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Pre-service Teacher Interpretations of Students' Mathematical Understanding

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***Abstract*—**Teachers do not have direct access to students' mathematical understandings. However, by observing students' mathematical activities, teachers can interpret students' mathematical understandings that include hypotheses about what students know and understand. We use the point of view "the teacher's perspective from the researchers' perspectives". This paper explores the types of pre-service teacher (PSTs) interpretations of students' mathematical understanding, defined as how PSTs to give the impression, opinion, or a theoretical view of the mathematical information in the form of a students' written work in solving problems. This studies aimed to examine the kinds of preservice teacher interpretation. These studies capture work done with 42 PSTs in mathematic education programs at Muhammadiyah University of Purworejo. We give them examples of students' written works and ask them to explain what they understood. Findings illuminate the types of interpretation of the students' mathematical understanding: naive, surface, and complete interpretation. Naive Interpretation characterized by PSTs do not provide a description of students' mathematical thinking and focus on other than the students' understanding. Surface nterpretation characterized by PSTs describe student thinking in form of operation / procedure / strategy undertaken by students without giving meaning to such thinking. Complete Interpretation characterized by PSTs identify the mathematical elements of a detailed students strategy and commented that are consistent with the students' strategies.

Keywords: *interpretation, preservice teacher, mathematical understanding*

I. INTRODUCTION

Problem solving played important role in mathematics and a prominent role in mathematics education. Problem solving become one of the objectives of mathematics learning in Indonesia [2]. In the Common Core State Standards for Mathematics [4] the first principle of standard centered on problem solving - making sense of problems and persevere in solving problems. Therefore, solving problems is relevant to teachers task in teaching mathematics with understanding. Teachers should understand students' thinking to be able to manage problem solving situation in the classroom. Teacher ability to make sense of student thinking can be seen from of how teachers identify key aspects of mathematics in students' thinking during solving problem. Developing ability to interpret students' thinking enable teachers to make appropriate instructional decisions.

According to [3] although analysis of students' thinking was highlighted as one of the main tasks of teaching mathematics, identifying the mathematical ideas that are inherent in the strategy used by a student during solving problem can be difficult for teachers. Teachers often work with routine tasks and complex teaching practices. However, teachers need to know how students understand mathematical concepts to help improve their students' understanding([5]; [25]). This approach is based on listening and learning from students ([26]; [30]). In this case, the teacher must make a decision based on the students' thinking.

Identify strategies that might be used by students in solving problem enable teachers to interpret why certain problem become difficult. It also allows teachers to pose a problem considering the characteristics of students' thinking. Teachers may be able to interpret students mathematical understanding accurately if they understand the mathematical ideas that relate to a specific mathematical domains. This knowledge can help teachers to understand the characteristics that make problem difficult for students and why[10].

Relating to the relevant role of student's mathematical thinking in teaching mathematics, mathematics teacher program set several goal that is the development of teacher's ability to interpret student's

mathematical thinking [27]. The idea to design learning based on the students' thinking increasing among researchers in all content area([9]; [30]; [6]; [21]). The researchers interested in examining the mathematics teacher noticed the phenomenon of the concepts they learned - noticing- in various ways. Different researchers incorporate different aspects of teachers' thinking and practices in their definition of noticing. Mason initiated noticing as potentially unintended actions than accidental actions (pottentially intentional rather than haphazard act) [13]. The center of this view is the idea that noticing is a collection of practices that are designed to sensitize ourselves so that they can see the opportunity in the future where the action is really new is not automatically out of the habit. Some researchers understand noticing as only involves a process that teachers initially see, or feel, the different aspects of classroom activities. For example, Star and Strickland [14] and Star, Lynch, and Perova [26] examined "what captured her attention (teacher), and what they miss ... when they see classroom lessons". The approach to teacher noticing, then, involves exploring what a teacher attend to as well as what the teachers decide to attend. Other researchers are interested not only in the initial screening classroom activities but also in teacher interpretation of the activity. This is a general stance taken in previous studies (eg, [1]; [18]; [19]). In particular, [20] have focused on noticing a professional vision in which teachers are selectively attend to events that happened and then draw their existing knowledge to interpret the observed events. For example, teacher noticing will include not only teachers pay attention to certain students' ideas but also what teachers understand of the idea based on their knowledge about students and mathematical content. Sherin, Russ, and Colestock [20] assumes that teacher expectations and knowledge affect how teachers view the events that occur in the classroom. Thus, understanding how teachers interpret what he felt.

