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Guided Discovery: A Method to Minimize The Tendency of Students' Rote-Learning Behavior in Studying Trigonometry

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***Abstract*—** In Indonesia, trigonometry is one of the topics that has been taught in senior high school level which contains a lot of formulas and concepts that have to be understood by the students. Formulas and concepts could be a cognitive burden for them, especially when the learning process does not involves them actively. So that, it will emerge a tendency for pupils to learn the formulas by rote. Teacher cannot take it for granted since this learning process will become meaningless. Therefore, Guided Discovery teaching method is considered to be needed as one of the alternative solutions to reduce the tendency of rote-learning behavior, since this method enables the teacher to engage students into a learning process of finding trigonometry formulas through reinvention steps provided by this method. In addition, this method require students' prior knowledge along the reinvention process, so that the learning process becomes more meaningful. The purpose of this article is to describe the implementation steps of Guided Discovery method in trigonometry learning process to minimize students' rote-learning behavior. The method used in this paper is literature review. This paper will explain the theories of Guided Discovery, rote-learning behavior, and how the method will minimize the learning behavior.

Keywords: *guided discovery, rote-learning behavior, trigonometry*

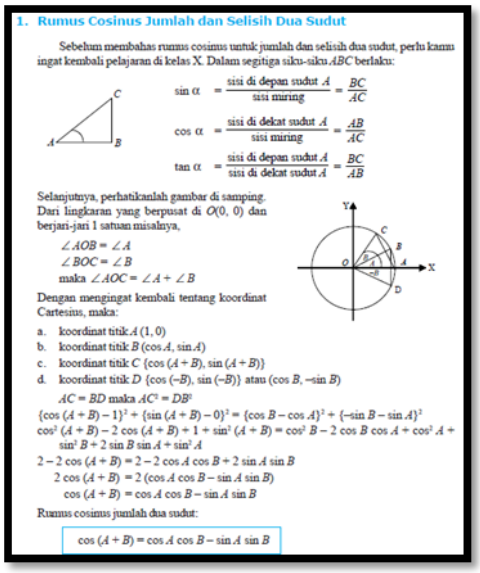
I. INTRODUCTION

Trigonometry is a branch of mathematics that concerns on the elements of triangle such as sinus, cosine, tangent, secant, cosecant, cotangent, and its applications [1]. Many concepts are developed from these elements to enable people in solving their daily-life problems, namely in a construction, in music, in economy, and etc. Therefore, we believe that trigonometry need to be understood as early as possible by the people, especially in high school level. In Indonesia, based on curriculum of 2013, topic of trigonometry is formally taught in senior high school level. At grade X Science Program, the students learn the topic of ratios, functions, equations, and identities of trigonometry. While for topics of trigonometry derivative formulas and its uses are taught at grade XII Science Program.

Specifically in the topic of trigonometry derivative formulas and it uses, the students will learn at least 25 formulas which whatsoever the way they have to understand these, plus the concepts and formulas that they have to remember when they learned this at grade X. Exactly, the demand to understand and to remember those number of formulas and concepts will be not easy for the pupils. So that, it emerges a tendency of the students will learn it by rote since this is the easiest way to put any information into their brain. This tendency appeared because some factors including the teacher factor, the student factor, and the learning resource factor.

From the teacher factor, the tendency can be caused by the way teacher explains, teaches, and delivers the material. There is a possibility the teacher uses a direct teaching method in explaining the way to find the formulas without giving any chance for the students to get involve during the learning process. From the student factor, as we mentioned before, it can be due to the cognitive burden arises in their mind when they are obliged to understand and to remember all of the concepts and the formulas, so they tend to use rote learning method to ease the way they understand the materials. While from the aspect of learning resources, a lot of mathematics books provided in bookstores merely give constructed steps without allocating any space for the students to be engaged in the process of finding the formulas. We can figure

this condition out by observing the following samples of Indonesian Mathematics Electronic School Book (ESB):



1. Rumus Cosinus Jumlah dan Selisih Dua Sudut

Sebelum membahas rumus cosinus untuk jumlah dan selisih dua sudut, perlu kamu ingat kembali pelajaran di kelas X. Dalam segitiga siku-siku ABC berlaku:

$$\sin \alpha = \frac{\text{sisi di depan sudut } A}{\text{sisi miring}} = \frac{BC}{AC}$$

$$\cos \alpha = \frac{\text{sisi di dekat sudut } A}{\text{sisi miring}} = \frac{AB}{AC}$$

$$\tan \alpha = \frac{\text{sisi di depan sudut } A}{\text{sisi di dekat sudut } A} = \frac{BC}{AB}$$

