Developing Students’ Mathematical Reasoning Through Learning Mathematics with Analogical Reasoning

Retno Kusuma Ningrum, S.Pd.¹, Nurul Husnah Mustikasari, S.Pd.²
¹Graduate Students Dept.of Mathematics Education, Yogyakarta State University
²Graduate Students Dept.of Mathematics Education, Yogyakarta State University
boenga16@gmail.com

Mathematics is not only about using numbers to fulfill the formulas or performing computations, but also a way of thinking. Based on NCTM, there are five standards in learning mathematics. Those are problem solving, reasoning and proof, communication, connection and representation. Mathematical reasoning is one of the ability that students must have after learning mathematics. Mathematical reasoning cannot be separated from learning mathematics. Mathematical reasoning have an important role in the process of understanding and applying mathematics. At the same time, mathematical reasoning also developed while we are learning mathematics. We often use deductive reasoning when we learn mathematics in school. Whereas, we also can use another kind of reasoning, inductive reasoning, to learn mathematics. One of components mentioned in inductive reasoning is analogical reasoning. Analogical reasoning is a process of thinking to obtain a conclusion or new knowledge by comparing analogical objects or their prior knowledge about something. Based on some research, inductive reasoning, especially analogical reasoning, can be used to develop students’ ability in mathematical reasoning. As a teacher, we can design a learning program which can fostering student’s analogical reasoning ability. Because of the goals of learning mathematics is not only about mathematics as the product of thinking, but also the process of learning and thinking, it is important for us to prepare and planned a good learning program. Focus on analogical reasoning, this paper is aimed to describe how mathematical reasoning can be developed through fostering student’s analogical reasoning ability and how we can bring it in our mathematics learning program.

Keywords: analogical reasoning, learning mathematics by analogical reasoning, mathematical reasoning

I. INTRODUCTION

Mathematics is one of lessons taught from elementary until secondary school. The aim of learning mathematics is to provide logical thinking, critical thinking, and creative ability. Mathematics is not only about using numbers to fulfill the formulas and performing computations, but also a way of thinking. Based on NCTM[1], there are five standards in learning mathematics. Those are problem solving, reasoning and proof, communication, connection and representation. Referred to this standards, mathematical reasoning is important things in learning mathematics. Reasoning is the process of thinking. Moreover, mathematical reasoning and mathematics are two things that cannot be separated. Mathematical reasoning is a fundamental ability which have an important role in the process of understanding and applying mathematics. At the same time, mathematical reasoning is also developed while we are learning mathematics.
Learning mathematics involve two aspects of reasoning. They are deductive and inductive reasoning. According to Fathima[2], inductive reasoning is generalization of the principles or conclusion of specific facts. One type of inductive reasoning is analogical reasoning. Several studies found a close relationship between student’s analogical reasoning ability and student’s mathematical reasoning ability. Alexander and Buehl[3] found evidence of relationship between them. Moreover, Goswami[4] stated that “experience in solving analogical problems and habit of thinking about the relationship between things that have similar properties can enhance one’s mathematical ability”. Alexander, White and Daugherty[4] suggested that the process of reasoning in mathematics has close correspondence with analogical reasoning process. They found that there is a strong relationship between analogical reasoning ability of someone with their mathematical reasoning ability. Viewed from that several studies, there is a close relationship between analogical reasoning ability with mathematical reasoning abilities. We can conclude that the role of analogical reasoning in the process of learning mathematics is very important because problems which involving analogical reasoning can be used to improve students mathematical reasoning abilities. Teachers can use learning process in the classroom to facilitate the development of student’s mathematical reasoning ability through analogical reasoning. This is accordance with the opinion of Mofidi[5] that one of the effective methods can be used by teachers to teach mathematics concepts is to use issues involving analogical reasoning. This paper will discuss how mathematical reasoning can be developed through fostering student’s analogical reasoning ability and how we can bring it in our mathematics learning program.

II. EXPLANATION

A. Mathematical Reasoning

Mathematical reasoning is one of the student’s ability that must be developed in learning mathematics. Steen[6] defined mathematical reasoning as “reasoning about and with the objects of mathematics”. On the other hand, Russell[6] defined “mathematics reasoning is essentially about the development, justification and use of mathematical generalization”. Based on NCTM[7], “Reasoning in mathematics is often understood to encompass formal reasoning, or proof, in which conclusions are logically deduced from assumptions and definitions. However, mathematical reasoning can take many forms, ranging from informal explanation and justification to formal deduction, as well as inductive observations”. According to that explanation, we can conclude that mathematical reasoning is the process of thinking about mathematics which can take many forms, they are justification and use of mathematical generalization, make conclusion from assumptions and definitions.

