ME – 20

The Student' Models For The Meaning And Procedure Of Multiply Two Fractions

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Abstract—Lortie-Forgues, Tian and Siegel (2015) suggests that students' understanding of the fractions is very important in the study of mathematics further and are also used in many professions, but according to Lortie-Forgues, Tian and Siegle (2015) and MA (1999), many students have great difficulty in understanding it. Furthermore, according to Ma (1999), the difficulty is not only the difficulties experienced by students in learning fractions, but also the difficulties experienced teachers to teach the concept of fraction. It was felt by teacher at one of the private elementary school in Yogyakarta, especially in teaching multiplication on fraction. The goals of this study were (1) finding the student'model that could be constructed when they learnend about the meaning and procedure of multiplication between an integer and a fraction, and (2) finding the student'model that could be constructed when they learnend about the meaning and procedure of multiplication of two fractions. There were two contexts used by the researchers in this study that is buying the ribbon and giving oranges. Lesson plan created by the researcher were for students of grade five. There were six models of multiplication between an integer and a fraction that could be constructed by students using that context. There were four models of multiplication of two fractions that could be constructed by students using that context. This type of research used by the researchers in this study was the design research developed by Gravemeijer and Cobb. According Gravemeijer and Cobb (in Akker, Gravemeijer, McKeney, and Nieveen, 2006) there are three phases in the research development, namely (1) the preparation of the trial design, (2) test the design, and (3) the retrospective analysis.

Key Words: *the multiplication of fractions, realistic mathematics education (RME), and design research.*

I. INTRODUCTION

In 2013 and 2014, the researcher developed some context and sequence of learning that can be used to teach the fractional multiplication in grade five of the elementary school. From the experience of two years, the researcher wanted to try to develop another context that will be used to teach the multiplication of the fraction in grade five of the elementary school. In this year, the researcher had the opportunity to develop and provide context about buying the ribbon, and giving oranges. The researcher also got the opportunity to pilot the lesson plan in one class on grade five in a private elementary school in Yogyakarta. In this paper, the researcher would present the student'models that could be built by students when problems were built with the context given to students.

Lortie-Forgues, Tian and Siegel (2015) suggests that students' understanding of the fractions is very important in the study of mathematics further and are also used in many professions, but according to Lortie-Forgues, Tian and Siegle (2015) and MA (1999), many students have great difficulty in understanding it. Furthermore, according to Ma (1999), the difficulty is not only the difficulties experienced by students in learning fractions, but also the difficulties experienced teachers to teach the concept of fraction. There were several studies that have been done related to fractions which explains why fractions into one material that is difficult to understand by students, namely:

1. According to Lamon (2001, in Ayunika, 2012), the development of understanding of the meaning of fractions in the teaching-learning process was a complex process because the concept of fraction had a

number of interpretations, namely (1) fraction as a part of the whole, (2) fraction as the result of a measurement, (3) fraction as an operator, (4) fraction as a quotient, and (5) fraction as a ratio.

2. According to Ross and Case (1999 in Shanty, 2011), on the process of learning fractions, teachers often emphasize on how to do the operation procedure than on the meaning of the operation.
3. Stafylidou dan Vosniadou (2004 in in Shanty, 2011) states that one of the reasons why the idea of mathematical fractions are systematically misinterpreted by students is an inconsistency with the principles of arithmetic used in operations involving natural numbers. For example in the operation of multiplication of natural numbers, if the two natural numbers multiplied, then the multiplicative result is a natural number greater than or equal to two natural numbers are multiplied. It was not always the case if the two fractions multiplied.
4. According Streefland (1991), in many textbooks the instruction of fractions is characterized by:
   a. Towards the concept of fraction.
   b. There are not meaningful contexts both as sources and domains for the application of fractions.
   c. The isolated use of models and patterns, which never extends to serve the process of algorithmization or mathematization.
   d. There are not connections with mathematically domains, such as decimal fractions, ratios, scale, and percentages (Vergnaud, 1981).
   e. Towards the algorithms.

There were two questions that will answer in this paper, namely (1) what were the student'model that could be constructed when they learnend about the meaning and procedure of multiplication between an integer and a fraction? and (2) what were the student'model that could be constructed when they learnend about the meaning and procedure of multiplication of two fractions?