Selanjutnya, perhatikanlah gambar di samping. Dari lingkaran yang berpusat di $O(0, 0)$ dan berjari-jari 1 satuan misalnya,

$\angle AOB = \angle A$
$\angle BOC = \angle B$
maka $\angle AOC = \angle A + \angle B$

Dengan mengingat kembali tentang koordinat Cartesius, maka:

a. koordinat titik A $(1, 0)$
b. koordinat titik B $(\cos A, \sin A)$
c. koordinat titik C $\{\cos (A + B), \sin (A + B)\}$
d. koordinat titik D $\{\cos (-B), \sin (-B)\}$ atau $(\cos B, -\sin B)$

$AC = BD$ maka $AC^2 = DB^2$

$\{\cos (A + B) - 1\}^2 + \{\sin (A + B) - 0\}^2 = \{\cos B - \cos A\}^2 + \{-\sin B - \sin A\}^2$

$\cos^2 (A + B) - 2 \cos (A + B) + 1 + \sin^2 (A + B) = \cos^2 B - 2 \cos B \cos A + \cos^2 A + \sin^2 B + 2 \sin B \sin A + \sin^2 A$

$2 - 2 \cos (A + B) = 2 - 2 \cos A \cos B + 2 \sin A \sin B$

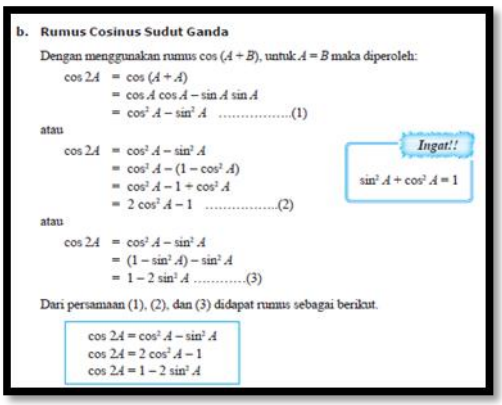
$2 \cos (A + B) = 2 (\cos A \cos B - \sin A \sin B)$

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

Rumus cosinus jumlah dua sudut:

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

FIGURE 1. THE SAMPLE OF ESB IN FINDING THE FORMULA OF COS (A + B)



b. Rumus Cosinus Sudut Ganda

Dengan menggunakan rumus $\cos (A + B)$, untuk $A = B$ maka diperoleh:

$\cos 2A = \cos (A + A)$
$= \cos A \cos A - \sin A \sin A$
$= \cos^2 A - \sin^2 A$ .....(1)

atau

$\cos 2A = \cos^2 A - \sin^2 A$
$= \cos^2 A - (1 - \cos^2 A)$
$= \cos^2 A - 1 + \cos^2 A$
$= 2 \cos^2 A - 1$ .....(2)

atau

$\cos 2A = \cos^2 A - \sin^2 A$
$= (1 - \sin^2 A) - \sin^2 A$
$= 1 - 2 \sin^2 A$ .....(3)

Ingat!!
$\sin^2 A + \cos^2 A = 1$

Dari persamaan (1), (2), dan (3) didapat rumus sebagai berikut.

$\cos 2A = \cos^2 A - \sin^2 A$
$\cos 2A = 2 \cos^2 A - 1$
$\cos 2A = 1 - 2 \sin^2 A$

FIGURE 2. THE SAMPLE OF ESB IN FINDING THE FORMULA OF COSINE 2A

When we observe the figures above, the book directly gives the way to find the formulas, then emphasizes it by "boxing" the formulas. The first concern is, as an appropriate textbooks in general, this book does not involve student participation in finding the formula in question. The next concern is the box sign given on the formulas that giving a possibility for the students to pay their attention not on the process of finding the formula, but straightly on the formula. So that if the students want to memorize the formulas, this book indirectly "facilitates" them by pointing some important parts to be memorized, when in this case is the formulas. From those concerns, this can be seen that the available learning resource still less engaging the pupils actively during the learning process of finding trigonometry formulas. Therefore, the tendency of rote-learning will be inclining.

