

Mathematical knowledge can be divided into two types, namely conceptual knowledge and procedural knowledge [1]. The core of conceptual knowledge is to understand the relationship between ideas and mathematical concepts. The purest form of procedural knowledge focuses on the symbolism, skills, rules and algorithms used step by step in solving a mathematical task. Learners should learn the concepts at once with the procedures so that they can see the connection.

Concepts, procedures or skills, along with facts and principles identified by Gagne as the direct object of mathematics [2]. While the one of indirect objects of the mathematical is problem solving. Problems are questions that can be understood by learners but can not be answered immediately with a routine procedure that has been known to learners. So a question can be clasified as a problem when the questions give a challenge to be answered and the answer can not be done by a routine strategy. Solve the problem is the process of accepting the challenge to answer the question that is the problem [3]. A problem for college students when administered to elementary level students excluding a problem because the problem will not be understood and will not give a challenge to be answered.

Bruner's connectivity theorem stated that in mathematics every concept related to other concepts. Similarly, between the argument and the other argument, the theory with the other theory, among the topics to the topic, and between branches of the branch of mathematics sholuld be related [4]. Therefore, in order to be more successful in learning mathematics, learners must be given the opportunity to make mathematical connections.

Connection mathematically described by [5] as part of the network is structured like a spider web, "*The junctures, or nodes, can be thought of as pieces of represented information, and threads between them as the connections or relationships*". Mathematical connections can also be described as a component of a scheme or a linked group of schemes in mental network. Reference [6] argued that the characteristics of the scheme is the connection. The more connections, the greater compactness and strength of the scheme.

Studies on the connection mathematical beside conducted in schools are also a lot done in universities [7], [8], [9], [10]. According [11], which examines undergraduate students in pre-calculus course concluded that the subjects had not yet established a connection between algebra procedures and the nature of the underlying numbers. Results of research on the connection in linear algebra carried out by [12] showed that subjects find it more difficult to make the connection between concepts like eigenvalues and eigenvectors and of other parts such as the conceptual basis and the dimension.

Research by [13] on middle school pre-service teachers has resulted five categories of mathematical connections, namely: (1) *categorical*, if the paticipant's explanation relied upon the use of surface features primarily as a basis for defining a group or category; (2) *characteristic/property*, if the participant's explanation for the sort involved defining the characteristics or describing the properties of concepts in terms of other concepts; (3) *curricular*, if the participant's explanation for the sort involved relating ideas or concepts in terms of the impact to curriculum, including but not limited to, the order in which one would teach concepts or topics; (4) *procedural*, if the participant's explanation for the sort involved relating ideas based on a mathematical procedure or algorithm possible through the construction of an example, which may include a description of the mechanics involved in carrying out the procedure rather than the mathematical ideas embedded in the procedure; and (5) *derivational*, if the participant's explanation for the sort involved knowledge of one concept(s) to build upon or explain other concept(s), included but not limited to the recognition of the existence of a derivation.

Results of another study suggests three types of connections, that are referred to the most commonly in relevant literature and in their formal curriculum documents, but in practice their development of "connected knowing" could have been stronger, more frequent and more consistent. The three kinds of connections are the connection between new information and current knowledge, the connection between mathematical concepts, and the connections to everyday experience [14]. The types of connections is in line with the scope of connection standards in the [15], which include recognize and use connections among mathematical ideas, understand how mathematical ideas interconnected and build on one another to produce a coherent whole, recognize and apply mathematics in contexts outside of mathematics.

Study of the source of theory and the previous research results presented above, gave rise to the idea of mathematical research with the theme of connections. The connections studied is considered by knowledge of mathematics that divided into two types of knowledge, namely conceptual knowledge and procedural knowledge. Therefore, research on the characterization of the mathematical connections when undergraduate students solve problems about calculus was conducted. The material of calculus is chosen because it is the core material in mathematics educations program, and a lot of good use in other disciplines as well as in everyday life.

