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Teacher's Pedagogical Content Knowledge Concerned To Students Knowledge On Quadratic Function

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Abstract—Teacher as an agent in teaching and learning process is one of five main factors in relation with the students' knowledge and comprehension in learning mathematics. Mastering the subject-matter is not enough for the teachers to make their students comprehend what are taught. The teachers need to know their students' thinking and the way to teach them in order to make them comprehend the materials easily. The participants of this study are senior high school teachers of mathematics subject whose different teaching experince: one novice teacher and one experienced teacher. The data were obtained through observation and interview which then were analyzed qualiatively. In this study, *Pedagogical Content Knowledge* (PCK) is defined as the teachers' knowledge which integrates the knowledge of mathematics content, pedagogy, and the students' thinking. The teachers' knowledge about the students, which is one of PCK components, is focused in this article, particularly the experienced teacher's knowledge about the students in mathematics subject on quadratic function materials. The teachers' knowledge about the students refers to their knowledge about students' conception and misconception. The result of this study shows that the experienced teacher identifies the possibilty of students' difficulties based on the students' responses during the learning process. The identification is still on analyzing the causes of the students' difficulties and misconception by using examples or contextual illustration which is then followed by re-explanation of the concept and the procedure to repair the students' mistakes and misconception.

Keywords*: pedagogical content knowledge, knowledge of students, misconception, quadratic function, teacher*

I. INTRODUCTION

Learning is a process of interaction among learners and educators with learning resources in a learning environment. National Council of Teachers of Mathematics (NCTM, 2000) recommended four principles of mathematics, namely (1) mathematics as a problem solving, (2) mathematics as a reasoning, (3) mathematics as a communication, and (4) mathematics as a relationship. Learning mathematics is a psychological process, the process is an active one's activity in an effort to understand and master mathematics.

Teachers must master the subjects that be taught and learned how to teach it so that students easily understand the material. Teachers who have not mastered their subject well, of course, does not have the knowledge needed to help students learn the content. Master subjects well is not enough for learning, but teachers need to master the mathematical content and know how to represent the lesson so it is easy to understand by students. Shulman (1986) explains that the Pedagogical Content Knowledge (PCK) as a special kind of knowledge that is the basic of knowledge for teachers involving linkage of various knowledge and skills about representations, analogies, examples, demonstration of a specific material that can be understood

by students. PCK involves knowledge of specific topics that is easy or difficult for students and conceptions or misconceptions that may be for students related to a particular topic.

An, Klum, and Wu (2004) considers that the PCK as the knowledge of how to teach a particular material content. Further explained that knowledge of the content and pedagogical knowledge is not enough to reach effective teaching practices without the knowledge of students, curriculum, educational objectives and teaching materials. While Ball and Bass (2000) identify student difficulties related teacher knowledge and learning strategies appropriate to address the difficulties of students in mathematics as part of the pedagogical content knowledge.

Based on the definition above can be concluded that the knowledge of content or knowledge of subject matter possessed by mathematics teachers is to be transformed by benefitted various sources such as textbooks with the presentation of the concept that is easy to understand the students. In addition, teachers in transforming content of knowledge should use different representations, helps students make connections between different representations to solve mathematical problems, recognize the error of students thinking and is able to respond to student questions.

Ball, Thames, & Phelps (2008) identified components of PCK they are (1) *Knowledge of Content and Students* (KCS), (2) *Knowledge of Content and Teaching (KCT), (3) Knowledge of Content and Curriculum (KCC).* Further Ball et al explained that KCS is a teacher's knowledge of students and teachers mathematics knowledge. Teachers must identify what it looks like and what the students thought that would make them difficult to learn mathematics. While KCT is a knowledge of mathematical content and teaching. Teachers evaluate the advantages and disadvantages of the use of certain representations to teach cerain ideas and identify the different procedures and methods which are valuable instructionally. Furthermore, Kilic (2011) divided the PCK into four components, namely (1) Knowledge of Subject Matter, (2) Knowledge of Pedagogy, (3) Knowledge of Learners, and (4) knowledge of curriculum.

Teacher's knowledge of students thinking, An, Klum, and Wu (2004) identified four aspects of PCK of students' thinking. These aspects are (1) develop students ideas in mathematics, (2) overcoming students misconceptions, (3) engage students in learning mathematics, and (4) support student thinking about mathematics. Pedagogical content knowledge is a special knowledge that integrates the knowledge of mathematics with knowledge of students, learning and pedagogic. Pedagogical content knowledge is important for teaching because this knowledge can help teachers anticipate students miskonseption and difficulties in learning and is ready to provide an alternative model or explanation to overcome students misconceptions and difficulties.