Basically, rote learning is the easiest way to take an update of what we learn. However, the information learned by the students will be easily forgotten since there is nothing "memorable" left in their mind. Hence, the learning process will become meaningless. Ausubel in [2] says, "…if the learner's intention is to memorize it verbatim, i.e., as a series of arbitrarily related word, both the learning process and the learning outcome must necessarily be rote and meaningless". By relating to Indonesian Curriculum 2013, rote-learning behavior does not support the aim of this curriculum which is the learning process of curriculum 2013 should be *to get to know*, not *to be told*. This means, in the purpose of making

the class becomes less rote learning but more meaningful, so the students should be actively finding out new information using their cognitive skill and prior knowledge, not in the opposite way.

Hence, we need a teaching method to minimize the students' tendency of memorizing the material they have learned so that the learning becomes more meaningful. The method should be able to engage the pupils actively in the process of constructing trigonometric derivative formulas. Therefore, they can experience the same condition as when the formulas were invented for the first time. This experience will be memorable in the minds of the students where they understand the way the formula was constructed. So when one day they forgot the formula they have learned, they will be able to reconstruct these formulas with the memory that sticks in their minds. Therefore, one of the teaching method that is appropriate to this condition is Guided Discovery teaching method. Guided Discovery is a teaching method that can involve the students actively to find new knowledge based on their prior knowledge under the guidance of the teacher. Through this method, the student involvement in finding the trigonometric formulas will be accommodated properly.

Based on the description of the background above, this paper offers a Guided Discovery teaching method in minimizing the tendency of students' rote-learning behavior in studying trigonometry. The aim of this paper is to explain the theories of Guided Discovery, rote-learning behavior and the way this method minimize the tendency of rote-learning behavior theoretically. The method used in this paper is literature rivew.

II. THEORITICAL FRAMEWORK

A. *Guided Discovery*

Before we discuss guided discovery teaching method, we need to explore the essence of this teaching method. Based on the history, [3] state that the first example of Discovery method was given by Plato about the dialogue between Socrates and a young slave where by the time this method is known as Socratic Method. According to Bruner in [3], he says, "discovery is a process, a way of approaching problems rather than a product or particular item of knowledge". In a short words, we can say that learning by discovery is learning to discover, where the students face a problem and they will find the way to solve it. According the definition [4], "Discovery learning is intentional learning through supervised problem solving following the scientific method of investigation". Whereas according to [5], "Discovery learning is hands-on, experiential learning that requires a teacher's full knowledge of content, pedagogy, and child development to create an environment in which new learnings are related to what has come before and to that which will follow". If we drag a correlation based on the three experts (Bruner, Moore, and Abruscato) statements, then we can conclude that Discovery learning is a learning process through a process of problem solving that is arranged from investigation steps to discover new information or skills for the students.

Furthermore, Moore explains that Discovery learning can be done in three steps, depends on the solution to the problem. First, a discovery that is guided carefully (Guided Discovery). Second, a discovery that is guided accordingly (Modified Discovery). Third, a discovery that is limited only on a supervision (Open Discovery). Since our focus only on the high school students ability, so the level of Discovery learning that is chosen to be discussed is Guided Discovery.

According to [6], "Guided Discovery was the name to hand-on activities and laboratory investigation that led the learner to a predetermined or a predictable data set or response". While [7] says that Guided Discovery is a teaching method based on inquiry, a constructive teaching theory within a problem solving situation where the student use their prior knowledge and experiences to find facts, correlations, and truths to be learned. Hence, based on the two experts' statement we can draw a conclusion that Guided Discovery method is a method to direct students to construct their knowledge through a discovery of new concepts and knowledge under teacher guidance.

As a teaching method, exactly Guided Discovery has advantages and disadvantages. The following table will mention some of them according to Marzano in [8]:

TABLE 1. ADVANTAGES AND DISADVANTAGES OF GUIDED DISCOVERY METHOD

Advantages	Disadvantages
1. Students will actively participate during the learning process. 2. Instill well as foster an attitude of inquiry. 3. Support students' problem-solving	1. Not all of the students can follow the lessons in this way since some of them are still familiar and easily understood by the lecturing method. 2. Not all topics are suitable delivered with

skill. 4. Provide a space for interaction among the students, as well as students to teachers, in order to train the students to communicate their idea properly and correctly. 5. The subject being studied can achieve a high level of capability and longer lasting because the students are involve to find new knowledge.	this method. Generally, topics related to the principles of the model can be developed with Guided Discovery.