This research is expected to be useful among others, to provide an overview of the undergraduate students about the characterization of mathematical connections, so it can be used as a benchmark to improve problem solving skills. In addition, it is expected to be useful as well as inputs for lecturers or teachers of the importance of undergraduate students making mathematical connections so that it can be taken into consideration in planning and implementing learning.

II. METHOD OF RESEARCH

Type of research is descriptive exploratory study. The participants were six undergraduate students of Mathematics Education Study Program, Faculty of Teacher Training and Education, University of Mataram in Academic Year 2015/2016. The main instrument in qualitative research was a researcher itself [16]. The support instruments are an assignment sheet and an interview guides. The assignment sheets gave a task of connection on calculus in the form some cards with information written on it. Informations on these cards can be facts, a concepts, principles, procedures, or problem about calculus. The cards are considered as nodes that shoul be connected.

The tasknumber 1, contain two cards, consist of information about The Pythagoras Theorem and a point (x, y) at Cartesius coordinate system that satisfy $x^2 + y^2 = r^2$, where $r > 0$.The task number 2 consist of three cards that each contain information about concepts, that are absolute value function, derivative of function, and graph of function. The problem structure of task number 1and number 2 is describedin Figure 1 and Figure 2.

Mathematical connections made by participants are grouped and identified its characteristics. Judging from the type of knowledge conceptual and procedural knowledge, this type of mathematical connection divided into two types, namely conceptual connections and procedural connections. While looking at the material, there are some topics related material, for example: Pythagoras theorem, Cartesius coordinate system, absolute value function, graph of functions, and derivative of fuctions.

In the Fig. 1, a participants that make connection inter nodes by similiraty formula is catogorized as procedural connections, meanwhile if a participants connect two nodes by plotting the right triangle at the

Cartesius coordinate plane that contain the circle with center O and radius r then it is categorized as conceptual connections.

