

# The Characterization Of Mathematics Students' Metacognition Process In Solving Mathematical Problems

by

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**Abstract-**It is an article as the result of a study to analysis happening of mathematical students' metacognition changes in solving and describing the characteristic of mathematical problem. The metacognition is based on the changes of awareness, evaluation, and regulation as the components of it. To make known of them, the writer has determined the indicators of each of the components which are spelled out into descriptors in order the process of metacognition changes can be seen. The process of the study involved 23 Students of Mathematics Department who finished Differential Calculus lesson as the subjects of the study. The subjects of the study were given mathematical problems about the application of function derivation to make known of the maximum area of a field. The data of the study was obtained consists of mathematical problem worksheet, record, metacognition questionnaire, and interview transcript. Then, the data was analysed by using Glaser's and Strauss' constant comparative models. Based on the analyzing, it can be inferred that the characterization of mathematical students' metacognition changes in solving mathematical problems can be classified by three categories: (1) metacognition changing process with complete-order indicator, (2) metacognition changing process with complete-disorder indicator, and (3) metacognition changing process with incomplete indicator. During the happening of the process of metacognition changes, the subject of the study, carried out the activities which are suitable to awareness, evaluation, and regulation indicators determined.

**Key Words:** *metacognition, awareness, evaluation, regulation, mathematical problem*

## I ACKNOWLEDGEMENT

Mathematics owns direct and indirect objects. According to Gagne (Soedjadi, 2000; Hudojo, 2008) direct object consists of fact, concept, operation, and principle. While indirect object consists of logical thinking capability, problem solving, analytical thinking, positive thinking against mathematics, carefulness, perseverance, discipline, and the other cases which will implicitly obtain if someone learn mathematics. One of the objects is concept. The orientation of the concept of mathematics can be done through learning activity. Djamarah (2008) states that someone who owns the concept could carry out the abstraction. So that, he or she is able to translate and invite awareness and form mental representation. During understanding the concept, someone needs a specific ability and strategy. Duffin and Simpson (2000) states that the ability owned by someone during the understanding the concept is expected to be able to re-express the subject communicated to the source of learning, so that could solve the problem.

Polya (1988) mentions four steps to solve the problem, e.g. understanding the problem, planning the action, carrying out the planning, and reflecting the action done. The step expressed by Polya in solving the problem is the activity which can be observed during learning activity and as the measurement of completeness of learning result. According to taxonomy Bloom, the completeness of learning result is differentiated in the field of cognitive, affective, and psychomotor. Anderson, Krathwhol (2001) improves taxonomy Bloom cognitive field into two dimensions: the process of cognition and knowledge. The process of cognition consists of remembering, understanding, applying, analysing, evaluating, and creating. While knowledge is divided into the factual, conceptual, procedural, and metacognition.

Metacognition consists of *meta* and *cognition*. *Meta*, Greek, means after or behind and *cognition* is the process of getting knowledge. (Zahmeister & Neyberg. 1982). Metacognition can be related to the activity of solving problem, knowledge, and cognition or the strategy used during the learning process. At first, the term of metacognition was used by Flavell in 1976. According to him, metacognition consists of knowledge, experience, and regulation. Besides, Flavell stated that metacognition is use as the important element and play an important role in the success of problem solving. However, someone often fail to solve the problem. The main cause of it is the lack of metacognition aspect, especially relating to the steps of problem solving. (Schoenfeld, 1992; Goos, 1995;). In its progress, metacognition can help mathematics

thinking process effectively. (Clarke, 2004). While Schoenfeld regards that the difficulty of solving problem is closely related to student's disability to observe and control their process of cognition (Schoenfeld, 1987). Some studies of the relationship between metacognition and problem solving had been carried out, such as Desoete (2001), Lioe (2003), Wilson & Clarke (2004), Cromley (2005), Efklides (2006), Lesh (2007), Panauorra (2009), Kuzle (2011), Molnar (2011), Karan & Irizary (2011), Magiera & Zawojewski (2011), In'am (2012), Praba (2013), Zarimah & Tajudin (2013). The studies have not expressed the characteristic the process of metacognition in detail. Relating to the process of problem, researchers did their studies to describe the characteristic of the process of metacognition to mathematical students in solving mathematical problems. The characteristic of the process of metacognition in the study is described based on *awareness*, *evaluation*, and *regulation*, the components of metacognition.