II. THEORETICAL FRAMEWORK

The philosophy of RME was mathematics as a human activity, which means that the learning process of mathematics first of all should not be connected with mathematics as a deductive system that was well organized and formal, but it should be connected with mathematics as a human activity (Freudenthal, 1971, 1973, in Gravemeijer, 1994). If the mathematics which was learned by the student was connected with a formal deductive system, then the student will view that mathematics was resulted by the human thinking; it was an abstract and was not related to real-life. So, they will think that they could not find mathematics and using mathematics in their life. Learning mathematics should be able to make the students thought that there was mathematics in human activities, and it was be used by them in real life.

There are four main principles in the RME (Gravemeijer, 1991 and 1994, Treffers, 1991, and Julie, 2014), namely:

1. Guided reinvention;

According to this principle, students were given the opportunity to be able to reinvention both concepts and procedures in mathematics, "like" the mathematicians to find it. In the reinvention process was done by the students, in addition there was the teacher guidance, there needs to be a student communication, and there was a negotiation process between one student and other students. The communication and negotiation process between one student and other students were intended to develop students' findings gradually until the students can achieve the mathematics formal knowledge.

2. The progressive mathematizing;

In RME, students learned to construct a formal mathematical knowledge through to solve the contextual problem series. In RME, this process is known as the mathematizing process. Students were expected to experience the development in every stage of problem solving from one problem to other problems. This development was happen in the translating problem and in the retranslating solution of the problem. The problem solving process evolved from informal strategies to more formal procedures. In the end, the solution for a kind of the problem becomes routine. In other words, the solution procedure on the similar problem can be simplified further and formalized through the problem series, so that at the end, a formal procedure can be found by students. Through this learning process, a formal mathematical knowledge can be reconstructed by themselves. This process is illustrated in figure 1. In the RME, this process is called a progressive mathematization.

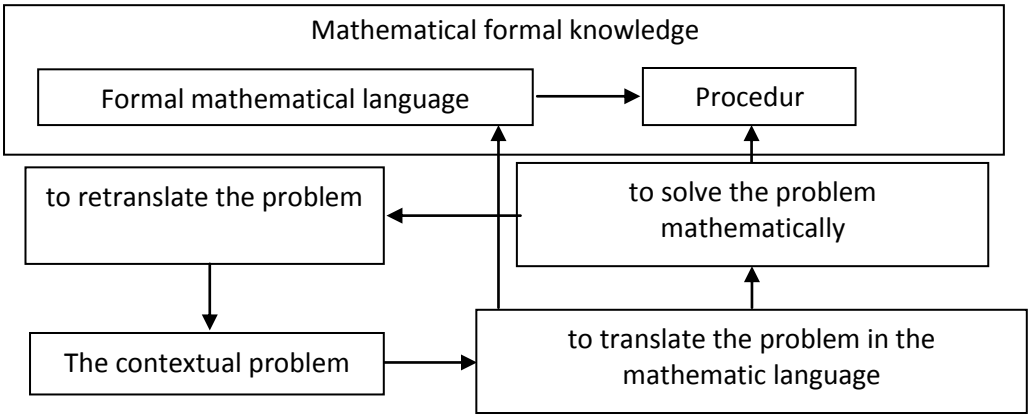


Figure 1: The reinvention process through the progressive mathematization process

3. Didactical phenomenology;

The students were given the opportunity to explore phenomena or situation series that can make students experience the process of establishing a formal mathematical knowledge in a sustainable manner. The purposes of the investigation of the phenomenon by students were to investigate the circumstances that approach to the particular phenomenon, and the results of the investigation can be generalized to generate solution procedures, so it would develop the formal mathematical knowledge.

4. Self-developed models.

In RME, models were interpreted as a representation of translating problems into the mathematics language and problem solving in the problem solving stages. A model in RME may involve a model of a situation, schematics, descriptions, or a way to express an idea or ideas. The modeling process by students played the role as a bridge between the informal and formal mathematical knowledge. In RME, the models must be built by themselves as a result of the exploration of the phenomenon by the students and the basis for forming a formal mathematical knowledge. It means that students should be given the opportunity to build models when the problem solving process was occured.