Besides the advantages and the disadvantages, we also can find cognitive benefits for the students by implementing Guided Discovery teaching method in class. Here are some of them [9]:

1. It encourages analytic learning.
2. It exploits learners' cognitive skills.
3. It improves critical thinking skills.
4. It involves students in problem solving tasks.
5. It helps learners become aware of and articulate their mental process.
6. Learner actively in the learning process.
7. Learners understand and remember better what they have work out for themselves.

Indeed, this method has main objective in term of developing students' capability i.e. according to [10], "The prime objective of these (guided discovery) activities is to have students discover, and/or self-construct, the scientific/technological concept embedded in the activities as students do the activities'. In addition in the same page, [10] reminds us that although this activity has been in the design as much as possible, there will be students who do not follow the lesson plan that has been designed by the teacher. Thus, the teacher has to be flexible during the learning process. Here are the steps of Guided Discovery teaching method [11]:

1. Teacher determines the task criteria such as giving problems. Then the students find solutions for the problems. The problem that has been given should contain clues about things students need to do, namely students find the solutions of the problems by themselves.
2. Smart students will have possibilities to find the answer by themselves without teacher's guidance. Otherwise, students who incapable to find the answer need to have their first guidance by the teacher. This guidance should be in form of simple questions.
3. Having first guidance, the students who able finish the problem should check their answer by using the provided data. While for the student who cannot find the answer, they get second guidance by the teacher. The guidance is in form of questions to arrange the data that already available. The aim of this guidance is to get samples of the answer from some of the provided problems.
4. After the second guidance has already given, the students who succeed getting the answer of the provided problems should check their answer using the data that already exist. The students who do not able to find their answer get the third guidance from the teacher which in form of additional data that is arranged into a list. The expected goal of this third guidance is to make students can find the answer. If by giving this additional data the student do not find the answer yet, the teacher have to give another short guidance verbally for the students to get the expected answer right away.
5. Students are required to check their answer after they get their third guidance.
6. After they check their answer, they use it to finish the task criteria.
7. Students find the answer from what they do in task criteria.
8. The answer from the task criteria is still conjectural, so a proof is needed to verify it. So the teacher must give the answer a confirmation whether the students are correct or not.
9. Teacher gives enrichment to the students in form of applicable problems. It is expected in solving the problems, they can use the concept they just have got.
10. If the student can answer the problems correctly, so it can be said that they succeed constructing new knowledge about a concept that they have learned.

While according to Soedjadi cited in [11], says that there are 6 steps that need to be done to complete a learning by Guided Discovery method:

1. Giving Problem
   Students required to understand the given problem.

2. Data Development
   Students required to seek or pointing the possibilities other data as the continuity of the known data.
3. Data Arrangement
   Students required to arrange the data obtained for the first step and second steps in a list.
4. Extra Data
   Students required to add extra data as the continuity of the known data if the expected pattern is not obtained yet.
5. Answer The Problem
   Students answer the problem in the first step.
6. Checking The Answer
   Students required to see the truth of the general pattern that is obtained with some available data.

In this instructional method, the teacher becomes a facilitator, who guides students in the right directions so as to avoid misunderstanding of the rules. Hence, there are some things that must be considered during the use of this method in the learning process, as outlined by Michael Swan [9].

1. The rules should be true.
2. The rules should show clearly what limits are on the use of a given form.
3. The rules need to be clear.
4. The rules ought to be simple.
5. The rules need to make use of concepts already familiar to the learner.
6. The rules ought to be relevant.

B. Rote-Learning Behavior

The definition of rote learning is a learning by memorizing information repeatedly [12]. The idea of this method is the more we repeat the information, the more information we will remember in verbatim. Experts generally agree to a certain point that rote learning is necessary and important. This method is commonly used when fast retention is required, such as memorizing phone number, someone's name, or post number. Or in education, when students need to remember the alphabets and how to spell the words.