## II. THEORITICAL STUDY

### A. The definition of Metacognition

Friedrichs & Hoyt (1976) calls metacognition in term of metamemory, while Veenman (2012) describes it as two main parts: knowledge of metacognition and regulation, and observing metacognition. Knowledge of metacognition is an offer of interaction between someone's knowledge and ability to do his or her duties, characteristic of duties, and strategy used to solve them. Regulation or observation of metacognition is the activities relate to one's planning, monitoring, evaluating, and the process of cognition to control the process. Metacognition is a process which has four important aspects. According to Baker & Brown (1984), the four aspects are self-controlling, planning, evaluating, and monitoring. Wellman (1985) states that metacognition is a form of cognition or a process more than second level of thinking which involves controlling of cognition activities. That's why, metacognition can be said as a someone's thought about thinking of her or his self or someone's knowledge of cognition.

Besides having four aspects, according to Schoenfeld (1992), metacognition is a process of someone's thinking about being thought and interaction among three important aspects: knowledge of thinking process, self-controlling, and intuition. The interaction is very important for the knowledge of process of cognition can assist and control cases around us and select the strategies to increase our further cognition ability. The process of metacognition, according to Schoenfeld, are the ability to question and answering the questions about case, topic and subject, duration of time used by the students to study a certain topic, strategy, method and tactic, level being studied, failure done by the students, and doing revision to the next plan.

Livingstone (1997) defines metacognition as *thinking about thinking*. In other word metacognition is someone's ability to think about he or she thinks, so that metacognition object is thinking process happens to her or himself. Biryukov (2003) expresses that metacognition is someone's prediction about his or her thinking consisting of knowledge, skill, and experience. The knowledge as awareness of what being known, skill of awareness of what being done, and experience as awareness of cognition ability owned.

Davidson & Sternberg (1998), states that metacognition has a very important function and contributes towards the success of problem solving which enable someone to identify and work strategically. Matlin (1998) states that metacognition is a knowledge relating to awareness and cognition process. Wellman (1985) states that metacognition is a form of cognition or two levels or more thinking process involves control to cognition activity. That's why metacognition can be said as someone's thinking of his/her-self or someone's cognition of self-cognition.

Tan (2003) states that metacognition is thoughtfulness refers to think of self-thinking, self-controlling, self-checking, and information processing and how to process the information effectively. Lioe (2003) states that metacognition is someone's awareness of cognition process and stand lone lines to reach a certain goal. Metacognition appears in problem solving whose components are attitude, skill, concept, process, and metacognition. Peirce (2003) defines metacognition in general and specific. In general, metacognition is thinking of thinking. While in specific, Peirce adopts Taylor's metacognition definition saying that metacognition is an appreciation of what is known relating to the ability to make a right conclusion of how to apply someone's knowledge of strategy in a certain situation, and to do it efficiently and perfectly. Taccas (2008) describes metacognition is a part of planning, monitoring, and evaluating the process of teaching and learning, thinking of what being known, or unknown and control the way to learn involving those two awareness and someone's learning awareness, so that learning will be effective. Mokos & Kafoussi (2013) states metacognition is someone's ability to observe and control her/him-self towards the known thing. During learning process, mathematics is an important thing of a study of students' metacognition process during solving problem focussed on the field of problem solving relating to mathematics.

Based on the definitions above, it can be identified the main meaning of metacognition, e.g. (1) metacognition is soul ability in cognition group, (2) metacognition is the ability to aware, know, the process of cognition happens in someone, metacognition is the ability to direct cognition process in someone, (3) metacognition is the ability to learn how to learn being done in the process of planning, observing, and evaluating, metacognition is a high thinking activity for the activity can control thinking process be doing on someone, and (4) metacognition relating to students thinking process of his/her thinking in order to find a suitable strategy to solve problem, (5) the skill of metacognition is very important to solve mathematical problem, so that the skill need to increase. To increase metacognition skill needs awareness owned by students in each steps of their thinking. Student's awareness is needed to think when solving the problem.