When teacher seek to build the formal mathematical knowledge of students, teacher need to do with the bottom-up approach. First, a model was related to real life activities, and it was called the **situational model**. After that, a model was a model of the specific context, and the model obtained in this way is termed **model of**. Then, the model was generalized to many similar situations, and the model was constructed in this way is termed the **model for**. At the end, the model becomes something truly lies within students, and can be used as a basis to achieve a formal knowledge of mathematics and it was called the **formal model**.

III. RESEARCH METHODOLOGY

The approach used to develop the students' learning materials and the teacher guide in this research activity was RME. This type of research that was used by the researcher in this study was the design research with three cycles. Things that were presented in this paper what was done by the researcher and what comes out of the third cycle. The data analysis was conducted by video data and the student's work. The steps undertaken by the researcher followed the phases in the development research were developed by Gravemeijer and Cobb.

IV. RESULTS

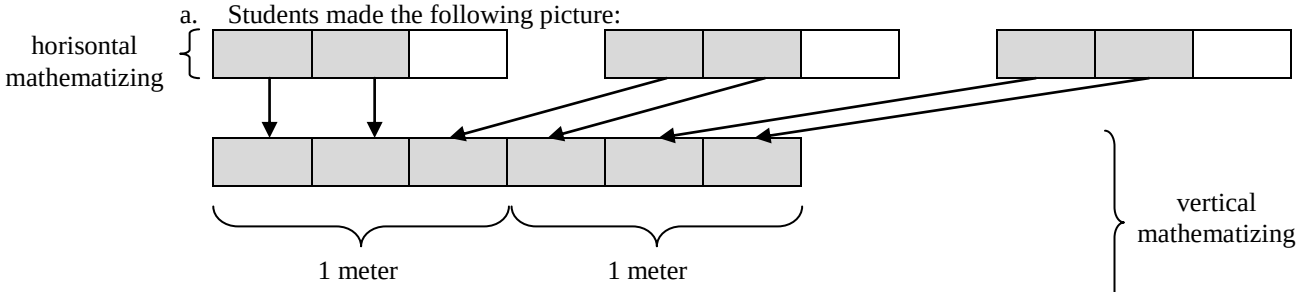
The research results presented in this paper were limited by the researcher on the third cycle. The aims of the design that was made by the researcher were that students could know about (1) the meaning and the procedure of the multiplicaton between an integer and a fraction, and (2) the meaning of multiplication of two fractions and the fractional multiplictation procedure. Before students experienced learning process designed by the researcher, students have learned about fractions in grade four, namely (1) the meaning of fractions, (2) the ordering of fractions, (3) the simplfying of fractions, and (4) the additing and subtracting of fractions. The problems were given to students inspired by the problems that

ISBN 978-602-74529-0-9

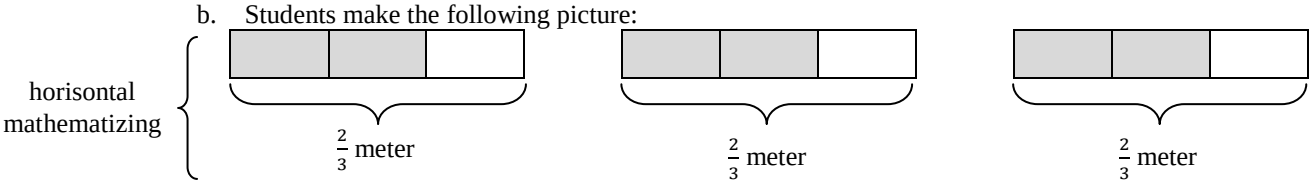
exist in the book that written by Fosnot, and Dolk (2002) and the teacher' idea who taught the students in grade five.