Student misconceptions occur in mathematics because of the lack of students' understanding of mathematics concepts, including on the quadratic functions. Teachers should have knowledge of errors and misconceptions of students on the material being taught so that teachers focus more on the learning process by using appropriate models, methods, strategies, or approach. Zevenbergen, Dole, Wright (2004) explained that good teaching involves the teacher's knowledge of student thinking related mathematical concept and know how to lead students to construct knowledge more complex, complete, and strong by using the activities, habits, and learning environment organized.

Novak & Gowin (1984) stated that the misconception is an interpretation of the concepts in a statement that can not be accepted. While Suparno (1998) stated that the misconception is the notion that inaccurate about the concept, the use of the wrong concept, classification examples are wrong about the application of the concept, the meaning of different concepts, chaos concepts are different, and a hierarchical relationship concepts not true.

According to Moore (2006) the cause of the misconception among others: (1) a scheme that has been owned by one and fail to change, (2) a scheme that is wrong in accommodating new information, (3) the

concept or the new scheme entered destructive the old schemes and cause confusion. The cause of misconceptions among other stage of cognitive development that is incompatible with the concept being studied, limit of student reasoning and wrong, the student's ability to capture and understand the concepts being studied, and motivate students to learn the concepts being taught.

According to Smith diSessa and Rochelle (Moore, 2006) that misconceptions can easily be fixed through the isolation. While Moore (2006) said "one method of remediation is by" explaining ". This means that one of the methods of remediation is to explain / clarify. While Lucariello stated that misconceptions can actually impede learning for several reasons. First, students generally do not realize that the knowledge they have is wrong. Additionally misunderstandings can be very strong in the minds of students. Besides new experiences are interpreted through a false understanding that it interferes in understanding new information.

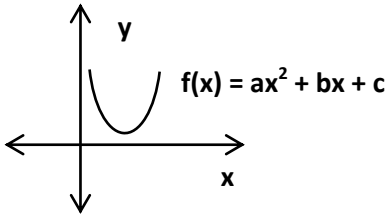
Refered to PCK of mathematics teacher in mathematics learning on this research is the kind of knowledge that integrate content knowledge, knowledge of pedagogic, and knowledge of student in learning mathematics. The purpose of this article is to describe the Pedagogical Conten Knowledge (PCK) of experienced teachers in teaching mathematics in high school related to the knowledge of students on the quadratic functions that include the teacher's knowledge of the possibility of students misconceptions, a possible source of students misconceptions, and alternative ways to overcome students misconceptions on quadratic functions.

II. METHODS

This research is a descriptive qualitative approach. The subjects were two high school mathematics teachers who teach in class X in the same school. Both subjects have qualified Bachelor of Mathematics Education. The criteria for determining the teacher as a research subject is (1) teach in class X in the same school, (2) have at least 5 years of teaching experience, (3) is willing to provide relevant data, including learning observations in each class. Subject 01 with 15 years of teaching experience (experienced teachers were given symbol S01, subjects 02 with 7 years of teaching experience (novice teachers) were given symbol S02, starting from rose to candidates for government employee.

Data were collected through classroom observations and interviews with the subject. Interview with the subject based of questionnaire of mathematics teaching adapted from An, Kulm, and Wu (2004). The questionnaire contains the results of the work of students who make mistakes in answering the question / problem given. Questions were raised on teachers with reference to the results of student work to dig teacher PCK related to teacher's knowledge of students. The questionnaire consisted of two issues that are designed to study in depth PCK related teacher knowledge of students on the topic of drawing graph quadratic functions. Teacher's knowledge of student thinking includes knowing their chances of students misconceptions, knowing the likely source of student misconceptions, and alternative ways to overcome students misconceptions.

Tabel 1 Questionnaire of Mathematics Teaching*

Problem 1	Question Items for teacher
Students class X are given a question as follow: Investigate the properties / characteristics of the values of a, b, and c of a quadratic function expressed in the image below!  	1. What do possible of each students think ? 2. If your student survive to his/her thinking. What steps will you do in order to student aware his/her error thinking ? 3. What the causes of misconception occured on each students ? 4. How alternative ways to address students miskonception that occured to each studets? 5. What questions/tasks will be given to each students to address his/her misconception?