### B. The Components of Metacognition

Magiera & Zawojewski (2011) find that metacognition happened during giving assignment in the class. Metacognition happening in the students has three components, e.g. awareness, evaluation, and regulation. During the process of metacognition, it can be seen the appearing activities in every components of metacognition called as types of metacognition activities. The types of awareness consist of what the students know, what the students need to solve the problem, what the students must do, where the students solve the problem. The types of evaluation consist of result evaluation, students' difficulty problem of evaluation, progress ability or understanding evaluation. The types of regulation consist of planning strategy, selecting strategy of problem solving, formulating the goal.

*Awareness*, according to Wilson & Clarke (2002, 2004), relates to someone's awareness in the process of learning or in the process of solving problem, the content of specific knowledge owned, and someone's knowledge in learning or strategy of solving problem. It is also about someone's knowledge about what is needed, what has done, what can be done in a certain learning or situation in solving the problem. *Evaluation* refers to evaluation made by someone about thinking process, ability and limitation, such as working in a certain situation or as a self-complication. For example, someone can make evaluation about thinking effectiveness done or strategy chosen. *Regulation* in metacognition happens when someone uses his/her skill of metacognition to direct knowledge and thought and refers to individual knowledge in the form of strategy, such as how and why using certain strategies, as well as skill, such as planning, self-correction, decide the goal to optimal the usage of their own cognition source.

Metacognition components stated by Wilson & Clarke (2002,2004) and Magiera & Zawojeski (2011) has indicators as variable and measurement. However, how the process of changes among the components of metacognition has not analysed deeper yet. Sriraman (2003) has considered students about the relationship between their knowledge and what is needed in problem situation given, as Stillman & Gabraith (1998). *Evaluation* has been described and studied concerning the students explicit reflection during the process of solving problem, the function of evaluation in determining the strategy in solving problem. Make a decision in systematic evaluation, alternative plan, and strategy insolving problem (Lester, 1980; Lester, Garafalo, & Kroll, 1989). *Regulation* has been clarified in the form of student flexibility in choosing a solution plan, choosing strategy, and plan implementation improved by Lester (1989), Zan (2000).

### C. THE METHOD OF THE STUDY

The study done is qualitative descriptive whose subject is 23 university students. The method and steps of study are (1) giving mathematical problem to the students who have got Differential Calculus as the subject of the study. The mathematical problems are about function downward application to determine maximum and minimum value. The problems have been validated by 2 experts in mathematics and mathematics education. During doing the problems, the subject do and being recorded. (2) Researchers correct the result of students' works based on the answer-key made before. Based on it, subjects of study are classified into high, middle, and low ability students. (3) Give questionnaire. It is given after the students finish doing mathematical problem. It consists of 14 items to measure the process of metacognition happened to the students. It is the improvement of research done by Biryukov (2001), Azsoy & Ataman (2009), Meriam & Idrus (2010), Panaoura (2010), Sengul & Katransi (2013). (4) Doing the interview. It is based on the result of students' works, record,questionnaire answers. It is done to make deeper known about the characteristic of the process of *awareness, evaluation, and regulation*. It is done after students carry out . Interview protocol is arranged by improving the indicators of *awareness, evaluation, and regulation*. (5) Doing transcription the record of and interview. The transcription is done for obtaining the data of each of the subject of the s tudy of the characteristic of metacognition process during finishing mathematical problem relating to students ability.(6) Reduce the data. Reducing the data

is done by making abstract in the form of summary of data core, process, and statements done by the subject of the study in finishing mathematics problem as well as arranging the data in parts which will be categorised by *coding*. (7) Data analysing, analysing metacognition process each of the subjects of the study through each indicators *awareness*, *evaluation*, and *regulation*. (8) Data validation, data validation is done using triangulation technique and checking classmates through discussion. Triangulation used is source one, that is comparing the result of observing during the subject finishing mathematical problems, the result of students works, , questionnaire answers, and interview.

#### **D. THE RESULT OF STUDY AND DISCUSSION**

The data of the study is in the form of the result of student works of finishing mathematical problems, the record of , the answers of metacognition questionnaire and the interview transcript is studied and analysed qualitatively. The theory design built by the researcher is analysing the characteristic of the student metacognition process in finishing mathematical problems through indicators of *awareness*, *evaluation*, and *regulation*. Based on the theory design, the result of analyse of the study is grouped into category of characteristic of student metacognition process of high, middle and low ability.