Here was presented problems that were given to students, and the student'model about (1) the meaning and the procedure of the multiplicaton between an integer and a fraction, and (2) the meaning of multiplication of two fractions:

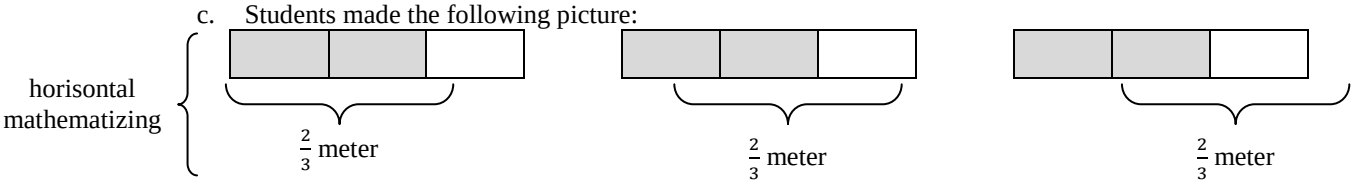
1. **The problem was given to students:**
   Kiki needed 3 pieces of ribbon for the gift decoration.
   The length of each ribbon was needed Kiki is $\frac{2}{3}$ meter.
   To fulfill the needs of a ribbon, Kiki would purchase the ribbon.
   How many meters of ribbon were to be purchased by Kiki?
   The Student'models:
   a. Students made the following picture:



   Thus, the length of the ribbon that needed to be purchased by Kiki was 2 meters.
   b. Students make the following picture:



   Students then made the following calculations: $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{2+2+2}{3}=\frac{6}{3}=2$. This step was vertical mathematizing.
   Thus, the length of the ribbon that needed to be purchased by Kiki was 2 meters. This step was hisontal mathematizing.
   c. Students made the following picture:

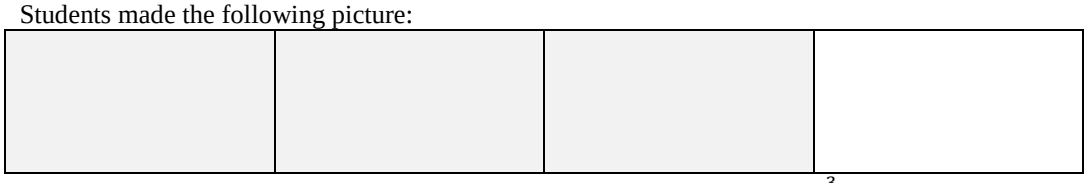


   Students then made the following calculations: $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=3\times\frac{2}{3}=\frac{3\times2}{3}=\frac{6}{3}=2$. This step was wertical mathematizing.
   Thus, the length of the ribbon that needed to be purchased by Kiki was 2 meters. This step was horisontal mathematizing.
   d. Students made the following calculations: $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{2+2+2}{3}=\frac{6}{3}=2$. This step was wertical mathematizing.
   e. Students made the following calculations: $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=3\times\frac{2}{3}=\frac{3\times2}{3}=\frac{6}{3}=2$. This step was wertical mathematizing.
   f. Students made the following calculations: $3\times\frac{2}{3}=\frac{3\times2}{3}=\frac{6}{3}=2$. This step was wertical mathematizing.

2. **The problem was given to students:**
   Gofil had $\frac{3}{4}$ kg of oranges.
   Gofil gave half part of oranges owned to Berto.
   How many kg of oranges would be given by Gofil to Berto?
   The Student'models:

a. Students made the following picture:

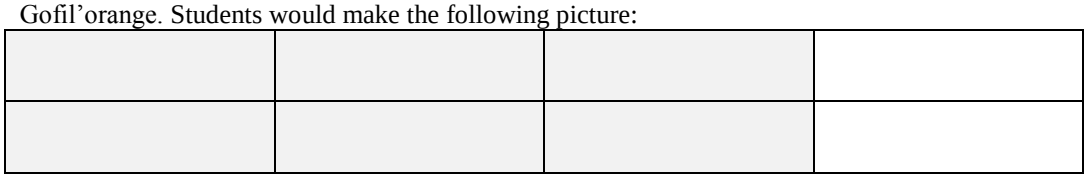
horisontal mathematizing



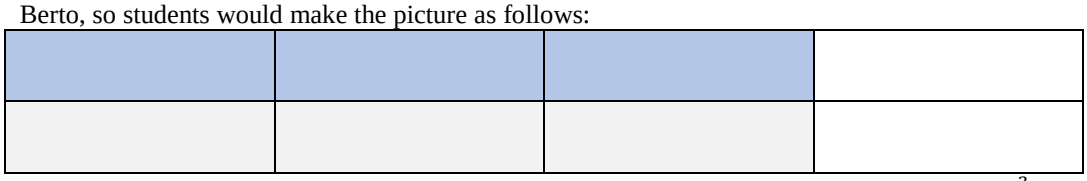
The gray shaded area was the heavy of the orange that owned by Gofil, i.e. $\frac{3}{4}$ kg.