Indeed, there must be pros and cons about the use of this method. For the cons, some believe that by having rote-learning method as a way to learn new information, some students may forget the facts that they have learn after doing test, and may not full understand the concepts to begin with [13]. For instance, when students are studying about trigonometric formulas for the sake of a test on the next day, whereas they did not study well before, so they realize that they need to remember all formulas in a short time. They will probably not have a very deep understanding of the actual meaning of the formulas. They will then likely forget all the other facts shortly anyway.

In the other hand, some pros believe that rote learning method will build the foundation, so students can learn more difficult concepts. For example, firstly students have to acknowledge the shape of triangle, square, trapezium, and circle, then they begin to find area of those shapes.

According to [14], the revised taxonomy includes six cognitive process categories, one most closely relates to retention (Remember) and the other five relates to meaningful learning (Understand, Apply, Analyze, Evaluate, and Create). This means that when the learning process merely focus on memorizing the material, so this will become meaningless.

C. Trigonometry in Senior High School

Based on Indonesian curriculum of 2013, topic of trigonometry has been taught at grade X and grade XII natural science program. At grade X, students will learn about angle, degree, radian, quadrant, ratios (sinus, cosine, tangent, cotangent, cosecant and secant) and trigonometry identities. While at grade XII natural science program, students will learn compound angles formulas, double-angle and half-angle formulas, multiplication formulas for sinus and cosine, addition and subtraction formulas of sinus and cosine. Those materials are formulated into basic competences presented as follows:

TABLE 2. BASIC COMPETENCES OF TRIGONOMETRY BASED ON CURRICULUM OF 2013

Grade	Basic Competences
X	3.15 Understanding the concept of trigonometric ratios in right-angled triangle through the investigation and discussion about the ratios of corresponding sides in some congruent right-angled triangles. 3.16 Determining the properties and relationship among trigonometric ratios in a right-angled

	triangle. 3.17 Understanding and determining the relation of trigonometric ratios from an angle in every quadrant, choose and implement it to solve problems in mathematics. 3.18 Understanding the concept of trigonometric function and analyze its graphic function as well as determining the relation of value of trigonometric function from special angles. 4.15 Applying trigonometric ratios to solve problems. 4.16 Presenting trigonometric graphic function.
XII	3.6 Describing the identity of sinus summation, identity of sinus subtraction, identity of cosine summation, identity of the different to be implemented in problem solving. 4.6 Presenting and analyzing the identity of sinus summation, identity of

III. DISCUSSION

As what has been described before, in which rote-learning is a method of learning that can lead students to a meaningless learning process, it is good if we discuss the definition and conditions of meaningful learning before we discus much about how the method of Guided Discovery can be an alternative solution in minimizing the tendency of rote learning.

Meaningful learning always be related to Ausubel, a psychologist who advance a theory which contrasted meaningful learning from rote learning. He believes that learning of new knowledge relies on what is already known [15]. This means that in order to construct new knowledge, it is required students' prior knowledge in order they can easily relate this new knowledge with what is already familiar in their minds. So that, the new knowledge will stick around longer in students' minds. Next, Piaget in his theory of cognitive development suggested that human's unable automatically understand and use information that they have been given because they need to construct their prior knowledge through prior personal experiences to enable them to create mental image [15]. Where in a short words, he says that human learn by constructing their own knowledge. Therefore, to construct a prior knowledge through prior personal experience, students need to be involved actively during the learning process. Because when they solely memorizing the material, they do not make any way to relate what they have known to what they going to know.

Following table explains the different between meaningful learning and rote learning according to Ausubel [16]:

TABEL 3. MEANINGFUL LEARNING CONTRASTED WITH ROTE LEARNING

Meaningful Learning	Rote Learning
1. Non-arbitrary, non-verbatim, substantive incorporation of new knowledge into cognitive structure. 2. Deliberate effort to link new knowledge with higher order concepts in cognitive structure. 3. Learning related to experiences with events or objects. 4. Affective commitment to relate new knowledge to prior learning.	1. Arbitrary, verbatim, non-substantive incorporation of new knowledge into cognitive structure. 2. No effort to integrate new knowledge with existing concepts in cognitive structure. 3. Learning not related to experience with events or objects. 4. No affective commitment to relate new knowledge to prior learning.