Mason [12] consider the ways in which noticing has produced insights and informed action in teaching, learning, and conduct professional development related to mathematics. Constructs such as attention and intention, awareness, and consciousness not only be investigated using a discipline noticing and informed how noticing actually works, but also contribute to their appreciation for the complexity of learning and teaching mathematics. Jacobs, Lamb, & Philipp [30] take more inclusive views about teacher noticing. They define professional noticing as involve not only teachers attend to and interpreting classroom activity but also teachers plan to respond to the activity. Jacobs, Lamb, & Philipp [30] conceptualized teacher expertise in professional noticing of children's mathematical thinking as a set of three skills that are interrelated: attending to children's strategies, interpreting children's understanding, and deciding how to respond underlying children's understanding.

Other studies indicate the relevance of PSTs interpretation about students' mathematical thinking to determine the quality of mathematics teaching ([22]; [16]; [26]; [18]). Therefore, the needs of prospective teachers to based their decision on their students' understanding underlines the importance of characterizing and understanding these skills [10], This confirms the need to focus our attention on how PSTs identify and interpret student's mathematical thinking in different domains ([7]; [11]). Research on the mathematics teachers' professional development underlines the importance of pre-service teacher noticing in teaching mathematics ([30]; [13]; [8]). Researchers and mathematics teacher educators consider developing noticing as a way to identify of how teachers understand complex situation in the classroom [21]. Mason [13] introduced the notion of awareness to characterize the ability of noticing as a consequence for the organization of teacher noticing on relevant teaching events. Jacobs [30] focus on identify key aspects for the students' mathematical thinking and interpretation to make decisions in teaching mathematics.

II. PRE-SERVICE TEACHER'S INTERPRETE STUDENTS' MATHEMATICAL THINKING

Interpreting the student thinking is an essential component of high-quality learning and assessment ([24]; [28]). Focus view of interpreting the student thinking is about what can be observed from the teaching skills of PSTs pay attention to w hat students do. A key factor in interpreting the students' work is the ability to see key aspects of students' mathematical thinking (eg, [30]; [21]; [8]). Skills around noticing can be learned [30] therefore it is important to know what is already owned by PSTs to work as they enter teacher preparation.

According to [17], of pre-service teacher skills in interpreting student's thinking can be evaluated by see whether: accurately describe the students' methods; accurately characterizes student's understanding; as well as accurately anticipate students' responses based on evidence of interaction with the students. The evaluation was conducted by analyzing students' written works and interaction with students with certain standards. If no interaction with the standardized students the skills can be evaluated by whether PSTs can precisely define the appropriate response to their interpretation. Noticing of children's mathematical thinking not only requires attention to students strategies but also the interpretation of mathematical understanding is reflected in these strategies. Identify the extent to which the evidence presented PSTs in interpreting the children's understanding is not looking for the single best interpretation but rather the

extent to which participants interpretation is consistent with the details of the strategy selected children and research on the development of mathematics children. Jacobs, Lamb, & Phlilipp [30] classifies the sample response within 3 scale: robust evidence, limited evidence, and lack of evidence of interpretation of the children's understanding.

Interpretation of students' mathematical thinking is giving impression, opinion, or a theoretical view towards mathematical information in the form of students' written work in solving problems. Researchers hyphotetize interpretation of PSTs into four categories: *Beyond Complete Interpretation*; *Complete Interpretation*; *Surface Interpretation*; and *Naive Interpretation*. Beyond Complete interpretation is the interpretation which is characterized by by identifying the mathematical element in the detailed students strategy and give comments relevant to the students strategy in various ways and make comments that are consistent with the strategy demonstrated and research on the development of students' understanding. Complete interpretation is the interpretation which is characterized by identifying the mathematical element in the detailed students strategy and give comments relevant to the students strategy. pre-service teacher's responses indicates strong evidence of interpretation but does not identify student's profile or relating with research on the development of students' understanding. Surface interpretation is the interpretation which is characterized by focusing on the interpretation of students' mathematical thinking by describing the student's operations / procedures / strategies without giving meaning. And then Naïve interpretations characterized by PSTs do not provide interpretations about student's mathematical thinking and have another focus (instead focus on the interpretation of students' understanding) like something they learned about the mathematical teaching and learning in general.

