The Role of Productive Struggle to Enhance Learning Mathematics with Understanding

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Abstract—Struggle is a natural part of the learning process that refers to a student’s intellectual effort to expand their understanding of the mathematical concepts that challenge but fall because of student’s capabilities. This paper discusses about the classification of student’s struggle and the teacher responses, the role of productive struggle in learning mathematics with understanding, and strategies to supporting productive struggle. Productive struggles are identified as an important component of effective mathematics teaching and learning, implying that good teaching involves placing students in problem solving situations where they would have difficulty and frustration experience that could serve as a learning opportunity. Student’s effort to learn mathematics, figure something out that is not immediately obvious can help students in their thoughts and play an important role in deepening the student’s understanding. Teachers can give appropriate guidance and support to maintain the mathematics understanding and opportunities to think more deeply about mathematical concepts. When students struggle, there are consequences of “incorrect” answers. It is not seen as failures but rather opportunities to explore, grow, and learn serve better support and motivate students to persist. Teaching that uses productive struggle leads to long-term benefits, with students more able to apply their learning to new problem situations. Mathematics teaching using student’s struggles can be good opportunities to deepening their understanding of mathematics so that it can be effective. This study suggests the productive role student struggle can play in supporting student learning with understanding.

Keywords: productive struggle, learning process, deep understanding

I. INTRODUCTION

Student’s struggle in learning mathematics is often seen in a negative light and viewed as a problem or learning difficulty [1]. Sometimes, the cause of it will be attempted to remove by diagnosis and remediation [2]. There is a negative belief that struggling with a mathematical task is not viewed as an opportunity to learn but as a weakness [3]. It would be hard to see that student’s struggle in mathematics could be viewed as positive thing and as a learning opportunity.

Recently, National Council of Teachers of Mathematics (NCTM) released policy document, Principles to Actions. It notes that supporting productive struggle in learning mathematics is one of the eight important teaching practices [4]. Struggle refers to student’s effort to make sense mathematics, find something out that is not immediately obvious. It is not used to refer needless frustration levels of challenge created by difficult problems [1].

Struggle is essential to intellectual growth has a long history. Dewey referred to the process of engaging students in some confusion or doubt as essential for building deep understanding. In the other hand, Piaget also thinks that student’s struggle as a process of restructuring their disequilibrium towards new understanding [2]. Prior study by Hiebert and Grouws suggest that struggle is a necessary component of learning mathematics with understanding [1]. Warshauer also suggests that productive struggle can support doing mathematics and its implications on student learning with understanding [2]. Teachers and instructional designers also can integrate student struggle into tasks and instructional practices rather than avoid or prevent it. Productive struggles are identified as an important component of effective mathematics teaching and learning, implying that good teaching involves placing students in problem solving situations where they would have difficulty and frustration experience that could serve as a learning opportunity [5]. This paper tries to discuss about the role of productive struggle to enhance learning mathematics with understanding.
II. DISCUSSION

A. Productive Struggle

Struggle is a natural part of the learning process. It refers to that a student’s intellectual effort to expand their understanding of the mathematical concepts that challenge but fall because of student’s capabilities [1]. Student’s struggle is an opportunity for investigating more deeply into understanding the mathematical problems and the relationships among mathematical ideas, instead of simply seeking correct solutions [4]. Thus, struggle is a student’s intellectual effort to expand their understanding of the mathematical concept by investigating more deeply the mathematical problems and the relationship among mathematical ideas, instead of simply seeking solution of a challenge problem but they fall because of their capabilities. There are some types of struggle [2][6].

1) Get started: Students feel confuse about what the task asked them to do (“I kind of understand it…but I’m a little confused”). Some students may even proclaim they did not remember doing problems of this type though it appeared vaguely familiar to them (“I have absolutely no idea….I don’t remember that far”). Then, they call for help (“I need help”) and gesture uncertainty and resignation (looks, thinks, sits back and then says, “I don’t know”) or show no work on their paper.

2) Carry out a process: Some students get difficult to carry out the procedure and the demonstrated or voiced some plan for achieving the goal of the task but encountered an impasse. These impasses tended to revolve around an inability to implement a process such as solving the problem.

