

# Increasing Higher Order Thinking Skill to Build Student's Character by Using Mathematical Reasoning

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**Abstract**— This paper discusses about mathematical reasoning can increasing higher order thinking skill which can build student's character while learning process. In the process of mathematics learning, students will be faced on routine and non routine problems, so they need critical and creative thinking skills to solve the problems. Mathematical reasoning is one of goals from learning mathematics. There are five various types of mathematical reasoning, such that; 1) Algebraic reasoning, 2) Proportional reasoning, 3) Statistical reasoning, 4) Geometrical reasoning, 5) Probabilistic reasoning. With mathematical reasoning, teachers can provide space for students to think logically in understanding mathematics concepts and consider all the possibilities that exist to solve problems. In the process of finding the solution of this problem, students will require unusual thinking skills are often referred to as higher order thinking skills. Improved way of thinking in the process of mathematical problem solving can build performance and moral student's characters such as discipline, honest, responsible; never give up etc. Because there are six pillars of character such that, 1) Trust, 2) Respect, 3) Responsibility, 4) Fairness, 5) Caring and 6) Citizenship. If this characters be applied to life, it can helping students to facing developments and global challenges.

**Keywords:** *Higher Order Thinking Skill, Mathematics Reasoning, Student's character*

## I. INTRODUCTION

The rapid growth of science and technology has made free flow of information accessible to everybody. The fact has shown that government is incapable of setting a certain limit to the accessibility and censorship of information which resulting in easy access to misbehavior. Weak parental and societal guidance in selecting information set the students as the only ones to decide for themselves of whatever information to be accessed. Brawling as an instance is a common reality presented by media technology. As this rampant phenomenon is shown, it then becomes a trend of the youth nowadays. Thus, like a tuberculosis, it spreads everywhere instantly from a certain place to another and from an educational institution to another.

Indeed there is a lot of accessible information either good or unhealthy ones whose unconfined accessibility can lead students astray. Consequently, the presence of the advantages and disadvantages of free access to this information influence the formation of students' characters. Aside from the problems arisen from the free access to information, the fact that human persons are relational being, there are indeed problems emerged in the relationship either with persons around the place where they live, with other students at school or with whoever they come in contact with. What is saddening nowadays is not anymore problems emerged from actual relationship with fellow students but rather problems encountered within virtual reality. The interaction and involvement of students with the latter reduce the necessary encounter with fellow students at school which basically formative.

However, regardless the above mentioned detrimental situations encountered by students, the government still has great hope for the schools which take into consideration the formation of students' characters. With the issue of Permendiknas No. 41 Tahun 2007 which talks about standard process, where in the learning process beginning from introduction, core substances, and conclusion, these three parts are taken into consideration so as to contain and give priority to expected values. This *Student Centered Principle* as it is called is above all aimed at the assimilation of values in life by students. Aside from that, teachers are expected to become living exemplars of how to live out of the values that they impart to the students.

Fundamentally in learning process the expected values to be achieved by students are integrated in every subject in the school not to exclude in Mathematics. The latter is the most common subject offered in school at any level. The main characteristic of this subject is not a usual skill as found in any other subjects. Its skill is called *Higher Order Thinking Skills (HOTS)*. *Higher Order Thinking Skills (HOTS)* is a thinking process of how to find answers or solutions in the midst of difficult and tricky situations by way of taking and connecting new and stuck knowledge so as to make them wider knowledge. HOTS in Bloom's taxonomy, falls on the category of cognitive analysis, evaluation, and creation. This cognitive analysis and evaluation falls under critical thinking while cognitive creation is under creative thinking<sup>[1]</sup>.

Indonesia is not included in the number of countries which are considered as well-versed in science and technology. This is because of the low achievements of our education. This low achievement is gauged from the ability of students in answering questions which demand higher thinking skills. Based on the datum given by PISA in 2012, from among 65 countries in the list, Indonesia is in the 64<sup>th</sup> rank when it comes to science and mathematical skills. Indonesian students are used only with common questions on first and second levels which consequently resulting in lower acquirement of grades when they are given questions based on PISA standard which are actually ranged from first to sixth level and basically contextual in their characters.