During the study, researchers gave mathematics problems to 72 students of Department of Mathematics Education who finished programming the subject of Differential Calculus. The process of metacognition was observed to describe and analysis. Description and analysis done relate to the characteristic *awareness*, *evaluation*, and *regulation*. The result of the initial observation and analysis show that 23 students did the complete metacognition as follows: 6 students are in high ability, 9 students are in middle ability, and 8 students are in low ability.

Based on the result of the study, each of the subjects of the study in each group has characteristic which are relatively same. So hat, there are 6 subjects of data explanation done by the researchers in describing the characteristic of the process of metacognition. They are 2 high ability students symbolized by S-1 and S-2, 2 middle ability students symbolized by S-3 and S-4, and 2 low ability students symbolized by S-5 and S-6.

Next, the characteristic of metacognition process of those three subjects of the study were examined through the indicators of *awareness*, *evaluation*, and *regulation*. The component of *awareness* has 5 indicators. They are the subject of the study re-think of what was known about given mathematics problems given (A1), re-think the question of mathematical problems and related it to the similar problem gotten and done before (A2), re-think the undone thing (A3), re-think the next steps to do (A4), and re-think the steps done (A5).

The component of *evaluation* has 5 indicators. They are the researchers re-think the way to solve the problems (E1), re-think the order and steps to do (E2), re-check the answers (E3), re-think whether the answers are right or wrong (E4), and re-think the failure done in answering mathematical problems which were given in the last way (E5).

The component of *regulation* has 4 indicators. They are the subjects of the study re-think and make a plan to finish mathematical problems (R1), re-think the different way done in answering mathematical problems (R2), re-think of what will do after finishing answering mathematical problems (R3), and re-think how to change the way. The process of the happening of metacognition can be described as follows:

#### **A. The Process of High Ability Students Metacognition**

The subject of the study in high ability S-1 and S-2 have same characteristics of metacognition process. During the metacognition process, S-1 and S-2 show the activity that describes the characteristic of component indicators *awareness*, *evaluation*, and *regulation*. When finishing mathematics problems, the indicators of mathematical problems appeared in order: A1, A2,A3, A4, A5, E1, E2, E3, E4, E5, R1, R2, R3, and R4.The order of appearing indicators on S-1 and S2 can be seen on the figure 4.1

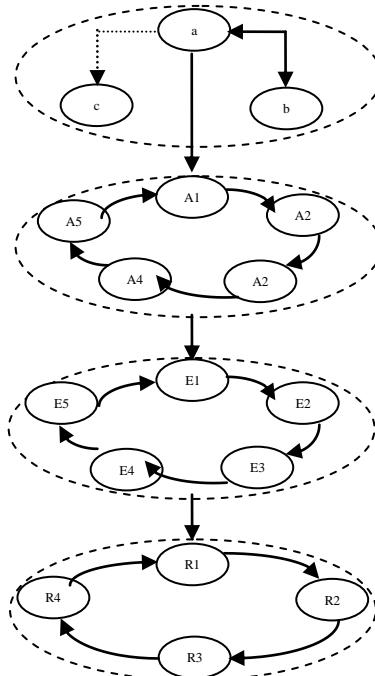


Figure 4.1: Metacognition Process S-1 and S-2

### B. The Process of Middle Ability Students Metacognition

The middle ability subject of the study, S-3 and S-4 carried out metacognition process with complete indicators but disorder. When finishing the problems, the indicator in S-3 appeared in order A1, A2, A3, A5, A4, E1, E2, E3, E4, E5, R1, R3, R2, and R4. The indicator in S-4 appeared in order A1, A2, A3, A4, A5, E1, E2, E3, E4, E5, R1, R2, R4, R3.

The order of appearing indicator in S-3 can be seen in figure 4.2 and the order of appearing indicator in S-4 can be seen in figure 4.3