horisontal mathematizing

Then, students shore the Gofil' orange into two equal parts. Students would get half part of the Gofil'orange. Students would make the following picture:



Then, students shaded with different colour to show the half part of the Gofil's orange given to Berto, so students would make the picture as follows:

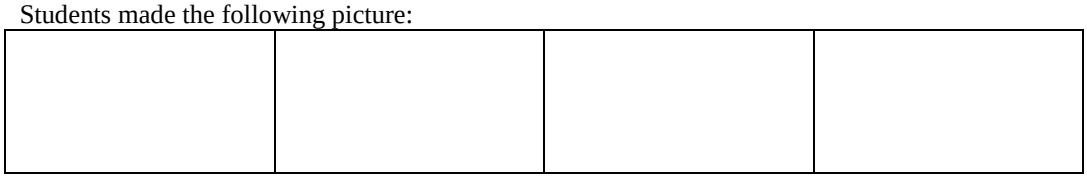


vertical mathematizing

1) The blue shade indicated the area of the Gofil'orange given to Berto, that is equal to $\frac{3}{8}$ kg. Because there were 3 blue shade parts of 8 parts of a whole.
2) The blue shade indicated the area of the Gofil'orange given to Berto, that is equal to $\frac{1}{2}$ part of $\frac{3}{4} = \frac{3}{8}$ kg. Because there were 3 blue shade parts of 8 parts of a whole.

b. Students made the following picture:

horisontal mathematizing

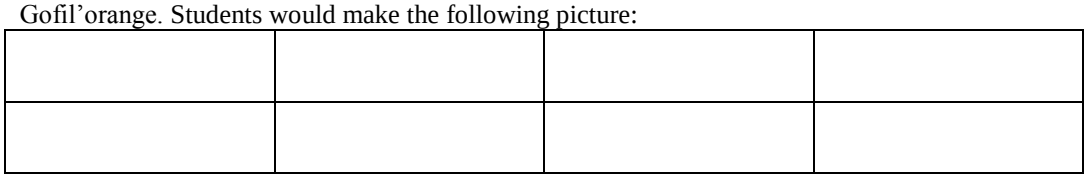


$\frac{3}{4}$

Students annotate the boundary area that showed $\frac{3}{4}$, so the weight of the Gofil' orange was represented by the left area of the boundary.

Then, students shore the Gofil' orange into two equal parts. Students would get half part of the Gofil'orange. Students would make the following picture:

$\frac{1}{2}$

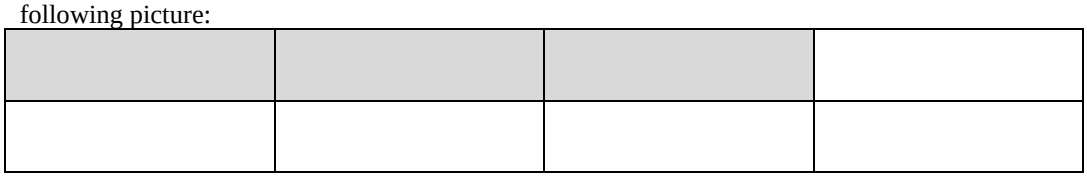


$\frac{3}{4}$

Students annotate the boundary area that showed $\frac{1}{2}$ part of $\frac{3}{4}$ kg, so half of the Gofil 'orange weight was represented by the upper area of the boundary.