The government then applies 2013 curriculum with the perspective of facing the demands of education in this globalization era. The theme of the development of the said curriculum is a curriculum which is capable of producing Indonesian individuals who are productive, creative, innovative, and affective through the strengthening of good positive values, skills, and integrated knowledge. Competent attitudes expected from the students are those attitudes that mirror piety, honesty, high regard for discipline, responsibility, care, courteousness, responsive and pro-active behavior. Besides, students are expected to acquire factual, conceptual, procedural and meta-cognitive knowledge in different disciplines for them to be knowledgably competent. The latter demands students to possess the ability to act effectively and creatively so as to put into practice in the daily life methods and theories acquired in school.

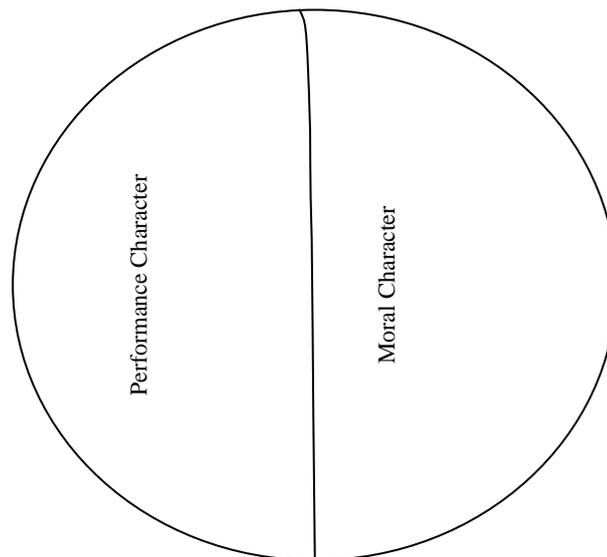
The learning system contained in 2013 curriculum stresses the indispensability of HOTS. Students with HOTS are not only those who memorize to the letter the information imparted at school but they also have the ability to put them into practice at daily bases. They possess analytic mind as well. Mathematical reasoning offers new efficacious ways to build into awareness and to express ideas about varied and wide phenomena in this globalization era. Thus, students with analytic minds tend to cling on pattern, structure, and regularity or irregularity either in concrete situations of life or hypothetical ones.

## II. STUDENT'S CHARACTERS

Human characters are different for each person, as well as student's character in the classroom. These differences are influenced by many factors; one of the factors is teacher's role in learning activities in the classroom. Teacher's role in the class may affect student's character, so teacher are expected to take more time to observe and learn student's character. Thus, teachers can choose appropriate learning methods to be able to build student's character. Characters are special markers that distinguish each person, or individual's qualities that made the difference. Naim [1] wrote in his book that character is a picture that shows attitude of right and wrong behavior, good or bad behavior. Characters can be classified into attitudes, behavior, motivation and skill, from those Naim wrote that characters include religious attitude, honesty, tolerance, caring, democratic, disciplined, hard working, independent, high curiosity, never give up, critical and creative. Hildebrandt & Zan [2] wrote that there are six pillars of character such that, 1) Trust, 2) Respect, 3) Responsibility, 4) Fairness, 5) Caring and 6) Citizenship. That opinion indicates that character will support human social life then can be used to deal with developments

and global challenges. Through learning, students are expected to have a character that can support their social life. So, mathematics teacher are expected help student to build their characters through the process of mathematics learning.

Characters cannot be formed in short time, when teacher want to building the student's character they must be patient. Build student's character through mathematics learning is not easy, it is requires long time and continuously. However, character which build in this learning will be rooted on student's life. For example, when teacher divides students into small groups and ask them to discuss an issue then the characters that appear are tolerance, caring, curiosity and creative. Then, when the students were asked to convey their discussion's result, the characters that appear are democratic, critical and creative. When teachers provide an opportunity for students to corparate their abilities then the characters are formed will be better. Summarize the Piaget's opinions [2] that students who intellectually passive then they do not have independent themselves, so teachers are asked to give them a chance to be active intellectually.



Picture 1st. Performance and Moral Character by Matthew

Learning mathematics which given by teachers can be used as bridge for students to build their good characters. When teachers provide opportunities for students, it will minimize rebel's attitude or minimize moral damage that happens now days. As written by Matthew Davidson [2] in his research that characters are defined into two sections interconnected, there are performance character and moral characters. Performance characters include persistence, tenacity, work ethic, positive attitude, ingenuity and disciplined which used to support academic's activities, work and life. While the moral characters include integrity, fairness, caring, respect and cooperation are needed in interpersonal relationship. So, to able to treat other students with good attitude, then a student must be has a good character too. Mathematics teachers can also manage their learning mathematics to be able to support characters building as exemplified advance.