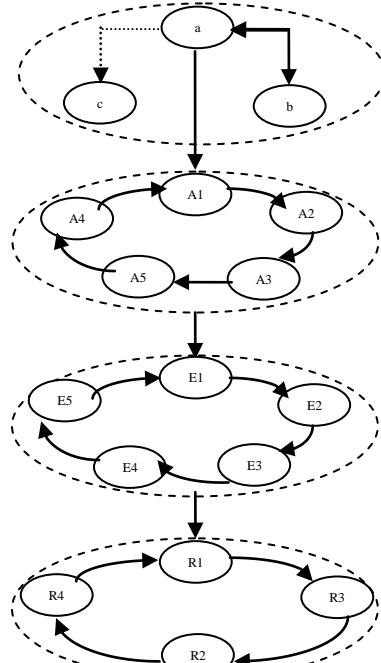


Figure 4.2: The process of Metacognition S-3

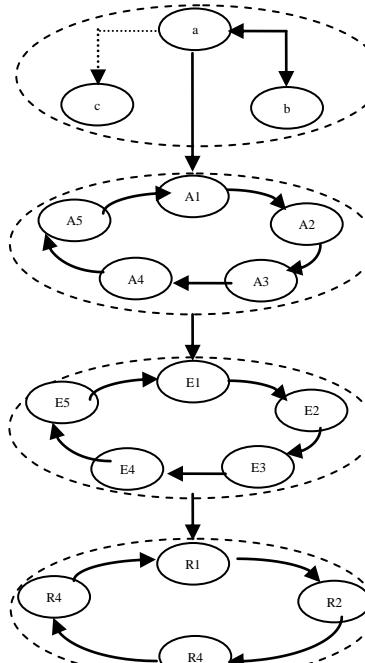


Figure 4.3: The Process of Metacognition S-4

### C. The Process of Low Ability Students Metacognition

The low ability subjects of the study are S-5, and S-6. The changes of metacognition process S-5 and S-6 showed the activities that described the characteristics of indicator *awareness, evaluation, and regulation*. When finishing mathematical problems, the indicator in S-5 appeared in the order A1, A3, A2, A4, A5, E1, E2, E3, E4, R1, R3, R2, R4, R3. The order of appearing indicator in S-5 can be seen in figure 4.4 and the order of appearing indicator in S-6 can be seen in figure 4.5

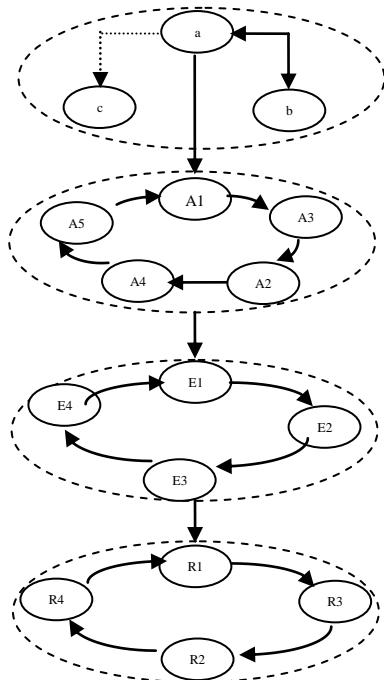


Figure 4.4: The process of Metacognition S-5

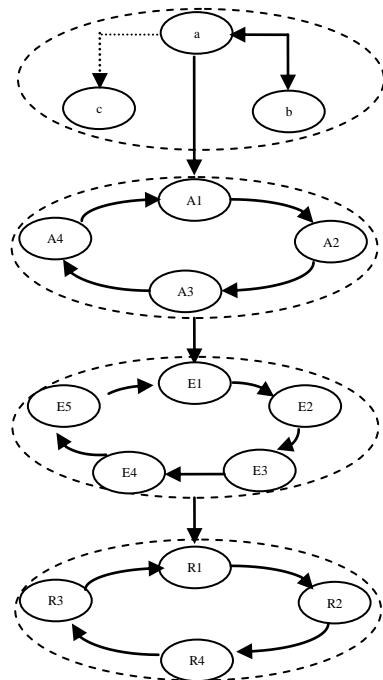


Figure 4.5: The Process of Metacognition S-6

The finding of the result of the study can be related to the previous theory as the basic of the study. If relating to the opinion of Wilson and Clarke (2002, 2004), each of the characteristic of the components are activities concerning with someone's ability to think of what he/she has done when finishing the problem. The problem of the study is how the subject of the study can find out the believed ways as the answers of the mathematical problems. The belief of the subject of the study is based on the process of evaluation and regulation done and as a part of abilities to observe and control him/her-self toward something known. Self-controlling can be done when finishing the problems (Mokos & Kafoussi, 2013). In a accordance with the indicator during the process of metacognition happened, it can be seen that the activities are the types of metacognition activities. The type of awareness is what the students know, what students need to finish the problems, what the student must do, where the students were. The type of evaluation activities result of evaluation, grading of students' difficulties problems, grading the improvement, ability, and understanding. The type of regulation activity is the strategy of planning, selecting the strategy, formulating the goal. Magiera & Zawojewski (2011).