Answer of student (Mr) If a>0, b≥0, and c ≥0 then the parabola opens upward and do not cut the x-axis and y-axis. Parabola is always on top of each x€R. Answer of student (Dg) If a>0, bc≥0 then the parabola opens upward and do not cut and do not tangent the x-axis and y-axis and on the positive side. Answer of student (Ag) Because the curve opens upward and do not intersect or tangent x-axis then a>0, D<0, b>0, c=0	
Problem 2	
Your students are trying to solve the following problem: a bullet was fired upward so that the trajectory forms a curved parabolic with the equation $h(t) = -t^2 - 6t - 8$ where t indicates a time in seconds and h shows height in meters. How is the maximum height of bullet's trajectory and how long it takes to reach the maximum height. Answer of two students are as follow : *Answer of student (Ag)* $h(t) = -t^2 + 6t - 8$ $-t^2 + 6t - 8 = 0$ $(t-4)(-t+2) = 0$ $t - 4 = 0$ atau $-t + 2 = 0$ $t = 4$ atau $t = 2$ $t = 4$ $\quad$ $t = 2$ $h(4) = -4^2 + 6(4) - 8$ $\quad$ $h(2) = -2^2 + 6(2) - 8$ $= 16 + 24 - 8 = 32$ $\quad$ $= 4 + 12 - 8 = 8$ Maximum height is 32 meters in t = 4 *Answer of student (Gn)* $h(t) = -t^2 + 6t - 8$ $t = 0$ then $y = -8$ $t = 1$ then $y = -1$ $t = 2$ then $y = 8$ Maximum height = 8 meter wit time used t = 2	1. What do possible of each students think ? 2. If your student survive to his/her thinking. What steps will you do in order to student aware his/her error thinking ? 3. What the causes of misconception occured on each students ? 4. How alternative ways to address students miskonception that occured to each studets ? 5. What questions/tasks will be given to each students to address his/her misconception ?

*Adapted of An, Kulm, dan Wu (2004)

III. RESULT AND DISCUSSION

Results of experienced teachers PCK related to the knowledge of students on quadratic functions that include the teacher's knowledge of the possibility of student misconceptions, causing student misconceptions, and alternative ways to overcome students misconceptions.

1. Knowledge of experienced teacher of the students possible misconceptions

Experienced teacher stated that for problem 1, generally the three students had understood that when the parabola opens upward then value of a or coefficient of $x^2 > 0$. But for the coefficients b and c are still errors. The concept is not well understood is the concept of the axis of symmetry, if the picture is on the x-axis and y-axis are positive, means the axis of symmetry is positive. This is related to the coefficient b, as to define the axis of symmetry $-\frac{b}{2a}$. So, nothing to do with the value of the coefficient x that is b and the coefficient of x^2 is a. It is known that so that if the value of the symmetry of axis is positive, then the value. Mn and Ag write means that both of these students do not understand the concept of symmetry of axis. Misconceptions experienced by students that is Mn determined values of b and c, Mn write that $c \geq 0$. To determine the value of c, there is a relation with a piece of graph y-axis. If the graph intersects the y-axis, the value x = 0. Furthermore, substituting the value of x = 0 to a quadratic function, then the result $f(0) = c$. If y = c, then y is above the x-axis, then y is positive, because y = c then c is also positive, it means that c> 0. While Mn answered, whereas c> 0. The answer is not clear Dg, and Pt writes that c = 0 Dg also wrote that if the parabola opens upwards, then it did not intercept and tangent the x-axis and y-axis. That means he does not understand this image. Then for Ag true for writes that D <0, mean Ag know that if D <0, then it does not intercept the x-axis, only wrong in determining the value of b and c.

While for the second problem, an experienced teacher, stated that both students Ag and Gn do not understand the concept of the turning points of a quadratic function graph, when describing the graph function, the maximum point is at the turning point of the parabola. That is, if students understand the picture, then the student can determine its maximum height. While the procedures performed mistakes that students Ag and Gn made a mistake in calculation, here the students wrote that -4 ^ 2 = 16, should result -16. Actually Ag understand that the maximum value is the highest value, but the problem when do substitution, Ag comparing the values 32 and 8, and then the students take the value t = 4 because the value is higher, at 32. They did not understand that the maximum value is at a turning point. Gn make the same mistake with Ag namely when subtitute, when had discovered the high value, then Gn stop there and not continue any longer.

Experienced teachers know possibilities of students misconceptions on quadratic function based on the results of the students' work can be identified by seeing whether the relationship among concepts are true or false. Possible students misconceptions often occurs in the quadratic functions material that students can not interpret picture by considering the coefficients and constants of the quadratic function, the students do not understand the relationship between the intersection point of the graph with the y-axis and a constant value, do not understand the relationship of the turning point of the parabola and equation axis of symmetry , According to Kilic (2011), teachers need to identify student misconceptions and difficulties by asking or using the right task.