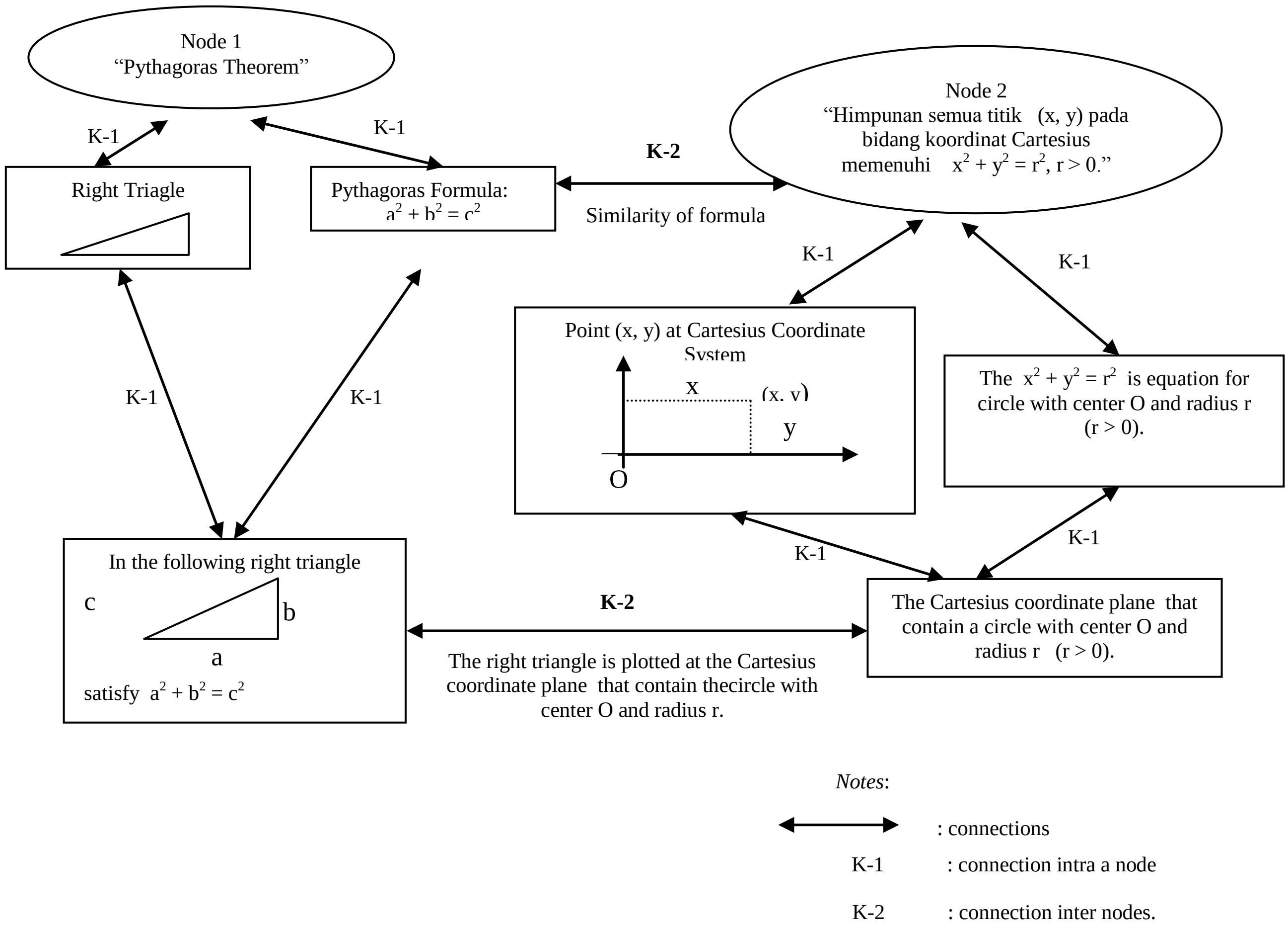

FIGURE 1. THE PROBLEM STUCTURE OF TASK NUMBER 1

In the Fig. 2 below, a participants that make connection inter nodes by compute derivative without consider geometrically interpretation is catogorized as procedural connections, meanwhile if a participants make connection inter nodes by compute derivatives, then consultated with geometrically interpretation is categorized as conceptual connections.The indicators of the type of mathematical connection can be expressed in Table 1.

TABEL 1. THE INDICATORS FOR TYPES OF MATHEMATICAL CONNECTIONS

No.	Type of Mathematical Connections	Indicators
1	Procedural Connection	• To connect the Pythagoras theorem with point (x, y) at Cartesian coordinate system that satisfy $x^2 + y^2 = r^2$by similarity formula. • To connect the absolute value function with derivative of function by count its derivative without consultated its graph. • To connect the absolute value funtion with graph of function by draw its graph without analize it. • Not make connection between derivative of function with tangent slope of function.
2	Conceptual Connection	• To connect the Pythagoras theorem with point (x, y) at Cartesian coordinate system that satisfy $x^2 + y^2 = r^2$by plotting the right triangle at the Cartesian coordinate plane that contain the circle with center O and radius r. • To connect the absolute value function with derivative of function by count its derivative and consultated its graph. • To connect the absolute value funtion with graph of function by draw its graph, then analize it.

		• Make connection between derivative of function and tangent slope of function, then look smoothness the graph.