III. METHODOLOGY

This study aimed to analyze of how interpretation of PSTs about students' mathematical thinking. The subjects were 42 students of the 3rd year Mathematics Education Muhammadiyah University of Purworejo, 37 females and 5 males. Researchers gave them four examples of students' written work about comparison and ask them to give an explanation of what they learned or understood about the students' understanding. Researcher using four samples of student work for each subject to obtain valid data on their interpretation. Furthermore, the researchers conducted a qualitative descriptive analysis towards their interpretation of students' mathematical thinking based indicators hyphotetized in each category. The indicators of each category are presented in the table below.

Table 1. Indicator and Descriptor of the Interpretations

Types of Interpretation	Indicator	Descriptor
1. Beyond Complete Interpretation	Interpretation characterized by PSTs identify the mathematical elements of a detailed students strategy in various ways and commenting that are consistent with the strategy presented and research on the students mathematics development.	1.1 Explains the detailed strategies of each students 1.2 Explains the process that students done in completing problems 1.3 Comment focused on the relevant mathematical details that reflected students' understanding 1.4 Recognize the strategies and understanding which did not demonstrated by children 1.5 Compare and contrast the answer / strategy students
2. Complete Interpretation	Interpretation characterized by PSTs identify the mathematical elements of a detailed students strategy and commenting that are consistent with the students' strategies.	2.1 Explains the detailed strategies of each students 2.2 Explains the process that students done in completing problems 2.3 Comment focused on the relevant mathematical details that reflected students' understanding
3. Surface Interpretation	Interpretation characterized by PSTs describe student thinking in form of operation / procedure / strategy undertaken by students without giving meaning to such thinking.	3.1 Describes the operation / procedure that appears on the child's responses without giving meaning 3.2 Explains the students' strategies in the common sense and sometimes undefined 3.3 Provide comments that are linked to the strategy of the child, but sometimes overgeneralized. 3.4 Using a common phrase and rarely mention the mathematical details in interpreting the child's understanding
4. Naive Interpretation	Interpretation characterized by PSTs do not provide a description of students 'mathematical thinking and focus on other than the students' understanding.	1.1 did not describe the students' understanding 1.2 describes what they learned or their own understanding 1.3 provide a response in the form of evaluation of learning 1.4 provide a response in the form of suggestions to improve learning 1.5 comment on the students, not towards student's understanding(less focus on the students as individu)

IV. RESULT AND DISCUSSION

A. The attention of PSTs towards students strategies

Researchers gave subjects are four examples of students' written works. This method is used to obtain valid data on their interpretation. If at least 2 of their interpretation of the interpretation consistent 4 shows the categories of interpretation then they can be grouped in that category. Subjects were asked to attend to student's work or strategies that students use in solving the problems of comparison. Their attention was categorized into two groups that understand students strateges and groups of students who do not understand the strategy. The results showed that 27 subjects more understand the strategies that students use and 15 subjects less understand the strategies that students use. Distribution of the attention of PSTs about the problems comparison are presented in the following table.

Table 2. Distribution of PSTs attention of four problems about Comparison

Problem	Subject Understand Students strategies	Subject less Understand Students strategies	Total
1. To build a multi-storey building, a building contractor takes 15 months with 120 workers. For one thing, the contractor requires accelerated work 3 months. If, ability to work everyone equally and that the project can be completed on time, how many workers should be added?	7	4	11
2. Rani and Diki make a drink that consists of a mixture of water and orange syrup. They use the same glass, but with a different recipe, namely: Rani use 3 cups water and 2 cups of orange syrup, while Kiki use 5 cups of water and 4 glasses of orange syrup. Whose drink is more orangey?	7	3	10
3. Three hectares paddy fields cultivated with rice is expected to generate as much as 13 1/2 tons of rice for one harvest. If Mr. Alan has a 3/4 hectares of rice fields were entirely cultivated with rice, how many tons of paddy rice produced Sir Alan for one harvest?	6	5	11
4. Mira and Indah use the same provider for their Hp numbers. Mira spent pulse IDR 5000 for 3 days, while Indah spent IDR 10000 pulses for 5 days. Which of them is more efficient in the use of pulses?	7	3	10

Students' understanding of the strategy is shown by evidence of their concern about the student's work. Subject can describe and show the students' line of thought. Here is an example of how they follow the strategy of the students about Comparison. For the third problem, here are two samples of student work.