3) Uncertainty in explanation and sense making: Students are uncertainty in explaining or sharing their work in small groups or with the whole class. In order for students to complete each task, they are expected to explain their work and solutions in writing and in many examples to each other or to the class. Students often struggled to verbalize their thinking and give reasons for their strategies even if their answer appears correct.

4) Express misconception and errors: Struggle is involved the students’ misconceptions appeared to be instances when deep-seated mistaken ideas were used as the basis for solving problems rather than student’s confusion or possible error due to carelessness.

Teacher must support student’s struggle, so it is become positive endeavor and not full of difficulties and frustration [2]. Teacher needs to be careful to select the best responses of the struggle. Teacher’s responses to student’s struggle generally divided into four types [2][6].

1) Telling: In a telling responses, teachers often provides sufficient information for the students to overcome the struggle, suggest a new approach or strategy, correct an error, evaluate students work, related to simpler problem and decrease process time.

2) Directed guidance: Directed guidance involves redirecting student thinking toward the teacher’s thinking, narrowed down possibilities for action, directing an action, breaking down the problems into smaller parts or altered problems to an analogous one such as from an algebraic to numerical ones. It is done by asking open-ended questions, breaking down the problem into smaller parts, and narrowing down what the student might try next.

3) Probing guidance: Probing guidance puts the struggle back into the student’s lap. Teacher offers ideas based on the student’s thinking, asks for an explanation that might surface an error, or asks for reasons and justifications. Then, teacher must seek explanation that could get at an error or misconception and ask for written work of student’s thinking.

4) Affordance: Affordance provides an opportunity for students to continue to engage in thinking about the problem and build on their ideas with little help from the teacher. Teacher is explicit in encouraging students to continue their effort in their task. The characteristics of these types of respond are ask for detailed explanation, build on students thinking, press for justification, sense making with group or individually, and afford time for students to work.
All of these approaches are useful as long as the level of cognitive demand remains high, and student thinking is supported [3]. When students show effort to solving the confusing problems or making sense of challenging ideas, they engage productive struggle process [7]. Productive struggle is fostered through what psychologists have termed desirable difficulties; challenges that compel the learner to repeatedly retrieve information over time, thereby strengthening long-term memory for flexible transfer of the information to new contexts later [7]. Productive struggle, similar with to other executive function like cognitive skills that help the brain organize and act on information, is supported by a developmental progression in thinking and learning [3]. It is a core component of effective mathematics teaching and learning, implying that good teaching involves putting students in problem-solving situations where they will feel the difficulties and frustrations that could serve as learning opportunities [5]. Students can experience productive struggle when given a task slightly beyond their abilities. Some factors influence productive struggle, that is:

1) **Mathematical self-image**: Mathematical self-image is related student’s perception about their mathematical ability. Students often think that mathematical ability is something that some students are skilled at rather that a behavior that everyone can develop [3]. Because of it, motivation is needed. Motivation for productive struggle requires a growth mindset. It is an understanding that success is a result of effort more than of raw ability. It makes students want to new challenges, and enthusiastic rather than be afraid about learning. Students who believe that their ability levels are fixed are less motivated to engage in productive struggle because they afraid of failure, resist the risks, and worry about the judgments of others, thwarting their own learning [7].

2) **Student’s disposition with challenging mathematical task**: Student’s disposition depends on whether they (1) find the task interesting, (2) believe that they know enough mathematics to be able to solve it, and (3) believe that solving it is worth the effort. Student’s belief that effort is more important than innate ability is the main factor. In order to persevere, one needs to view the struggle that may be a part of problem solving as an opportunity to learn. Motivation enables a solver to see struggle as a natural part of the learning process, and to see that confronting and working through struggle can ultimately be helpful [5].

3) **Support and feedback**: Effective feedback makes clear about what the goal is, what progress they are making toward that goal, and what they need to do next to make better progress. The durability of students’ motivation to do in struggling to achieve a goal is mediated by the quality of the teacher-student relationship and the scaffolding provided through feedback. Correcting students’ errors, effective feedback guides students to develop better strategies for processing and understanding the material so that they get mastery, confidence, and motivation to continue to support effort in productive struggle [6].

Struggle is not productive when students become frustrated because the goal is unclear or far out of reach, they do not feel safe to fail, or they do not receive adequate, appropriate support. Struggle can be destructive, and teachers need to intervene after finding that students are not making any progress and feeling that their efforts are pointless [6]. Struggle is productive if student’s can maintain the initial goals and cognitive demand of the task, support their thinking by acknowledging effort and mathematical understanding, and move forward in solving the task through their actions [2].