If we associate it with topic of trigonometry taught in schools, the formulas and concepts being taught is easy to "rediscover" by the students. This is because students already have their prior knowledge enough to find those formulas and concepts. For instance, in the material of trigonometric ratios in right-angled triangle at grade X, students already studying the properties of right-angled triangles earlier when they were in junior high school. So as to introduce the concept of sine (as a comparison of the length of front side of the corner and the hypotenuse of right-angled triangle), or cosine (as the length ratio of the next side of the corner and the hypotenuse of right-angled triangle) can be done easily. Another example, the material of trigonometric derivative formulas to formula of sin ($\alpha \pm \beta$) which can be found easily using the trigonometric formula to determine the area of any triangle they have learned at grade X. Therefore, it is very unfortunate if students only know (by memorizing process) without understanding the meaning and the origins of the trigonometric formulas and its concepts. The impact is that when they are oblivious to these formulas they cannot re-construct these formulas, or they merely know the formulas and concepts but they cannot use it to solve problems. So, meaningful learning is a must in studying trigonometry by the students.

Based on those reasons, Guided Discovery is considered to be an alternative solution to lead the learning process into a meaningful learning process by minimizing the tendency of students learn by rote. This is because:

1. Guided Discovery addresses some of the drawbacks associated with both deductive and inductive instruction as it is essentially learner-centered. With the combination of inductive-deductive instruction, this will encourage analytic learning and students' cognitive skills will be exploited.
2. It makes learning memorable since learners are actively involved in the process, so the material will be long lasting in their mind.
3. It is meaningful because it involves learners' own reasoning. With this, they can reconstruct the trigonometric formulas whenever they forget the formulas or whenever they will use it to solve problems.

From two version of Guided Discovery presented before, Hirdjan's steps of Guided Discovery is considered to be preferred to Soedjadi's steps as his steps are more flexible. This means in the Guided Discovery steps we may provide some loops in some steps. For example, in Hirdjan's steps, for some smart learners or for them who able to finish the first steps may skip the second and the next steps. Therefore, the Guided Discovery steps using in this paper is an adoption from Hirdjan's steps of Guided Discovery teaching method. As a result of the modifications are as follows:

1. Giving Problems
   The teacher gives a problem, and learners seek resolution of the problem. The given problem should contain clues to the direction and the objectives about what they have to do. Such as they find the solution by themselves from the given problem.
2. Development of Data
   In this phase, students are required to find/pointing to the possibility of other data as a continuation of the data that is already known. Students who are smart enough will finish the problem without guidance. Otherwise, they will get a guidance in the form of developed questions from the simplest way.
3. Data Preparation
   In this phase, teacher guides the students by giving them a more specific ways to find the formula using the data in step 1 and step 2. This way is in form of the steps to find the formula, but not in a general way.
4. Extra Data
   Teacher gives students extra data that will direct them to the targeted formula. It is expected that with this guidance, students can determine the formula.
5. Verification
   In this step, the students are required to verify the formula they have found by themselves. If the verification is correct, so they can continue to the next step. Otherwise, they need to recheck their work in the previous step, or they can consult with the teacher of their friends who already finish their work correctly.
6. Application Exercises
   Teacher gives the students some problems, and it is expected that they do the problems use the formulas they found.

The followings are the sample of using the modification steps in finding the formula of sin ($\alpha \pm \beta$):

1. Giving Problems
   In this step, students are given a question whether the students are able to determine the value of sin ($\alpha + \beta$) and sin ($\alpha - \beta$) by providing a reason or an explanation. If they sure with their answer, they may continue to the step of Verification with a condition if they have a problem, they have to back to the second activity. But if they do not find the problem, they may continue to the last step.
2. Development of Data
   Students get a guidance in the form of new data by letting α and β in the form of numbers such as "let α =45° and β = 30°, is it true if sin ($\alpha + \beta$) = sin 75°? And is it true if sin ($\alpha + \beta$) = sin 15°?". If the students are able to answer those question, so they may continue to the verification phase. Otherwise, they have to continue to the third steps.
3. Data Preparation
   The sample of this step can be seen in the following illustration:

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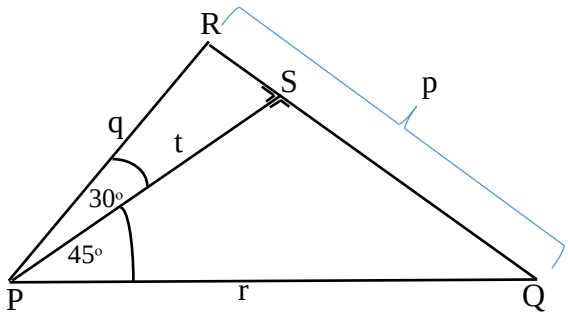


Use the trigonometric formula to find the area of triangle to determine the area of ΔPRS, ΔPQS, dan ΔPQR.