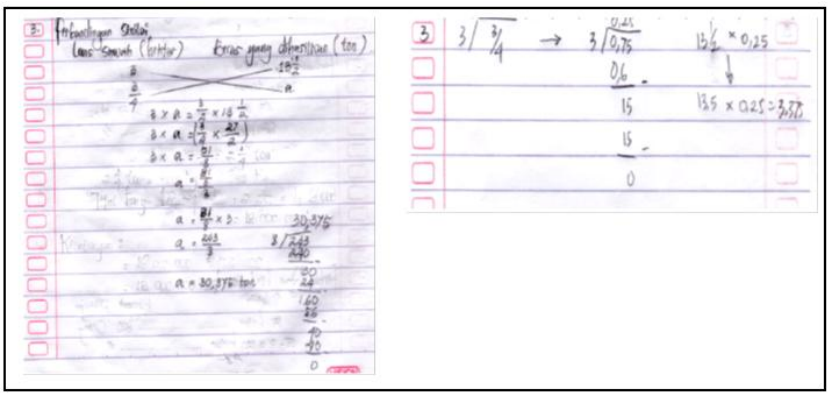

Figure 1. Sample of Students Works

Subject S9 demonstrated her understanding of strategy samples 1 and 2. She seemed to attend to students' thinking in solving the problem. Here is evidence of his attention.

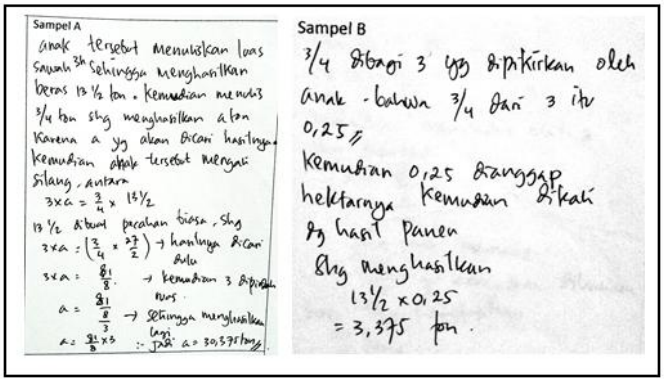

Figure 2. Subject S9 Attending to Students strategies

For sample A, she expressed her concern about the strategy to attend to students' strategies with the student thinking at every step in the students's responses. For example

$3 \times a = ¾ \times 13\ ½$

$13\ ½$ made common fraction so

$3 \times a = (¾ \times 27/2)$ → look for the results first

$3 \times a = 81/8$ → then move 3 to other side

$a = \frac{81}{\frac{8}{3}}$ → so resulting

$a = \frac{81}{8} \times 3$

So a = 30,375 ton

Although the student answers incorrectly, she can follow student's strategy and show students' mistakes in his interpretation. She knew that sample A make a mistake when dividing $= \frac{81}{8}$ by 3. And make a correction that it should be $= \frac{81}{8} \times \frac{1}{3}$. She was also able to follow the thought of sample 2. Sample 2 dividing ¾ to 3 in advance and multiplying the result with the harvest for 3 hectares. Then he get the result is 3,375 ton. It showed her understanding about unusually students' work.

B. The Interpretation of PSTs towards students understanding

To describe the interpretation of PSTs about student understanding, we chose subject that shows evidence he attend to students strategies in solving problems. This is determined by consideration that subjects did not show evidence of attending to students strategies because of lack of their mathematical understanding of mathematics about problems. To be able to make interpretations, they must have an adequate mathematical understanding. Researchers hyphotetize four categories of interpretation also illustrates stage of development of interpretation. The stages of interpretation is shown in the following figure.

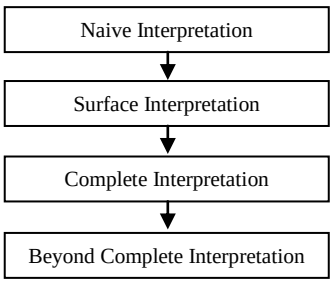

Figure 3. Development Stage of Interpretation

When the subject is asked to interpret the students' understanding, mostly fixated on the instruction "Explain what you learn or understand about understanding each child". This is evident in most of the answers: the students know enough, students have good understanding of the subject , the students' understanding is still lacking, and others. Nevertheless, their interpretation can still be categorized into the following categories. That showed the early interpretation toward students' understanding.