Then, students shaded to indicate the area of Gofil'orange given to Berto, as shown in the following picture:

$\frac{1}{2}$



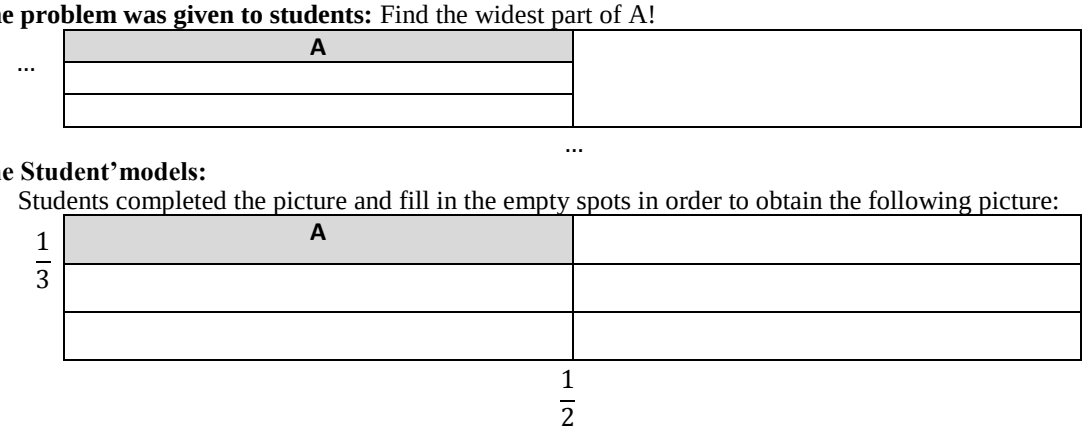
$\frac{3}{4}$

ISBN 978-602-74529-0-9

vertical mathematizing

1) The gray shade indicated the areas of Gofil'orange given to Berto, that is equal to $\frac{3}{8}$ kg. Because there were three blue shade parts of eight parts of a whole.
2) The gray shade indicated the areas of Gofil'orange given to Berto, that is equal to $\frac{1}{2}$ part of $\frac{3}{4} = \frac{3}{8}$ kg. Because there were 3 blue shade parts of 8 parts of a whole.

3. **The problem was given to students:** Find the widest part of A!



The Student'models:

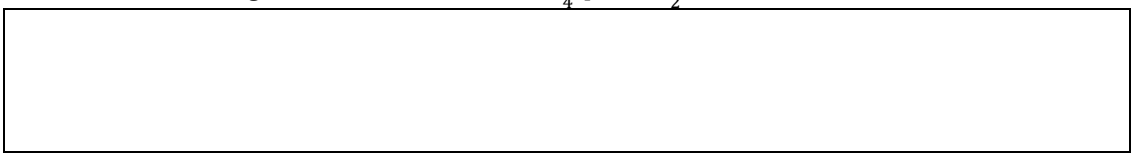
horisontal mathematizing

a. Students completed the picture and fill in the empty spots in order to obtain the following picture:

vertical mathematizing

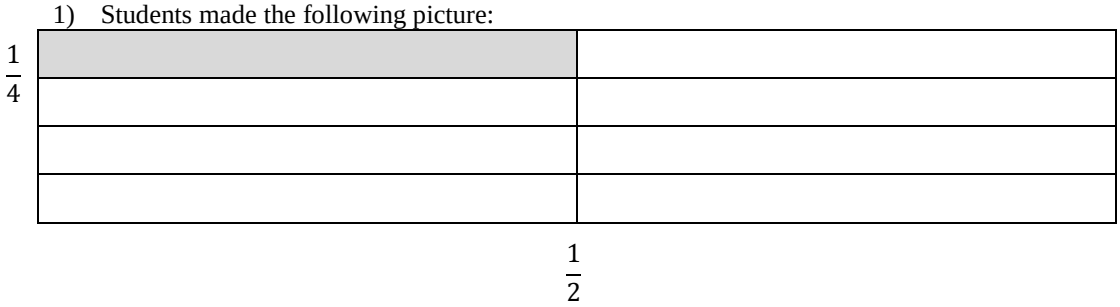
b. Then, students made calculation to find the widest part of A, i. e.:
   1) Students calculated, the widest part of A = $\frac{1}{3}$ part of $\frac{1}{2} = \frac{1}{6}$.
   Because there was one gray shade part of six parts of a whole.
   2) Students calculated, the widest part of A = $\frac{1}{3}$ part of $\frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6}$.
   3) Students calculated, the widest part of A = $\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6}$.

4. **The problem was given to students:**
Use the follow rectangle to illustrate the statement $\frac{1}{4}$ part of $\frac{1}{2}$ and calculate the results.