**B. The Role of Productive Struggle in Learning Mathematics with Understanding**

Struggle is a necessary component of learning mathematics with understanding [1]. Understanding is defined as the mental connections among mathematical facts, ideas, and procedures, and then struggling is viewed as a process that reconfigures these things [1]. Relationships among facts, ideas, and procedures are reformed when new information cannot easily be assimilated or when the old relationships are found to be inadequate to make sense of a new problem [9].

Understanding is crucial because learning with understanding can be used flexibly, adapted to new situations, and used to learn new things. It is the most useful things to know in a changing and unpredictable world. Students who lack understanding and must resort to memorizing are likely to feel little sense of satisfaction and are likely to withdraw from learning. Understanding gives student confidence and engagement; not understanding leads to disillusionment and disengagement [11].

Understanding is participating in a community of people who are becoming expert at doing and making sense of mathematics, then struggling is vital because it can be an essential aspect of personal meaning making within the community [14]. Understanding is also important because it is one of the most intellectually satisfying experiences, and, on the other hand, not understanding is one of the most frustrating and ultimately defeating experiences. Learning mathematics with understanding has
increasingly received attention from mathematics educators and psychologists and has progressively been elevated to one of the most important goals of the mathematical education of all students [10].

Students who are given opportunities to understand, from the beginning, and who work to develop understanding are likely to experience the kind of internal rewards that keep them engaged. Student’s effort to learn mathematics, find something out that is not immediately obvious can help students in their thoughts and play an important role in deepening the student’s understanding. In summary, struggling with important mathematics is implicated in both definitions of understanding by identifying the common processes hypothesized to develop understanding. Mathematics teaching using student’s struggles can be good opportunities to deepening their understanding of mathematics so that it can be effective.

C. Strategies to Supporting Productive Struggle

To support productive learning, students must realize that they have an ability of doing well in mathematics with their effort and perseverance in reasoning, sense making, and problem solving. Teachers provide support and guidance for students, individually and collectively, to work through uncertainties as they struggle with representing a mathematical relationship, explaining and justifying their reasoning, or finding a solution strategy for a mathematical problem. The table below summarizes teacher and student actions supporting struggle as a natural aspect of learning in the mathematics classroom [4].

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
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</thead>
<tbody>
<tr>
<td>Ancipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.</td>
<td>Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.</td>
</tr>
<tr>
<td>Giving students time to struggle with tasks, and asking questions that scaffold student’s thinking without stepping in to do the work for them.</td>
<td>Asking questions that are related to the sources of the struggle and will help them make progress in understanding and solving tasks.</td>
</tr>
<tr>
<td>Helping students realize that confusion and error are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.</td>
<td>Persevering in solving problems and realizing that is acceptable to say, “I don’t know how to proceed here,” but it is not acceptable to give up.</td>
</tr>
<tr>
<td>Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.</td>
<td>Helping one another without telling their classmates what the answer is or how to solve the problem.</td>
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</tbody>
</table>

Teacher and student must cooperate to support struggle in mathematics classroom. Teachers need to select tasks carefully that require students to struggle and provide the support that students need without giving students too much help. Students need sufficient time, not only to solve difficult mathematical problems, but also to develop genuine curiosity and durability [3]. They also influence how students perceive and approach struggle in the mathematics classroom. They can give appropriate guidance and support to maintain the mathematics understanding and opportunities to think more deeply about mathematical concepts. In the below, there are some strategy to support student struggle and make it productive [3].

- Set goals at the beginning of the lesson and keep track of student progress during the lesson.
- Set problems in a familiar setting whenever possible, such as a sport or a familiar everyday task.
- Support students by providing appropriate tasks, tools, and representations.
- Group students heterogeneously, which helps struggling students.
- Establish high mathematical expectations (i.e., doing mathematics requires effort).
- Use good questioning techniques, such as asking students to explain how they solved a problem and why a strategy works or ask them to describe another way to solve the same problem.
- Provide time for group reflection during problem-solving activities. This can help students recognize unproductive strategies.
- Compare student outcomes at the end of the lesson to your original goals.