2. Knowledge of experienced teachers of the causes of student misconceptions

Experienced teachers stated that the probabilities cause of misconceptions students in problem 1 is the three students do not understand the beginning of knowledge related to graph of quadratic functions, for example the coordinate axes, coefficients, variables, drawing graphs of functions ever learned before. In addition, students are less precise in the applying the concepts that have been studied for instance the concept of the axis of symmetry, the point of intersection with the graph of the coordinate axes. As for the second problem of misconceptions that occur on both the students caused the students do not understand about the story that was given and did not understand the initial knowledge as the possible cause of the problem 1.

Knowing the possibilities of students misconceptions by experienced teachers can ask questions, for example, for problem 1, questions to ask coefficient what coefficient related to the x-axis, what the coefficients a, b, or c constants? As it relates to the y axis, whether the coefficients a, b, or c constants? According to Kilic (2011), teachers need to identify student misconceptions and difficulties by asking question or using the appropriate task. Moreover, teachers need to be able to determine the source of difficulties and errors so that students can be corrected appropriately.

3. Knowledge of experienced teachers of alternative ways of overcoming students misconceptions

Experienced teachers stated that alternative ways to overcome the students misconceptions for problem 1 is explained again by the students way when given a quadratic function in general, to describe exactly, students simply replace the value of a, b, c with numbers. So, to see if the picture is true or not, it should be noted the rules are a> 0, b <0, and c> 0, then the student was asked to subtitute a, b, c with value. Then the students were asked to find the value of a > 0 to subtitute the values of a. values of b <0, and c> 0. Then the students were asked to create an image of the parabola, whether the pictures are really like this. So in determining the value of a, b, and c, students could write differently for in accordance with the terms given. After the students to draw a graph of the function is by itself the students will know the characteristics of a given function of graph (in question). The problem leads to understanding of the concept, which presented a graph of the function and then the students were asked to identify the characteristics of the values of a, b and c. On the problem given a parabola opens up means the value of a> 0, then on the graph of a function can also be seen that the axis of symmetry is on the right of y-axis (positive x-axis) means the value of b was negative (b <0) because formula axis of symmetry is $\frac{-b}{2a}$ and value a> 0 so that the value of b should be negative so that the axis of symmetry obtained is positive. Furthermore, the value of c associated with the point of intersection of the y-axis, because the point of intersection of the y-axis is above the x-axis (positive y-axis) so that when x = 0 then y = c means that the value of c must be greater than zero (c> 0) for the value of y is positive. Thus, the characteristics a, b, and c for the graph on the problem 2 is a> 0, b <0 and c> 0.

To overcome problem 2, experienced teacher explain again prosedure of solving by first step determine what is known from the question, then draw it with x-axis represent time and y-axis have a height. then from pictue, equation of height parabola is $-t^2+6t-8$, then determine what asked. From the picture of parabola, student asked to determined the heighest of bullet. Of course student can detemine it, then we link to concept of graph before. Then shown the turning point, how to find it? Before had been learned that to find the turning point with axis $-\frac{b}{2a}$ ond the ordinate $\frac{-D}{4a}$, it will represent point of maximum height. Later, determined value of a=-1, b=6, dan c=-8 then subtituted to $-\frac{b}{2a}$ obtained 3. To find the height can be done by 2 ways, they are by using $\frac{-D}{4a}$ such that obtained 1 meter and the second way is to subtitute the value of t=3 to the eqution $-t^2+6t-8$ hence obtained 1 meter. So, the maximum height is 1 meter with t=3 second. If student knows concept, then they will think that way 1 and way 2 will result the same value. Because symetry of axis is point of center from the two factor means same when added $\mathbf{x_1}$ and $\mathbf{x_2}$ then divided by 2 $(x_1+x_2)/2$. Wile the material before had explained that $x_1+x_2=-b/a$. Such that formulation for symmetry of axis is $x=\frac{-b}{2a}$ and because formulation of $y=\frac{b^2-4ac}{4a}$ truely obtained when we subtiute symmetry of axis $x=\frac{-b}{2a}$ to the equation then the result is $y=a\left(\frac{-b}{2a}\right)^2+b\left(\frac{-b}{2a}\right)+c$ or when simplified the $y=\frac{b^2}{4a}-\frac{2b^2}{4a}+\frac{4ac}{4a}$ or $y=\frac{b^2-4ac}{4a}$. Means equals when value of symmetry of axis we subtitute directly by using formula of $y=\frac{b^2-4ac}{4a}$.