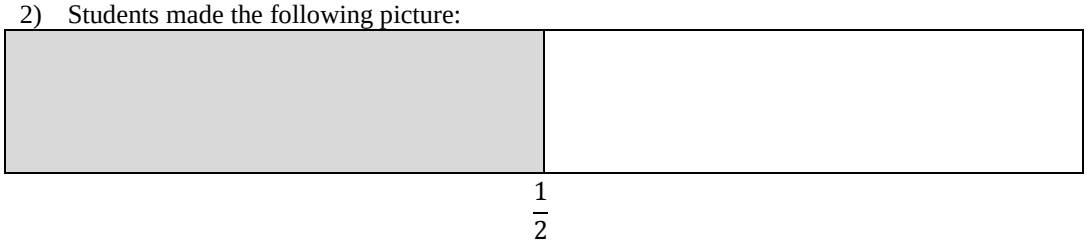


The Student'models:

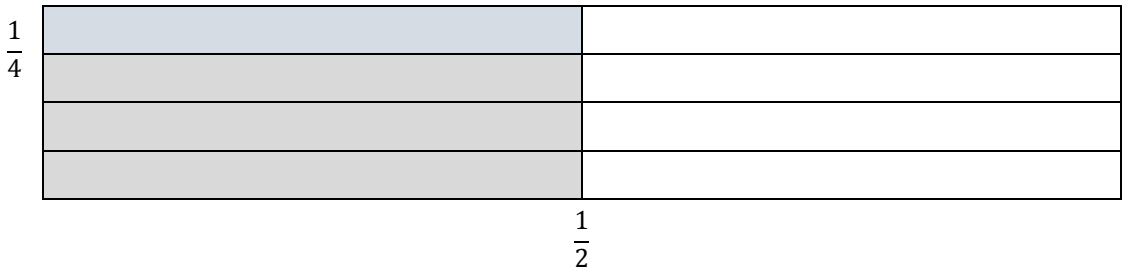
a. The possible answers were made by the student to describe $\frac{1}{4}$ part of $\frac{1}{2}$. This step was horisontal mathematizing.
   1) Students made the following picture:



   Students stated that the shaded area was $\frac{1}{4}$ part of $\frac{1}{2}$.
   2) Students made the following picture:

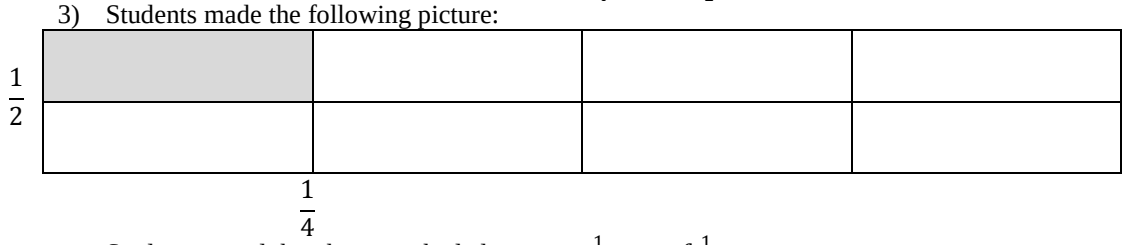


Students stated that the gray area shaded was $\frac{1}{2}$.
After that, the students subdivide the rectangle to obtain the following picture:



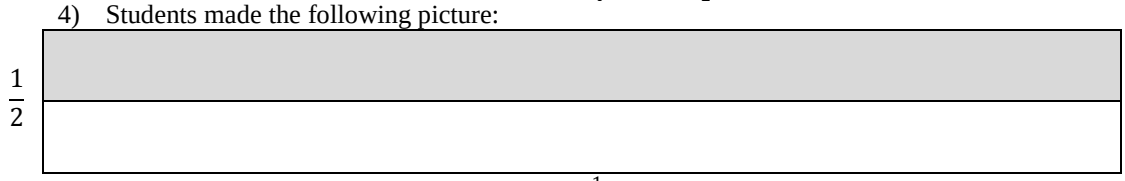
Students stated that the blue shaded area was $\frac{1}{4}$ part of $\frac{1}{2}$.