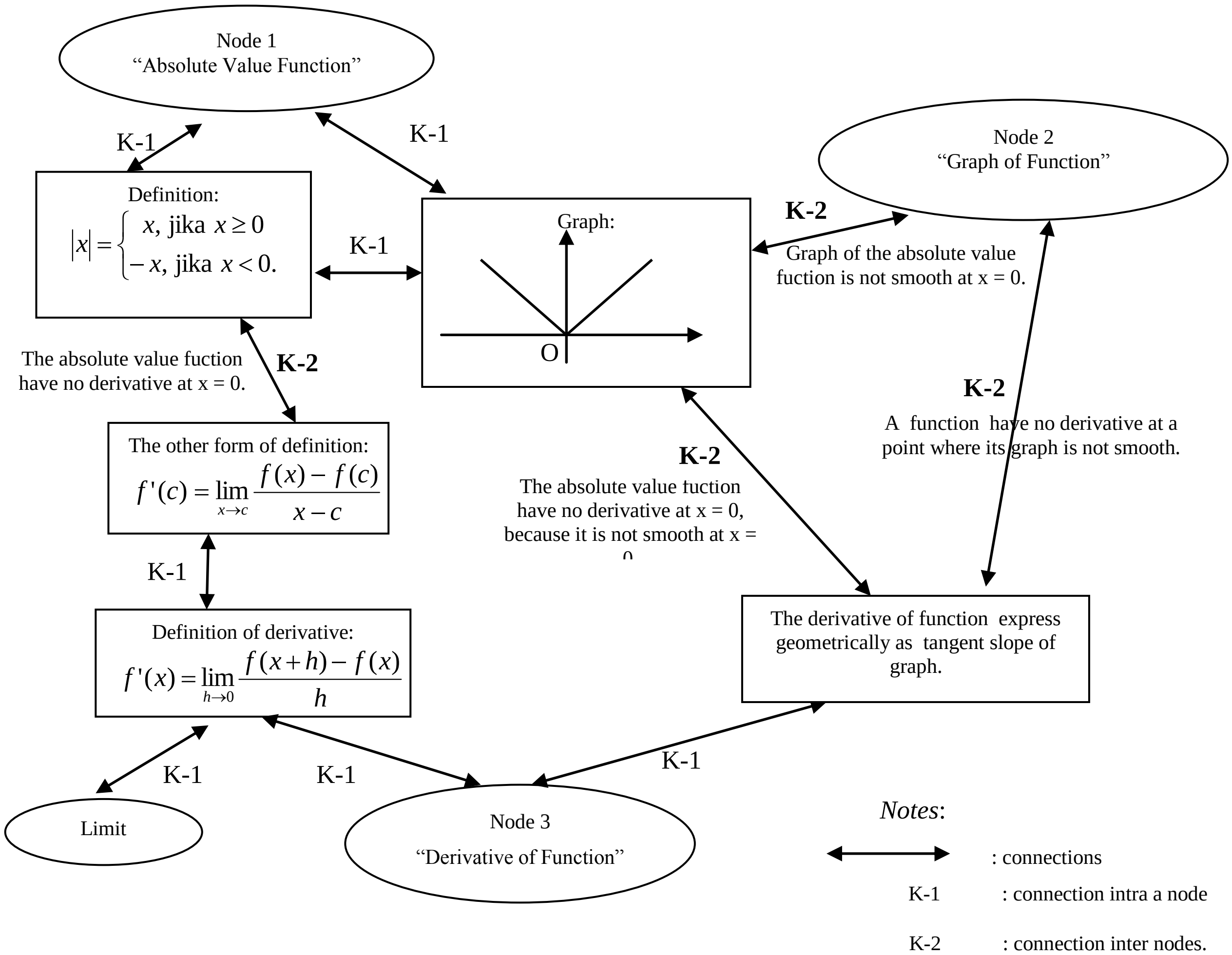

FIGURE 2. THE PROBLEM STUCTURE OF TASK NUMBER 2

III. RESULT AND DISCUSSION

The results of the participants' answers to the connection task recapitulated based on aspects of conceptual connection and procedural connection. There are some topics related to the material, which arePythagoras Theorem and Point in Cartesian coordinate system at task 1, meanwhile material in task 2 include absolute value function, derivatives and graph. Responses of participants is presented in Table 2 below.

Results of analysis of the data resulted in grouping type of mathematical connection, which can be divided into two types, namely conceptual connections and procedural connections. The conceptual connections occurs for participants S1 when make connections between "Absolute Value Function", "Graph of Function", and "Derivative of Function", because after count its derivative,then she draw its graph then analize it, so that can make connection between derivative of function and tangent slope of function or smoothness the graph, as shown in Fig. 3.