### E. CONCLUSION

Based on the data analysis, it can be inferred that high ability students are in complete and order of their characteristic of metacognition process, middle ability students are in complete and disorder of their characteristic of metacognition process, while low ability students are in incomplete of their characteristic of metacognition process.

The complete and order of characteristic of metacognition process is in S-1 and S-2, the complete and disorder of characteristic of metacognition process is in S-3 and S-4, and the incomplete characteristic of metacognition process is in S-5 and S-6 can be in detail as follows: The component of awareness with A1 brings out 6 characteristics, A2 brings out 7 characteristics, A3 brings out 5 characteristics, A4 brings out 8 characteristics, and A5 brings out 4 characteristics. The component of evaluation with indicator E1 brings out 6 characteristics, E2 brings out 6 characteristics, E3 brings out 6 characteristics, E4 brings out 4 characteristics, E5 brings out 4 characteristics. The component of

regulation with the indicator R1 brings out 5 characteristics, R2 brings out 3 characteristics, R3 brings out 4 characteristics, R5 brings out 6 characteristics.

The explanation of the order of indicator of 6 subjects of the study can be clarified as in the following table.

**TABLE 1: The Order of Metacognition Process of Subject of Study**

No.	The Subject of the Study	The Component of Metacognition	The Order of Indicators
1.	S-1 and S-2	<i>Awareness</i>	A1, A2, A3, A4, A5
		<i>Evaluation</i>	E1, E2, E3, E4, E5
		<i>Regulation</i>	R1, R2, R3, R4
2.	S-3	<i>Awareness</i>	A1, A2, A3, A5, A4
		<i>Evaluation</i>	E2, E1, E3, E4, E5
		<i>Regulation</i>	R1, R3, R2, R4
3.	S-4	<i>Awareness</i>	A1, A2, A3, A4, A5
		<i>Evaluation</i>	E1, E2, E4, E5
		<i>Regulation</i>	R1, R2, R4, R3
4.	S-5	<i>Awareness</i>	A1, A3, A2, A4, A5
		<i>Evaluation</i>	E1, E2, E3, E4
		<i>Regulation</i>	R1, R3, R2, R4
5.	S-6	<i>Awareness</i>	A1, A2, A3, A5
		<i>Evaluation</i>	E1, E2, E3, E4, E5
		<i>Regulation</i>	R1, R2, R4, R3