Mathematical reasoning have some component. According to NCTM[1], Instructional programs from prekindergarten through grade 12 should enable all students to
1. recognize reasoning and proof as fundamental aspects of mathematics;
2. make and investigate mathematical conjectures;
3. develop and evaluate mathematical arguments and proofs;
4. select and use various types of reasoning and methods of proof.

Ball and Bass[6] state that “there are two key practices involved in mathematical reasoning – justifying and generalizing – and other mathematical practices such as symbolizing, representing, and communicating, are key in supporting these”. According to English[8], nowadays, mathematical reasoning is viewed as gathering evidence, analyzing data, making conjectures, constructing arguments, drawing and validating logical conclusion, and proving assertions. Moreover, English[8] stated that “the ability to see connections and relationships among mathematical ideas and to apply this understanding to the solution of new problems is a basic component of mathematical reasoning”. According to that explanation, we can conclude that the components of mathematical reasoning are
1. make conjectures
2. develop and evaluate mathematical arguments and proofs
3. generalizing
4. draw conclusions from evidence

B. Analogical Reasoning

Fathima[2] described reasoning as a thinking process, including the ability to interpret various forms and concept formation. So we can conclude that the reasoning is a process of thinking that organizes knowledge to make a new form, concept or a conclusion. There are several kinds of reasoning, one of which is inductive reasoning. Inductive reasoning is a process of generalizing the principle or a conclusion based on the specific facts that exist[2]. One kind of inductive reasoning is analogical reasoning.

Spiers[9] defined analogy as a set of problems which contains the initial problem and the target problem, where each problem has a relevant knowledge or information that can be mapped from the initial problem to the target problem. According Keraf[10], the analogy is comparing two things that have a lot in common. Gentner, Holyoak & Kokinov[8] defined analogical reasoning as one of reasoning ability by using the relationship of a pattern, include the ability to find patterns, identify repeat pattern with variations of each element, concluded from the patterns and communicate their conclusions as the goal of the process. Basically, analogical reasoning is the part of cognitive abilities and closely linked with someone’s representation ability.

According to those opinions, we can conclude that the analogy is compare a few things based on the similarity or difference. Analogical reasoning is a process of thinking that aims to get a conclusion or new knowledge by using analogy or comparison between analogical objects with the prior knowledge about something. There are two types of the problem which use analogical reasoning.

1. Classical Analogy

Classical analogy, a type of analogical reasoning, involving some similarities in the nature of things, at least from the 4 things that will be compared. These linkages between the A and B terms and between the C and D terms[11]. The relationship in classical analogy is usually denoted by A: B :: C: D, which means the relationship owned A and B is similar to the relationship that is owned by C and D. There are two types of relation between terms in classical analogy, there are "lower order relation" and "higher order relation".

English[8] provides an example of the "higher order relation" in classical analogy is in branches: tree :: hand: human. The relation can be found between the branches of trees is similar with human hands, which branches are parts of a plant, as well as a hand which is part of the human body. The examples of the "lower order relation" in classical analogy is in goats: lung :: fish: gills. The relation can be found between goat and lungs is similar with fish with gills, the goats have lungs that function as a respirator, so do the fish, have gills that serves as a breathing apparatus.

According to English[8], there are three phases which always passes in classical analogy,

a. encoding phase. There is a process of identifying each object analogy to assess the properties or characteristics of each given object analogy. In this phase, the characteristics of each object analogy would be identified and the possible relationships between objects analogy;

b. infering phase. There is the process of comparing a pair of things that become the factor of analogy to determine the relationship between each object analogy. In this phase, we identified the possible relationship of each pair of given object analogy. In this phase, it is possible to appear more than one relationship between object pairs analogy can be identified;

c. applying phase. There is the process of generalizing or selection of the most appropriate relationship to complete an analogical process. In this phase, some relationships obtained in infering phase will be choose one of the most appropriate as a relationship. It will be used to determine the object analogy to complete the process of analogical reasoning.

Lunzer[8] have done a research which related with mathematical reasoning. In his research, Lunzer represents the classical analogy into a mathematics problem that includes three pairs of given objects
analogy and an object analogy to look for a partner based on the relationship analogy indicated by three pairs of objects analogy that has been given.

Example question adapted from the classical analogy questions used in Lunzer research is:

\[
18:12; 27:18; ?: 6; 24:16
\]

What number is the most appropriate to fill the question mark?