Supporting from teacher, in one side can enhance and in other instances diminish the level of student learning. It depends on the circumstances, including the goals of the task, the student’s prior knowledge, and the student’s willingness to attempt to do the problem. Teachers can incorporate into their practice explicit reminders to students that struggling to make sense of mathematics is an important and natural part of learning. Rather than avoiding this phenomenon, teachers can integrate struggle as part of doing...
mathematics by acknowledging students’ consternation, encouraging perseverance, asking questions, and offering time to work through problems. Table 3 shows the outlines teaching strategies that remind students of the positive aspects of struggle and student actions that indicate productive engagement [6].

TABEL 2. STRATEGIES AND INDICATIONS OF PRODUCTIVE STRUGGLE

<table>
<thead>
<tr>
<th>Teaching Strategies</th>
<th>Student indicators of a Productive Struggle</th>
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<tbody>
<tr>
<td>Question</td>
<td>Teacher asks question that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem.</td>
</tr>
<tr>
<td></td>
<td>Students ask questions to identify the source of their struggle, write down their ideas, clarify ideas with others, and consider alternative strategies or representations to address their struggle.</td>
</tr>
<tr>
<td>Encourage</td>
<td>Teacher encourages students to reflect on their work and support student struggle in their effort and not just in getting the correct answers.</td>
</tr>
<tr>
<td></td>
<td>Students use their effort to solve problems and try to make sense of their work, not only satisfied with a correct answer or that they perceive themselves as smart or not.</td>
</tr>
<tr>
<td>Give time</td>
<td>Teacher gives time and support for students to manage their struggle through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students.</td>
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<tr>
<td></td>
<td>Students use their time to develop and follow through on their strategies, evaluate their progress, and understand what they can do and what still remains to be done.</td>
</tr>
<tr>
<td>Acknowledge</td>
<td>Teacher acknowledges that struggle is an important part of learning and doing mathematics.</td>
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<tr>
<td></td>
<td>Students persist in their work to make sense of and to solve their problem and not give up or get discouraged easily.</td>
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</tbody>
</table>

When students appear confused, unable to make sense of an answer, or reach an impasse in working on their task [6]. This is not obstacle in learning but it is an opportunity for develop student’s understanding of mathematics. When students struggle, there are consequences of “incorrect” answers. It is not seen as failures but rather opportunities to explore, grow, and learn serve better support and motivate students to persist. One of cognitive approach to learning mathematics with understanding is using student’s error or incorrect answer as sources of information about their understanding [9]. By incorporating instructional approaches that acknowledge student’s struggles and effectively support, guide, and feedback the student’s thinking toward a productive resolution, student given opportunities to strengthen their disposition toward challenging tasks and they will persist through their struggle to make sense of and understand important mathematics.

III. CONCLUSION

Struggle is a student’s intellectual effort to expand their understanding of the mathematical concept by investigating more deeply the mathematical problems and the relationship among mathematical ideas, instead of simply seeking solution of a challenge problem but they fall because of their capabilities. There are some type of struggles, i.e., get started, carry out a process, uncertainty in explanation and sense making, and express misconception and errors. Teacher’s responses to student’s struggle generally divided into four types, i.e., telling, directed guidance, probing guidance, and affordance. Struggling to make sense of mathematics is a necessary component of learning mathematics with understanding. Understanding is the mental connections among mathematical facts, ideas, and procedures and learning with understanding can be used flexibly, adapted to new situations, and used to learn new things. Students who are given opportunities to understand, from the beginning, and who work to develop understanding are likely to experience the kind of internal rewards that keep them engaged. Teachers provide support and guidance for students, individually and collectively, to work through uncertainties as they struggle with representing a mathematical relationship, explaining and justifying their reasoning, or finding a solution strategy for a mathematical problem. Teachers can incorporate into their practice explicit reminders to students that struggling to make sense of mathematics is an important and natural part of learning. Rather than avoiding this phenomenon, teachers can integrate struggle as part of doing mathematics by acknowledging students’ consternation, encouraging perseverance, asking questions, and offering time to work through problems. Teaching that uses productive struggle leads to long-term benefits, with students more able to apply their learning to new problem situations. Mathematics teaching using student’s struggles can be good opportunities to deepening their understanding of mathematics so that it can be effective.

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