#### REFERENCES

- [1] Anderson, L.W. & Krathwohl, D.R. 2001. *A Taxonomy for Learning, Teaching, and Assessing (A Revision of Bloom's Taxonomy of Educational Objectives)*. New York: Addison Wesley Longman, Inc.
- [2] Baker, L. & Brown, A. L. 1984. Metacognitive Skills and Reading. In Douglas J. Hacker, John Dunlosky and Arthur C. Graesser (Eds.) *Handbook of Metacognition in Education*. (p. 7-25). New York: Routledge.
- [3] Biryukov, P. 2003. *Metacognitive Aspects of Solving Combinatorics Problem* Kaye College of Education. (Online), ([www.cimt.plymouth.ac.uk](http://www.cimt.plymouth.ac.uk)), accessed in February, 13<sup>th</sup>2014.
- [4] Charter, E. 2003. The Use of Methods in Qualitative Research an Introduction to Method. *Brook Education*. (Online), Volume 12 Number 3 2003. p. 68-82, ([www.brock.scholarsportal.ca](http://www.brock.scholarsportal.ca)), accessed in August, 22<sup>nd</sup> 2015.
- [5] Cromley, J.G. 2005. *Metacognition, Cognitive Strategy Instruction, and Reading in Adult Literacy*. (Online), ([www.ncsall.net](http://www.ncsall.net)), accessed in March, 1<sup>st</sup> 2014.
- [6] Davidson, J. E. & Sternberg, R. J. 1998. Smart Problem Solving: How Metacognition Helps. In D. J. Hacker., J. Dunlosky, A. C. Graesser (Eds.), *Metacognition in Educational Theory and Practice* (pp. 47-68). Mahwah, NJ: Lawrence Erlbaum Associates.
- [7] Desoete, A., Roeyers, H. & Buysse, A. 2001. Metacognition and Mathematical Problem Solving in Grade 3. *Journal of Learning Disabilities*; Sep-Oct 2001; 34, 5; Academic Research Library.p 435. (Online), ([www.fi.uu.nl](http://www.fi.uu.nl)), accessed in September, 16<sup>th</sup> 2014.
- [8] Djamarah, S.B. 2008. *Psikologi Belajar*. Jakarta: Rineka Cipta
- [9] Duffin, J.M. & Simpson, A.P. 2000. A Search for Understanding. *Journal of Mathematical Behavior*. 18(4): 415-427.
- [10] Efklides, A. 2005. *Metacognition and Affect: What Can Metacognitive Experiences Tell Us about the Learning Process?* (Online), ([www.researchgate.net](http://www.researchgate.net)), accessed January, 24<sup>th</sup>2014.
- [11] Flavell, J. 1976. Metacognitive Aspects of Problem Solving. in L. Resnick (Ed), *In the Nature of Intelligence*. (Online), ([www.library.edu](http://www.library.edu)), accessed in July, 12<sup>th</sup>2014.
- [12] Goos, M., Galbraith, P. & Renshaw, P. 2002. Socially mediated metacognition: creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics*, 49, 193-223.
- [13] Howard, B.C., McGee, S., Shia, R. & Hong, N.S. 2000. *Metacognitive Self-Regulation and Problem-Solving: Expanding the Theory Base Through Factor Analysis*. (Online), ([www.cet.edu](http://www.cet.edu)), accessed in September, 12<sup>th</sup> 2014.
- [14] Hudojo, H. 1988. *Mengajar Belajar Matematika*. Jakarta: Departemen Pendidikan dan Kebudayaan.
- [15] In'am, A., Sa'ad, N.& Ghani, S.A. 2012. A Metacognitive Approach to Solving Algebra Problems. *International Journal of Independent Research and Studies*. University Pendidikan Sultan Idris, Malaysia. (Online), ISSN: 2226-4817; EI ISSN: 2304-6953 Vol. 1, Number 4, October2012, p162-173, ([www.aiars.org/ijirs](http://www.aiars.org/ijirs)), accessed in May, 4<sup>th</sup> 2014.
- [16] Livingstone, J.L.1997. *Metacognition: An Overview*. (Online), ([www.gse.buffalo.edu/fas](http://www.gse.buffalo.edu/fas)), accessed In June, 2014.
- [17] Kuzle, A. 2011. *Patterns of Metacognitive Behavior During Mathematics Problem Solving in a Dynamic Geometry Environment*. (Online).Volume 8, Number 1, January 2013, ([www.jwilson.coe.uga.edu](http://www.jwilson.coe.uga.edu)), accessed InSeptember, 2014.
- [18] Larkin. S. 2000. *How Can We Discern Metacognition in Year One Children from Interactions Between Students and Teacher*. (Online), ([www.tlrp.org](http://www.tlrp.org)), accessed in July, 2014.
- [19] Lioe, L.T., Ho Ka Fai & Hedberg. 2006. *Students' Metacognitive Problem Solving Strategies in Solving Open-ended Problems in Pairs*. (Online), ([www.math.ecnu.edu.cn](http://www.math.ecnu.edu.cn)), accessed in February, 3<sup>rd</sup> 2014.
- [20] Magiera, M. T. & Zawojewski, J. S. 2011. Characterizations of Social-Based and Self-Based Contexts Associated with Students' Awareness, Evaluation, and Regulation of Their Thinking During Small-Group Mathematical Modelling. *Journal for Research in Mathematics Education*. Number 5, Volume 42 November 2011. p. 486-516.