In this case, we can refers to three phase in classical analogy by English[8] and identified how the reasoner thinking when solving problems, among others:

a. Encoding phase.
   In this phase, reasoner thinking about the characteristics of each number of the object of analogies and possible relationship.
   Suppose that when attention to numbers 18, 27 and 24. reasoner may identify some linkage between numbers, such as 18 and 27 are both multiples of 9 consecutive numbers, 18, 27 and 24 numbers are divisible by 3 or may arise identifying the nature of the other. Likewise, when attention to numbers 12, 18, 6 and 16. It is possible to identify some linkage reasoner numbers, such equally an even number, numbers 12, 18 and 6 is the number that is divisible by 3 or might appear identifying other properties.

b. Infering phase.
   In this phase, reasoner began comparing each other factors that have been identified in the encoding process.
   For example, the relationship 18:12 and 27:18. Reasoner possibly find the difference between the two pairs of these numbers, which is the difference between 18 and 12 is 6 and the difference between 27 and 18 is 9, so it is possible reasoner concluded that the relationship of the two pairs of these numbers are multiples of three consecutive numbers.
   Other relationships may arise between the 18:12, 27:18 and 24:16. One of them saying the numbers are added to the product terms 3 and 2, ie 6x3: 6x2, 9x3: 9x2, 8x3: 8x2, so it is possible reasoner concluded the pair's relationship of these numbers are the numbers that are on the left and the number that appears to the right has a fixed value ratio, which is 3: 2.
   In addition to the two relationships above, it is possible reasoner find other relationships between numbers.

c. Applying phase
   In this phase, reasoner seek the most appropriate relationship with the numbers of analogical object with several possible relationships that have been found in the process infering.
   For example, for the first relation, we can subtract the number in each pair of the number to find the difference. In this connection, it is possible reasoner think that the right number to fill the question mark is 18, obtained from the sum of 6 and 12 as a difference in the next number. But in the next pair of the numbers, the difference between 24 and 16 are 8, not 15. So it can’t be concluded that the first possible relationship of the number can not be used to complete the analogical reasoning problem.
   For the next possible relationship, that each pair of numbers has a consistent comparisons, which is 3: 2. If the object of the right is 6 that can be expressed in the form of multiplication by 2 is 3x2, number 9 is obtained by multiplying 3 by 3 as numbers can be used to fill the object analogy to the left. Since the three pairs of these numbers qualify the relationship, then the relationship is the right relationship in analogical reasoning problem. So, the answer of the analogical reasoning problem is 9.

2. Problem analogies
   Problem analogies is used to determine an analogical reasoning ability in problem solving. This type of analogy depends on the solved problem to solve new problems that become a target problem[11].
Problem analogies is presented in the form of word problems. To solve the problems of the target, we must know how the first problem, initial problem, has been solved. The steps to solved the initial problems then be applied to the next problem, the target problem[8].

The example of problem analogy that has been written by the English[8] is: “Sarah has 52 books on her self. Sue has 4 times as many as Sarah. How many books has Sue? If Mary has72 books and it 3 times as many as Peter has, how many books has Peter?”. To solve this problem, the number of Peter books resolved by the number of Sue books. Having obtained the completion of the initial problem, with the same procedures, concerns about the number of books Peter can be completed.

According to Clement[12], there are four phases which always passes in problem analogies.

a. generating the analogy, is the process of representing the conditions and possibilities of compatibility between the initial problems with the problems of the target. Reasoner identified the suitability of the things that are given as initial conditions in the initial problem and the target problem;

b. evaluating the analogy relations, is a process to re-examine, with the detail, the suitability of analogical relation between the initial problem with the target problem and determine appropriate analogical relation between the two object of analogy. In this phase, reasoner doing more detailed analysis of the relation or suitability of the problems which has been found in the stage of generating the analogy to identified the corresponding problem in the initial and target problems;

c. understanding the analogy case, is the process of testing and analyzing each component in the initial problem to understand the target problem. In this phase, reasoner analyzed the method used to solve the initial problem and the suitability of the initial problem with the target problem to determine the right method which is used to solve the target problem;

d. transferring findings, is the process of transferring the suitable conclusion or method which is used to solve the initial problem to the target problem. In this phase, the target problem solving by the method which is obtained in the understanding the analogy case phase.

C. Learning Mathematics

The ability of each student to learn actively is determine how the student obtain the purpose of learning. Hewit[13] stated that "learning is an active process of constructing knowledge." This definition implies that learning is a process that is done by the students in constructing their own knowledge. Joice, Weil, and Calhoun[14] stated that "Learning is the construction of knowledge. In the process of learning, the mind stores information, organizes it, and revises previous conceptions. Learning is not just a process of taking in new information, ideas and skills, but the new material is reconstructed by the mind ". Learning is an activity to construct knowledge. Learning is consists of saving information activities, organize and refine their prior knowledge. Learning is not only a process of receiving information, ideas or new abilities, but also how the new knowledge constructed. Based on the description above, we can said that learning is an activity in which students actively build their own knowledge to achieve a competence, skill and a certain attitude.