Students have different characteristics and should be used as one of the focus of attention in learning, so that optimal learning results obtained. Therefore, at least the teacher must know the character of each student. By knowing the character of their students, teachers will gain some of the benefits that would be used as one of the cornerstone in the implementation of learning activities. The benefits of such teachers can recognize the ability of the students, the student learning experience, the social background of students, the level of development and the needs of students, as well as the level of mastery of previous students.

Learning mathematics is a process organized by the teachers to teach the students to acquire science and math skills. When we ask a student for expressing his opinion about learning math, then it

will be a lot of complaints that sounds boring math lesson, not interesting, even too abstract, so that leads to the results of learning math. Teach mathematics is not easy due to the fact that the students experience difficulties in learning mathematics. To avoid those things, math learning should be based on the characteristics of the students.

### III. HIGHER ORDER THINKING SKILL

Bloom's taxonomy is considered as the basis for the classification of thinking skills. In revised Bloom's taxonomy, analytic, evaluative and creative skills are categorized as *transferring* or processing which is part of higher order thinking skills while the ability to remember, to comprehend and to apply the learning are categorized as *recalling* which is part of lower order thinking skills [3].

The school's curriculum which focused to develop the human resources (HR) includes the cognitive, affective and psychomotor. Higher Order Thinking Skills is one of the resources so that knowledge and skills must be improved and developed. Therefore, one indication the improvement success of human resources in the education field are students have a high level of skill is good, because the primary purpose of learning in the 21st century is to develop and enhance the student's HOTS. Especially in the math lesson HOTS is one of the priorities of developed, following Permendikbud No 64 Tahun 2013 about standard contents for mathematical subjects stated that math needs to be given to all students ranging from elementary school to equip learners with the ability of logical thinking, analytical, systematic, critical and creative & ability of problem solving and teamwork.

However, based on the results of TIMSS written by Mullis et al (2012) the position of Indonesia on the domain of cognitive reasoning and domain content numbers each are on the last ranking and rating to 37 from 43 countries. This suggests that the ability of the learners on the low, so the reasoning needs to be improved by making the process of learning mathematics based HOTS. In addition, based on the results of the survey conducted Musfiqi [4] to 20 Junior high school mathematics teacher in Jepara regency, found that 75% of respondents initiated their math learning with the introduction of definitions and formulas without connecting it to solving problems in a variety of contexts. While it 90% of the respondents had never planned nor implement learning that emphasizes on thinking skills. As a result of students' thinking ability has not been directed to the level of the higher thinking skills, including the ability to think critically and creatively in problem solving.

Based on the above facts, it can be concluded that the ability of higher-order thinking students remains relatively low; this is due to the HOTS have not applied in the process of learning mathematics processed. Therefore, the perceived need for a learning process oriented enhancement as well as granting HOTS questions based HOTS. In line with that, based on the principle of teaching from NCTM (NCTM, 2000, p. 20) "effective mathematics teaching requires understanding of what students know and need to learn and then give the challenge and support them to learn them well".

According to Bloom [5], Higher Order Thinking Skills is a process that involves mental, such as classification, induction, deduction, and reasoning. Stein and Lane [6] defines a high level thinking is the complex thinking, non-algorithmic thinking to solve a task in which there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instruction, or a worked out example. According Stein, higher-order thinking using a complex thinking, non algorithmic to complete a task, there is that unpredictability, uses a different approach with existing tasks and different from the example. Higher-order thinking is the ability to complete tasks where there is no algorithm that has been taught, which require justification or explanation and have more than one possible solution [7]. High level thinking skills learners will produce: proficiency students in problem-solving strategies to become good, confidence level learners in mathematics increased, and the learning achievements of students in non-routine problem that demands high level thinking skills increase.

Mc Loughlin and Luca [8] state that HOTS means the ability to comprehend the information beyond what is given, to adopt critical disposition, to possess the meta-cognitive awareness, and to solve problems. HOTS is an ability to connect, manipulate, and transform knowledge as well as already possessed experience so as to think critical and creatively in order to decide for the necessary things to do and to solve problems in any new situations. According to Susan M. Brookhart [9], HOTS is classified

into three categories. Those categories are which define HOTS as *transferring* in matter of critical thinking and in problem solving.