- 
- [21] Mokos, E. & Kafoussi, S. 2013. Elementary Students' Spontaneous Metacognitive Functions Different Types of Mathematical Problems. *Journal Research in Mathematics Education*. (Online), volume 2 number 2, June 2013. p 242-267. ([www.hipatiapress.com](http://www.hipatiapress.com)), accessed in April, 14<sup>th</sup> 2014.
- [22] Moleong, L.J. 2008. *Metode Penelitian Kualitatif*. Bandung: PT Remaja Rosda Karya.
- [23] Panaoura, A., Gagatsis, A. & Dimetriou, A. 2009. An Intervention to the Metacognitive Performance: Self-Regulation In Mathematics and Mathematical Modeling. *Acta Didactica Universitatis Comenianae Mathematics*, (Online), Issue 9, 2009, p. 63–79, ([www.researchgate.net](http://www.researchgate.net)), accessed in June, 2<sup>nd</sup> 2014.
- [24] Polya, G. 1988. *How to Solve It, A New Aspect of Mathematical Method*. Oxford: Princeton University Press Princeton and Oxford.
- [25] Praba, G. J. 2013. Metacognitive Instruction and Cooperative Learning Strategi for Promoting Insightful Learning in Science. *International Journal on New Trends in Education and Their Implications*. (Online), January 2013 Volume: 4 Issue: 1 Article: 15 ISSN1309-6249.January 2013, ([www.ijonte.org](http://www.ijonte.org)), accessed in March, 30<sup>th</sup> 2014.
- [26] Purnomo, D. 2014a. *Proses Metakognisi Matematis Siswa dalam Pemecahan Masalah*.  
The paper presented in National Seminar of Mathematical Education in The University of Ronggolawe, Tuban, In May, 24<sup>th</sup> 2014.
- [27] Purnomo, D. 2014b. *Profil Metakognisi Matematis Siswa Kelas 3 Sekolah Dasar pada Pemecahan Masalah Bangun Datar Berdasar Kerja Kelompok*.  
The paper presented in National Seminar of Educational Research in The Kanjuruhan University of Malang.
- [28] Purnomo, D. 2014c. *Proses Metakognisi Matematis Siswa Sekolah Dasar pada Pemecahan Masalah dalam Kelompok Kecil*.  
The paper presented in The XII Mathematics Nasional Conference in The Sepuluh Nopember Technology Institute, Surabaya, In June, 11<sup>th</sup> 2014.
- [29] Purnomo, D. 2014d. *Proses Metakognisi pada Kelompok Kecil Siswa Kelas III Sekolah Dasar dalam Memahami Konsep Bangun Datar*. The paper presented in National Seminar of Mathematical Education in The University of Sanata Dharma, Jogjakarta, in September 2014.
- [30] Schoenfeld. 1994. *Mathematical Thinking and Problem Solving*. New Jersey: School Mathematics. Reston: NCTM
- [31] Soedjadi, R. 2000. *Nuansa Kurikulum Matematika Sekolah Di Indonesia*. Dalam Majalah Ilmiah Himpunan Matematika Indonesia (The X Prosiding Mathematical Nasional Conference in ITB, July,17-20<sup>th</sup> 2000
- [32] Tan O. S., Parsons, R. D., Hinson, S. L.,& Sardo-Brown, D. 2003. *Educational Psychology a Practitioner-Researcher Approach*. Australia: Thomson.
- [33] Veenman, M.V.J., Wilhelm, P.,& Beishuize, J. J. 2004. *The Relation Between Intellectual and Metacognitive Skills from a Developmental Perspective*. (Online), ([www.elsevier.com](http://www.elsevier.com)), accessed in September, 14<sup>th</sup> 2014.
- [34] Wellman, H. M. 1985. Origins of Metacognition. In D. L. F. -Pressley, G. E. Mc Kinnon and T. G. Waller (Eds.), *Metacognition, Cognition and Human Performance* (Volume 1). Orlando, Florida: Academic Press.
- [35] Wilson, J. & Clarke D. 2002. *Monitoring Mathematical Metacognition*. Paper presented at the Anual Meeting for the American Education Research Assosiation, New Orleleans, LA.
- [36] Wilson, J. & Clarke D. 2004. Towards the Modelling of Mathematical Metacognition. *Mathematics Education Research Journal*.(Online), 16(2) p. 25-48, ([www.files.eric.ed.gov](http://www.files.eric.ed.gov)), accessed in July, 12<sup>th</sup> 2014.
- [37] Zawojewski, L.R. 2007 Problem-Solving and Modeling. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 763-804). Reston, VA: NCTM.
- [38] Zechmeister, E.B. & Neyberg, S.E. 1982. *Human Memory: An Introduction to Research and Theory*. Monterey, CA: Brook/Cole Publishing Company.