1. Beyond Complete Interpretation

Beyond Complete interpretation is the interpretation which is characterized by by identifying the mathematical element in the detailed students strategy and give comments relevant to the students strategy in various ways and make comments that are consistent with the strategy demonstrated and research on the development of students' understanding. Responses indicates strong evidence of interpretation. According to [30]) responses that indicates strong evidence interpretation understand the details of the strategy in different ways but all are consistent with the strategy demonstrated and research on the development of mathematics of children. The response also noted of how these details reflect what the children do not understand and recognize the strategy and understanding that are not demonstrated by the students. Researcher viewed that when the response identify mathematical elements and identifying child's thinking profiles [3] and accurately describes students methods, accurately characterize the students' understanding [17], the interpretation is belong to this category . There is no subject in this research meet those criteria. but I think that there must be a teacher candidates who meet these characteristics. this is indicated by some the subjects showed several characteristics. For example Subject 9 to problems 3, he attend to student's strategies and also shows which are not carried students the so the answer become wrong.

Figure 4. Interpretation S9 to Sample A written works

He understands that the sample A made mistakes in the division of fractions which cause errors in the final answer.

2. Complete Interpretation

Complete interpretation is the interpretation which is characterized by identifying the mathematical element in the detailed students strategy and give comments relevant to the students strategy. pre-service teacher's responses indicates strong evidence of interpretation but does not identify student's profile or relating with research on the development of students' understanding. S9 showed complete interpretation case.

3. Surface Interpretation

Surface interpretation is the interpretation which is characterized by focusing on the interpretation of students' mathematical thinking by describing the student's operations / procedures / strategies without giving meaning. This interpretation is conform with the interpretation that shows limited evidence of student understanding. Limited evidence, include responses in which subjects maintain a focus on interpreting the children's understanding but with a depth of less than a response that shows strong evidence [30]). Subject describes the children's understanding, but often in a broader sense that sometimes undefined. A special connection to the strategy of kids is there, but they are limited, and the conclusions that are sometimes overgeneralized, exceeded evidence presented. Responses are common, sometimes coupled with overgeneralization. The response is limited evidence still focused on interpreting the children's understanding. Subject indicates that students understand or not understand the material and observed that students are less conscientious in solving problems. subjects using the phrase student already understand or do not understand, but he does not show the mathematical details as evidence of his interpretation. For example, S2 in first problem only said that “sample A did not understand the subject and problem given”. He also said that “sample C is sufficient understand matter being taught, but students was less conscientious in solving problems”.

4. Naive Interpretation

Naïve interpretations characterized by PSTs do not provide interpretations about student's mathematical thinking and have another focus (instead focus on the interpretation of students' understanding) like something they learned about the mathematical teaching and learning in general. This interpretation does not show any evidence of their attention to the mathematical aspects of the student's work. This interpretation is consistent with the lack of evidence that the pre-service teacher / teachers interpreting. [30] indicates that some of the responses did not provide evidence of interpretation of the children's

understanding, even though the participant has explicitly requested to do so ("Can you explain what you learned about the children's understanding"). This response has an alternative focus, such as something about the teaching and learning of mathematics and learning in general. This category is dominated by the subject that does not show evidence of that he understand students' strategies. For example, S4 for problem 2 interpret sample B written works by judging that Sample B seems student are lazy writing and likes to draw conclusions without reasonable cause.

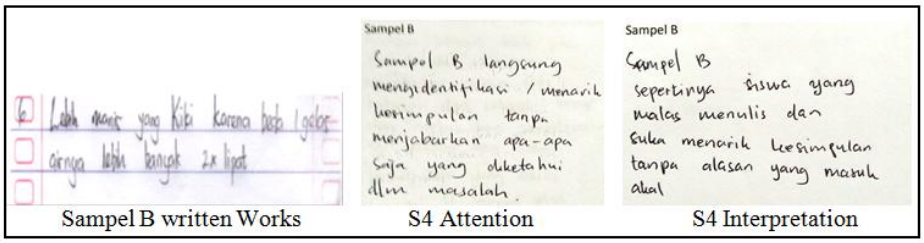

Figure 5. Comparing sample B written works, S4 Attention, and S4 Interpretation

We can compare students' understanding with understanding the subject. Although the students' answers is short, but these students demonstrate the ability to reason in comparing the two mixed drinks. Subject view that students do not understand the problem.

V. CONCLUSION

Based on the analysis, researchers concluded that pre-servive teacher interpretation are on three stage of interpretation, i.e complete interpretation, surface interretation, and naive interpretatin. However, researcher believe that beyond complete interpretation is possible bellonging by pre-service teacher. A lot of PSTs' interpretations about students' mathematical understanding at the stage of surface interpretation and complete interpretation. Their presence on this stage is not separated from their understanding about the subject matter, their pedagogical knowledge, as well as their reasoning ability. Sentence order to explain what they have learned and understood from each students understanding makes the subject stuck to interpret students' understanding to understand or not understand the problems.

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