Area of ΔPRS = ……. (i)

Area of ΔPQS = ……. (ii)

Area of ΔPQR = ……. (iii)

From the figure above, you can determine:

In ΔPRS, cos 30° = $\frac{t}{q}$, t = ...(iv)

In ΔPQS, cos 45° = $\frac{t}{r}$, t = ...(v)

Substitute equation (v) to equation (i), and substitute equation (iv) to equation (ii):

Area of ΔPRS = ……. (vi)

Area of ΔPQS = ……. (vii)

Use the equation of (iii), (vi), and (vii) to determine the formula of sin (a + b) and sin (a - b) with given:

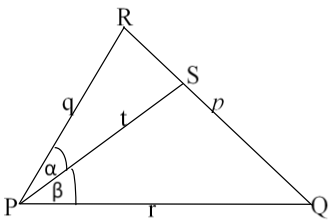
Area of ΔPQR = area of ΔPQS + area of ΔPRS
..... = ...
sin 75° = sin (45° + 30°) = ...

With the same way, you can determine the value of:
sin 15° = sin (45° - 30°) = sin (45° + (-30°) = ...

Can you determine the value of sin (a + b) and sin (a – b)? If yes, go to step 5. If not, continue to the next step.

4. Extra Data
The sample of this step can be seen in the following illustration:

Let a = α and b = β, then observe the following figure:



Use the trigonometric formula for area of triangle and determine the area of ΔPRS, ΔPQS, and ΔPQR.

Area of ΔPRS = ……. (i)

Area of ΔPQS = ……. (ii)

Area of ΔPQR = ……. (iii)

From the figure above, you can also determine:

In ΔPRS, cos α = $\frac{t}{q}$, t = ...(iv)

In ΔPQS, cos β = $\frac{t}{r}$, t = ...(v)

Substitute the equation (v) to (i), and substitute equation (iv) to (ii):

Area of ΔPRS = ……. (vi)

Area of ΔPQS = ……. (vii)

Use equation of (iii), (vi), and (vii) to determine sin (a + b) and sin (a - b) with given:

Area of ΔPQR = area of ΔPQS + area of ΔPRS

..... = ...

Sin ($\alpha + \beta$) = ...

As you already determine the formula of sin (α + β), then determine the formula of sin (α - β) using the formula of sin (α + β) that you have found by considering that $\alpha - \beta = \alpha + (-\beta)$.

sin (α – β) = sin (α + (– β))

= ……

sin (α – β) = ……

Remember:
cos (–α) = cos α
sin (–α) = –sin α

Of course you have found formula of sin (a + b) and sin (a – b). Then you may continue to the next step.

5. Verification
   Students are required to rewrite the formula of sin (a + b) and sin (a – b) they have found. Then they have to find the value of sin (a + b) and sin (a – b) if given a = 30º dan b = 30º, so they are able to prove whether the formula they have found is correct or not which is seen from its calculation. Because for sin (a + b) = sin 60º = $\frac{1}{2}\sqrt{3}$, while for sin (a – b) = sin 0º = 0. If their answer is correct, so they can continue to the sixth step, otherwise they can discuss to their friend until they get the right answer.
6. Application Exercises
   Teacher can provide them some enrichments such as find the value of sin 125º, sin 15º, etc.

CONCLUSION

Trigonometry as a branch of mathematics plays a big role in assisting people solve their problem, such as in economy, technology, architecture, music, etc. That makes experts believe that trigonometry must be taught as soon as possible especially in school, so that students can get to know the application of trigonometry earlier. Hence, in the process of learning trigonometry, the meaningfulness of this learning should be prioritized so that students' understanding can be more comprehensive. Guided Discovery teaching method is chosen in this paper as an alternative solution to support this condition since this method prioritizes students' involvement and reasoning ability during the discovery process of trigonometric formulas, thus material will be long lasting in their minds and become more meaningful. So that, the possibility of students choose rote learning will be minimized.

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