Based on the definition, higher order thinking skill leads to problem solving skill. According to Woolfook [10], problem solving skill is a skill of a student in directing his/her thoughts to solve problems through the gathering of facts, analysis of information, putting together the many alternatives of solving them, and selecting the way the most effective means to solve the problems. Thus, HOTS is a one's skill to criticize, to find solution to problems which are complex and to be able to give several alternative solutions over a certain problem by way of manipulating different information acquired. Human persons are not the only ones who can solve problems but solving problems is identical to human activities. On the basis of the opinions above, then it can be inferred a high level thinking skills is thinking skills at a higher level than just memorizing content and involve diverse thought processes of analyzing, evaluating and creating a conditioned in the process of mathematical problem solving.

According to Krathworl & Anderson in Arifin [11] Practice, indicators to measure higher-order thinking skills include: (1) analyze: (a) Distinguish (differentiating) include the ability to distinguish the parts of the overall structure of the form accordingly. Distinguish happens when students discriminate information relevant and irrelevant, what's important and not important and then pay attention to the information that is relevant and important. (b) Organize (Organizing) include the ability to identify the elements together into a structure. The process of organizing occurs when students build relationships a systematic and coherent (related) between pieces of information. (c) Attributing is the ability of students to mention about viewpoints, refraction, the value or the intention of concerns. (2) Evaluate: (a) Checking is the ability to test the internal consistency or error in operation or the outcomes as well as detect the effectiveness of the procedures used. (b) Criticize is the ability to decide the result or operation based on specific criteria and standards, and detect whether the results obtained on the basis of a procedure are approaching the correct answer. (3) Created: (a) Formulating hypothesis or make hypothesis, involves the process of describing the problem and make choices that meet certain criteria. (b) Planning process involves plan a method of settlement of an issue that is in accordance with the criteria of the problem. (c) Producing involves the process of implementing a plan to solve a problem that meets certain specifications.

Newman and Wehlage [12] state that HOTS require students to requires students to manipulate information and ideas by way of transforming their meaning and implications, such as when students combine facts and ideas in order to synthesize, generalize, explain, hypothesize, or arrive at some conclusion or interpretation. Realizing that knowledge is wide, with HOTS students will learn more profoundly and comprehend the concept better. This is definitely in accordance with substantial character of every subject when students are expected to be able to express their understanding over the matter with profundity. With HOTS, students will be able to differentiate ideas with clarity, to stand the arguments soundly, to solve problems, to construct due explanations, to hypothesize, and to comprehend complex matters clearly. HOTS is learnable, it can be taught to students, and with it, students' character can be formed accordingly. Furthermore it is said that there is indeed difference between rote learning with that of HOTS which applied higher order thinking. To think means to use one's analytical, creative, and applicable skills. This sort of thinking is what is needed in one's daily life.

It is also said that with HOTS students can become independent thinkers, hence the arguments uttered indicates the quality of his/her abilities. Applying HOTS as one of the means of learning leads to the productive learning especially in socio-cognitive interaction, such as: (1) giving and receiving help, (2) changing and completing information, (3) elaborating and explaining concepts, (4) sharing of knowledge with co-students, (5) mutual giving and accepting correction, (6) collaboratively doing assignments, and (7) rending necessary contribution in overcoming challenges. With the approach brought by HOTS, students are invited to actively think especially in the problem solving [13].

#### IV. MATHEMATICS REASONING

Learning school mathematics is assumed to be given to a student who never know mathematics before. According to Joyce and Well [14] that the teacher's duty in the learning process is to assist students to finding information, ideas, skills, value, way of thinking, as well as looking for good way of learning and express themselves. Having regard to learning's rule, teachers may be able to evoke right

way to makes mathematics reasoning. In Permendikbud No.22 Tahun 2006; that students are expected to use reasoning, logical thinking, analytic, systematic, critical and creative in solving mathematical problems. Thus, the reasoning ability is one of important focus in learning mathematics, because if the students have not been able to reasoning, then they will difficult to solve the problems presented. Mathematics reasoning is not an ability that can be seen in a short time. Students, who are able to reasoning properly, usually trained in completing various forms of problems presented by teachers, both problems are simple to complex problems. Given the problems with the conditioning routine, non mathematical reasoning students will be expected to build and can be applied to the problems faced in the future.