Learning process occurs in students mind and the results of the learning process are the performance or product which is produced by the students. Nitko & Brookhart[15] stated that "instruction is the process you use to provide students with the conditions that help them Achieve the learning targets". From these opinions, we can said that learning is a process used to help students achieve their learning targets. Dick, Carey, and Carey[16] stated that "instruction is that it is a systematic process in the which every component (i.e., teacher, learners, materials, and learning environment) is crucial to successfull learning". Learning is an activity that involves teachers, students, learning materials and learning environment to achieve the purpose of learning. Based on of those explanation, we can concluded that learning is a process that involves students, teachers and teaching materials. It is aimed to construct students knowledge by theirselves to achieve the purpose of learning. Suherman[17] defined the learning process as an educational
process within the scope of the school. Mathematics as one of the subjects taught in school, the curriculum of Elementary and Secondary Education, is a school mathematics[17]. School mathematics consists of the parts of mathematics which is chosen to develop abilities and personality of students. Based on principles and standards for school mathematics proposed by the NCTM[1], the study of mathematics should be able to equip the students in achieving the five standards process namely problem solving, reasoning and proof, communication, connection, and representation. Based on these standards, learning mathematics is not just activity to memorize formulas or perform calculations.

III. DISCUSSION

A. Relationship between mathematical reasoning and analogical reasoning

It has been explain before that the components of mathematical reasoning are make conjectures, develop and evaluate mathematical argument, generalization, and draw conclusion from evidence. Now we will discuss about how mathematical reasoning can be developed through fostering student’s analogical reasoning ability.

One of the type of analogical reasoning is classical analogy. As stated before, there are three steps of classical analogy. When students do classical analogy about mathematics, they must use those three steps. In encoding and infering steps, they may see some relation that match the analogical object. Of course in this case these relation is about mathematics. Then, they must make conjecture which relation that really match the pair of analogical objects. Afterwards, they must evaluate their arguments why that relationship is correct. Then, they must draw conclusion to decide what relationship the most correct. So, when students do classical analogy about mathematics, they use their mathematical reasoning. It means while students do classical analogy about mathematics, they use and develop their mathematical reasoning.

The other type of analogical reasoning is problem analogies. There are four steps in problem analogies. It means when students do problem analogies about mathematics, they must do those four steps. When they do generating the analogy step, they must think all of the possibilities of compatibility between the initial problems with the problems of the target. In this step, they must make conjecture about what relationship that match. Then, as they do evaluating the analogy relation step and understanding the analogy case step, they must evaluate their conjecture that have made before. Afterwards, they do transferring finding steps. In this step, they must make conclusion from the evidence that have been gathered. From those explanation, it means when students do the problem with analogical reasoning about mathematics, the also use their mathematical reasoning. So, using analogical reasoning when learning mathematics will help students develop their mathematical reasoning.

B. Learning mathematics using analogical reasoning

Based on theoretical studies, it was concluded that the analogical reasoning is a thought process that aims to get a conclusion or new knowledge by way of analogy or comparison between objects with the knowledge that has been there before. In mathematics, we can use the analogical reasoning in the learning process. Not only in the process of transfer of knowledge, but also use to reinforce the concept that has been given to the students. Here's an example of instructional design of learning mathematics using an analogical reasoning.

1. Preliminary Activity

In preliminary activity, we can use the analogy reasoning in apperception phase. In this activity, the teacher can show the relationship of the material prerequisites or their prior knowledge to the material they will be learning, then asking them to give a similar example. In this case, we use the principle of analogy reasoning is about two things that have similar properties. Students are asked to give an example of a problem that is almost the same or may be similar to that given by the teacher.

2. Core activities
In core activity, teachers can use some types of problems that involve reasoning analogy in the process of transferring knowledge. The teacher can providing stimulus material to students using the analogy of classical or analogy problems. For example in the study of geometry in grade VIII, we can take advantage of a classical analogy to direct students find the concept of giving name of prism. Teacher can give some pair of the picture of prism with different base side and their name. And then, teacher give the picture of prism with n-side and ask them to give the name of that prism. Moreover we can take advantage of the problems of analogy, to lead students to find a concept to define the terms of a general formula for the surface area of prism-n.

It just an example for using two kinds of analogical reasoning. Moreover, we can give the problem and the solution, more times, and then give a problem to the student which is analogous with example problem and then ask them to solve the problem based on the example. It use the principal of analogy, that compare two things that have the similarity.

3. Closing activity

In the closing activity, we can use the analogy of reasoning in the stabilization phase of the material they have learned. We can provide exercises using the two types of analogical reasoning above to reinforce the concepts they have learned.

IV. Conclusion

Analogical reasoning and mathematical reasoning has strong relationship. Mathematical reasoning can be developed through fostering student’s analogical reasoning ability in learning process. Analogical reasoning can be used in preliminary activity, core activity, and closing activity in learning mathematics. When students do classical analogies and problem analogies in learning mathematics, students use the components of mathematical reasoning. It means by using analogical reasoning, students mathematical reasoning can be developed.

References