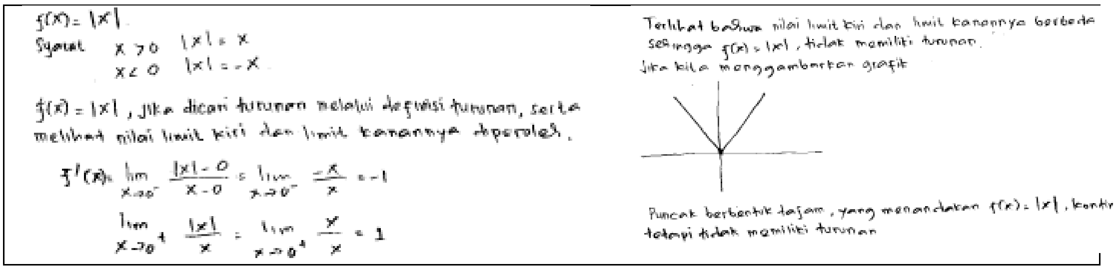

FIGURE 3. PARTICIPANTS' ANSWER THAT CATEGORIZED AS CONCEPTUAL CONNECTIONS

TABEL 2. RESPONSES OF PARTICIPANTS

Participants	Node of Connections	Explaining of Connections
S1	"Pythagoras Theorem" and "An arbitrary point (x, y) at Cartesian Coordinate System satisfies $x^2 + y^2 = r^2$, where $r \geq 0$"	Pythagoras theorem entitled at a right-angled triangle. The right triangle is drawed with O (0,0) and (x, y) as the vertex and sides are respectively x, y, and r.Therefore, the equation $x^2 + y^2 = r^2$ is the form of Pythagoras Theorem, where $r \geq 0$ is distance from original point (0, 0) to point (x, y).
S1	"Absolute Value Function", "Graph of Function", and "Derivative of Function"	Graph of the absolute value function depicted shaped sharply, indicating continuous but has no derivative. The absolute value function has no derivative at x = 0. It is calculated using the definition of derivative obtained limit value of the left and right limits are different.
S2	"Absolute Value Function", "Graph of Function", and "Derivative of Function"	Expressed $f'(x) = \mid x \mid / x$, but can not give a reason.Graph illustrates the absolute value function sharp angle, signifying nothing derivative in that.But for the general functions, such as quadratic functions, she can not give the interpretation of the derivative in terms of the graph of a function.
S3	"Pythagoras Theorem" and "An arbitrary point (x, y) at Cartesian Coordinate System satisfies $x^2 + y^2 = r^2$where $r \geq 0$"	Pythagorean theorem is the distance of points (x, y) in Cartesian coordinates satisfy $x^2 + y^2 = r^2$, $r \geq 0$. The right triangle is drawed in the Cartesian coordinate system with O (0,0) and (x, y) as a point angles and sides are respectively x, y, and r.
S4	"Absolute Value Function", "Graph of Function", and "Derivative of Function"	The absolute value function is a function that its graph is above the x-axis. Derivative of a function can be illustrated in a graph of a function. Absolute value function is a function that can be derived and the function can be described in a function.

The procedural connections occurs for participants S1 and S3 when make connections between "Pythagoras Theorem" and "An arbitrary point (x, y) at Cartesian Coordinate System satisfies $x^2 + y^2 = r^2$, where $r \geq 0$", because they only look similarity formula without plotting the right triangle at the Cartesian coordinate plane that contain the circle with center O and radius r.Connections between "Absolute Value Function", "Graph of Function", and "Derivative of Function" maked by participants S2 and S4 is categorized as the procedural connections too, because S2 expressed $f'(x) = \mid x \mid / x$, but can not give a reason and S4 can not make connection between derivative of function with tangent slope of function. The examples of procedural connections is shown in Figure 4.

FIGURE 3. PARTICIPANTS' ANSWER THAT CATEGORIZED AS CONCEPTUAL CONNECTIONS

IV. CONCLUSIONS

From the results of this study concluded that the type of mathematical connections when undergraduate students solve problems in topics of calculus can be divided into two types, that are conceptual connections and procedural connections. Characterization of each type of mathematical connections in terms of topics related, that are Pythagoras theorem and point at Cartesian coordinate system,as well as absolute value function, derivatives and graph. The implications of this study are expected that lecturers need to provide problem-solving issues that are non-routine and requires undergraduate students to explore mathematical ability as a whole in order to make the mathematical connections which is a tool in problem solving.

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