Each student needs reasoning in order to achieve the desired learning objectives. Reasoning has not been clearly defined, but if makes hypothetical analytic reasoning means controlling and developing the thought of a thing until became clear. Then, when drawn in mathematical it is called mathematical reasoning. Mathematical reasoning is an ability to solve mathematics problems. There are five various types of mathematical reasoning, such that; 1) Algebraic reasoning, 2) Proportional reasoning, 3) Statistical reasoning, 4) Geometrical reasoning, 5) Probabilistic reasoning. Which all of those component are in learning school mathematics. When students are learning mathematics, teacher must be direct students to build their reasoning.

Walle [15] makes a conclusion that algebraic reasoning is ability to representing, generalizing and formalizing pattern and regularity in all aspect of mathematics. There are levels within learning algebra. Learning school mathematics is not always use abstract symbol, because teachers must be developing mathematics reasoning gradually. Lesh, Post, and Behr [16] claimed that proportional reasoning is ability to compare different comparison and mental ability to store and process the information pieces by using sensory. So, when students learn about ratio and they can compare all of different comparison, its mean students have a proportional reasoning. Scaffer [17] stated that statistical reasoning is ability to solve mathematics problems related to corporate data, sampling and experimentation, anticipating patterns and statistical inference. Then Jeanette [18] says that “geometrical reasoning is the process defining and deducing the properties of a geometric entity using the intrinsic properties of that entity, its relationships with other geometric entities and the rules of inference that kind such properties together in geometric space”. The last of various types of mathematical reasoning is probabilistic reasoning. Have the same principle with statistical reasoning, probabilistic reasoning is ability to see pattern in data.

## V. RELATION BETWEEN MATHEMATICS REASONING, HOTS AND STUDENTS CHARACTER

Students who give non routine mathematical problems in continue ways; will give the effect of improved thinking skill. Thinking improvement is characterized by the reason ability or gives reasons over the settlement they earn. This ability is called mathematics reasoning. Mathematics reasoning needs critical and creative thinking skills, which is both an indicator of higher order thinking skills. So it is understood that reasoning, critical and creative thinking skills are indicators of Higher Order Thinking Skill such as formulated by the Bloom that Higher Order Thinking Skills is a process that involves mental, such as classification, induction, deduction, and reasoning.

Thus, it is known that in order to improve high order thinking skills required learning mathematics with mathematical reasoning. But note that mathematical reasoning include analyze, differentiate, simplify, presenting and illustrating with accompanying reasons. And this would not be realized in learning just a few times, but it takes a long time to let those skills appear in students. The exposure of previously known that high level thinking skills can foster student’s character, it is seen from the student's persistence in solving mathematical problems presented by the teacher. To solve mathematical problems in diverse, it is not possible for students to find an alternative solution in short time, which could indicate towards a critical and creative. In the face of diverse mathematical problems, the non routine, students need quite a long time so that it will appear in the process traits of character that leads to students as thorough, diligent, sensitive, honesty, tolerance, caring, democratic, disciplined, hard working, independent, high curiosity, never give up, critical and creative.

## VI. CONCLUSION

Higher Order Thinking Skills is a process that involves mental, such as classification, induction, deduction, and reasoning. And if we drawn in mathematics there are five various types of mathematical reasoning, such that; 1) Algebraic reasoning, 2) Proportional reasoning, 3) Statistical reasoning, 4) Geometrical reasoning, 5) Probabilistic reasoning. With mathematical reasoning, teachers can provide space for students to think logically in understanding mathematics concepts and consider all the possibilities that exist to solve problems. In the process of finding the solution of this problem, students will require unusual thinking skills are often referred to as higher order thinking skills. Improved way of thinking in the process of mathematical problem solving can build performance and moral student's characters such as discipline, honest, responsible; never give up etc. Because there are six pillars of character such that, 1) Trust, 2) Respect, 3) Responsibility, 4) Fairness, 5) Caring and 6) Citizenship. When learning mathematics that uses mathematical reasoning as one of focus learning can increasing higher order thinking skill. This can be achieved when in learning process; teachers provide opportunities for students to explore their ability. Then, with non routine problems, students will be used to looking for creative solutions, critically analyze, unyielding seek a proper, honest and tolerance in the difference solutions. And teacher's belief when provide an opportunity for their students can affect their moral characters, because students will find that their mathematics teacher is an open-hearted person. So, students will apply these characters in their life.

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