3) Students made the following picture:

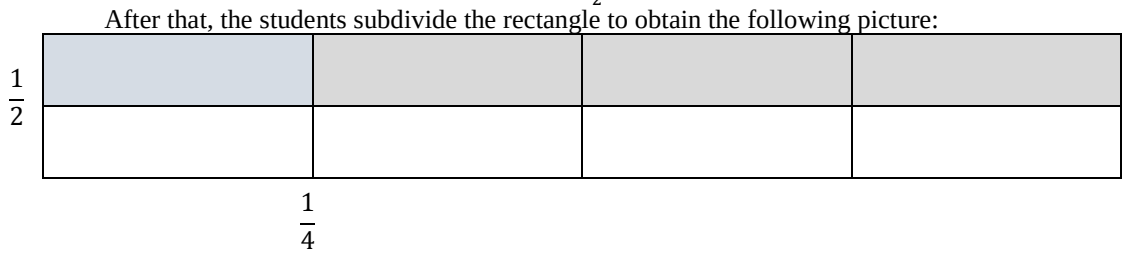


Students stated that the gray shaded area was $\frac{1}{4}$ part of $\frac{1}{2}$.

4) Students made the following picture:



Students stated that the gray shaded area was $\frac{1}{2}$.
After that, the students subdivide the rectangle to obtain the following picture:



Students stated that the blue shaded area was $\frac{1}{4}$ part of $\frac{1}{2}$.

b. Then, to calculate the amount of $\frac{1}{4}$ part of $\frac{1}{2}$, the possibility undertaken by students were as follows: (this step was vertical mathematizing)
   1) Studensts answered $\frac{1}{4}$ part of $\frac{1}{2} = \frac{1}{8}$. Because there was one gray shade parts of 8 parts of a whole.
   2) Students calculated that $\frac{1}{4}$ part of $\frac{1}{2} = \frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$.
   3) Students calculated that $\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$.

5. **The problem was given to students:** calculate the follow multiplication $5 \times \frac{3}{7}$.
   The Student'models:

vertical mathematizing
a. $5 \times \frac{3}{7} = \frac{3}{7} + \frac{3}{7} + \frac{3}{7} + \frac{3}{7} + \frac{3}{7} = \frac{3+3+3+3+3}{7} = \frac{15}{7} = 2\frac{1}{7}$.
b. $5 \times \frac{3}{7} = \frac{5 \times 3}{7} = \frac{15}{7} = 2\frac{1}{7}$.

6. **The problem was given to students:** calculate the follow multiplication $\frac{5}{6} \times \frac{12}{15}$
   The Student'models:

vertical mathematizing
a. $\frac{5}{6} \times \frac{12}{15} = \frac{5 \times 12}{6 \times 15} = \frac{60:30}{90:30} = \frac{2}{3}$.
b. $\frac{5}{6} \times \frac{12}{15} = \frac{5}{6} \times \frac{4}{5} = \frac{5 \times 4}{6 \times 5} = \frac{20:10}{30:10} = \frac{2}{3}$.

V. CONCLUSIONS

The student learning materials has been tried out on students in the 5th grade at a private elementary school in Yogyakarta. The results of the trial were as follows:

1. Kiki problem could lead students to develop the situational model on the meaning of multiplying an integer by a fraction and on the calculation of multiplying an integer by a fraction.
2. Gofil' orange problem could lead students to develop the situational model on the meaning of multiplying two fractions and the calculating two fractions.
3. The problem about calculating $5 \times \frac{3}{7}$ could lead students to develop the formal model on the meaning of multiplying an integer by a fraction and on the calculation of multiplying an integer by a fraction.
4. Problem (a) seek the widest part, (b) describe and calculate the results of the $\frac{1}{4}$ part of $\frac{1}{2}$, and (c) calculating $\frac{5}{6} \times \frac{12}{15}$ could lead students to develop the situational model on the meaning of multiplying an integer by a fraction and on the calculation of multiplying an integer by a fraction.
5. The context of the Kiki' ribbon and the Gofil'orange could help students to construct about (a) the meaning and the procedure of multiplication of an integer and a fraction, and (b) the meaning and the procedure of multiplication of two fractions.

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