Experienced teachers have some alternatives to overcome the students misconceptions that explains again procedures to resolve the problem properly, using context illustrations, and use the bridge analogy, for example, problem 1, experienced teachers provide one alternative to improve the students misconceptions that

is students asked to change any value of a but with the condition a> 0 or a<0. Likewise for the value of b and c. Later it would appear that although the values of a, b, and c are different, but if the condition is the same, is greater than zero or less than zero then the graph of the function will have special characteristics in terms of its location, open to the direction or intersection point toward an axis. According to Richland (2004) that reasoning analogy has an important role and is an effective strategy in learning mathematics. Using the example of the analogy is a way for teachers to improve student misconceptions. As for the second problem, an experienced teacher explains the intent of questions and troubleshooting procedures correctly. According to Moore (2006) that one of the methods of remediation is to explain / clarify.

Observing one aspect of PCK in this research is the knowledge of students which includes the teacher's knowledge of the possibility of student misconceptions, causing student misconceptions, and alternative ways to overcome students misconceptions on the quadratic functions material, especially experienced teachers. Results of research of Lee (2010) showed that teaching experience is also an important factor in the PCK of mathematics teacher. PCK contains elements of subject matter and how to teach it, as well as the knowledge of students. Ball and Bass (2000) defined that PCK is a special kind of knowledge that integrates the knowledge of mathematics teachers with the knowledge of students, learning and pedagogic. While Fennema & Franke (1992) stated that the knowledge of students is knowledge about specific student characteristics and built a classroom environment and planned an appropriate learning as needs of students.

IV. CONCLUSION

Pedagogical Content Knowledge (PCK) is the expert knowledge and special knowledge that integrates the mastery of subject matter, how to teach it, as well as the knowledge of students. Teaching experience is one of important factor in teacher's PCK. PCK of experienced teachers related to knowledge of students on the quadratic functions material, namely: (1) determine the possibility of students misconceptions on quadratic function material that is checked weather the relationship among concepts are right or wrong; (2) investigate the possible causes of the misconceptions that students do not understand taht is the prerequisite knowledge related quadratic functions and students are less precise in applying the concepts that have been studied; (3) know the various alternative ways of overcoming misconceptions on the quadratic functions, consisted of retell the procedure to solve the problem correctly, using context illustrations, and using the example of an analogy.

REFERENCES

[1] An, S., Kulm, G., & Wu, Z. 2004. The Pedagogical Content Knowledge of Midle School,Mathematics Teacher in China and The USA. *Journal of Mathematics Teachers Education* 7: 145-172

[2] Ball, D. L., Thames, M. H., & Phelps, G. 2008. Content Knowledge for Teaching: What makes it special? *Journal of Teacher Education* 59(5), 389-407.

[3] Fennema, E. & Franke, M.L. (1992). Teachers knowledge and its impact. In D.A. Grouws (Ed.), *Handbook of mathematics teaching and learning* (pp. 147–164). New York: Macmillan Publishing Company.

[4] Kilic, Hulya. 2011**.** Preservice Secondary Mathematics Teachers' Knowledge of Students. *Turkish Online Journal of Qualitative Inquiry*, April 2011, 2(2).

[5] Lee, Joohi, 2010. Exploring Kindergarten Teachers' Pedagogical Content Knowledge of Mathematics: *Department of Curriculum and Instruction-EC4, College of Education and Health Profesional, University of Texas at Arlington, Science Hall*.

[6] Ma, Liping, 1999. Knowing and Teaching Elementary Mathematics:*Teachers' Understanding of Fundamental Mathematics in China and the United States*. New York: Routledge.

[7] Moore, Gordon March, 2006. *Mathematical Misconceptions in Schooll Children*. Identification Impact and Remediation with a Computer Based Assesment System Master's Dissertation.

[8] Novak & Gowin. 1984. Learning How to Learn. New York : *Cambridge University Press*

[9] NCTM. 2000. *Principle and Standards for School Mathematics.* Reston: The National Council of Teacher Mathematics, Inc.

[10] Richland, Lindsey E., Holyoak, Keith J., and Stigler, James W. 2004. Analogy Use in Eighth-Grade Mathematics Classrooms. *Cognition and Instruction*, 22(1), 37–60

[11] Shulman, L. S. 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4 – 14.

[12] Shulman, L. S. 1987. Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review*, 57(1), 4 – 14.
[13] Suparno. 1998. Penelitian Tindakan Kelas, Makalah disajikan dalam Lokakarya Nasional Instrukur PKG.Depdikbud Bogor
[14] Zevenbergen, Dole, Wright. 2004. Teaching Mathematics in Primary Schools. Sydney : Allen & Unwin.