4th ICRIEMS Proceedings

Published by The Faculty Of Mathematics And Natural Sciences

Yogyakarta State University, ISBN 978-602-74529-2-3

Stability Analysis of SEIR Model (Susceptible-Exposed-Infected-Recovered) with Vaccination on the Spread of Measles in Sleman Yogyakarta

Septina Wahidah Indrayani[1a], Nikenasih Binatari[b]

Department of Mathematic Education, Faculty of Mathematics and Natural Science, Yogyakarta State University
Jl Kolombo No 1, Karangmalang, Depok, Sleman, Yogyakarta, Indonesia

[1]septinawahidah@gmail.com [2]nikenasih@uny.ac.id

Abstract. Measles is an infectious disease that is still happening in Sleman Regency. This disease is caused by infection of measles virus, *paramyxovirus,* due to direct contact with infected individuals. This research aims to analyze the spread of measles disease by establishing mathematical model, *SEIR (Susceptible-Exposed-Infected-Recovered)*, interpreting the model by performing a simulation model, and analyze the effect of vaccination on the behavior of spreading measles in Sleman. Stage to analyze the spread of disease in the case of disease free is (1) determine the assumptions based on the characteristic of measles, (2) constuct the measles disease spread models, (3) determine the equilibrium point of the disease, (4) find the basic reproduction number, (5) analyzes the stability of the model around the free disease equilibrium point, and (6) simulate the spread of the disease using the parameters. The results obtained can be in the form, the percentage of the grant amounting to 95% vaccine hasn't been able to prevent the occurrence of endemic measles in Sleman Regency, Yogyakarta, however, it can shorted the period of the spread and the rate of individuals infected with measles allowing for less than 100 days will decrease the spread of disease in the population.

Keywords: measles, SEIR model, disease free equilibrium point, the basic reproduction number, stability.

INTRODUCTION

Measles is an infectious disease that is found in the world and is considered a public health issue that must be resolved. Early symptoms of measles be fever, runny nose cough and small spots with the middle section is white or bluish white with a reddish base in the cheek area. A typical sign of redness in skin patches on the third day to seven, starting in the area of the face, then, takes about 4-7 days, and sometimes end up with a brownish colored skin exfoliation.

Prof.Dr.m.Juffrie, Ph.d., SP. (K), a professor of Health Sciences section of the Faculty of medicine, Gadjah Mada University, vaccination of measles is the most effective way to prevent transmission of the measles virus. Immunization is given by way of delivering vaccines (antigenic material used to produce active immunity to a disease so they can prevent or reduce the influence of infection by an organism into a person's body to give immunity to disease. (Nugroho, 2009).

Based on data centers and health information (2015), Yogyakarta included areas with coverage of immunization which belongs to high. However, cases of measles still occur in some areas in Yogyakarta. At DIY, measles vaccine coverage reached 96.7%. Nevertheless, Sleman is an area with categorized of immunization are low. Sleman's vaccine still low than Kulonprogo Regency, Yogyakarta Regency and Bantul Regency. Therefore, it will be done analysis of the stability of the SEIR model with vaccination on disease spread measles in Sleman Regency of Yogyakarta. This analysis aims to find out the spread of measles in a population with the influence of vaccination.

The problem is limited to the analysis of the spread of the disease in the case of measles disease free equilibrium point that is when the disease does not spread in population, the spread of disease in population that is only the Sleman Regency.

The purpose of this research is to find out the influence of vaccination on the model behavior of the spread of disease measles in Sleman Regency, Yogyakarta.

EXPERIMENTAL

Materials and Method

The data in this study were taken based on health data and information center by 2015 the Province of DIY.

Assumptions of the Model are

a. The population is assumed to be large enough so it can be referred to as a continuous variable.
b. The population is assumed to be closed.
c. Birth and death note.
d. Any individual born is assumed to be susceptible.
e. Any individual having the same possibilities in contact with other individuals.
f. Individual who is infected with a disease can be cured of the disease and may die of the disease.
g. It is assumed there is only one disease that spread in the population.
h. The vaccine is given in a new-born child.
i. Vaccine efficacy was 100%.
j. Immunity that occurs because the vaccine is permanent.

Model Notation

b = birth date

μ = mortality rate

β = contact rate

σ = infectious rate

γ = recovery rate

δ = differential mortality due to measles

p = proportion of those successively vaccinated at birth

Every individual who is newly born, get into susceptible class. Then out of the susceptible class, as it enters the exposed class (individuals who contracted the make contact directly with the other individual) or allowed to die naturally (of death not due to measles).

Someone is going to get into the exposed class when a virus invades a human on a susceptible classes, and individuals who contracted the make contact directly with other individuals in the population. Then out of the classes exposed, because the virus is evolving and infecting the individual from a class then the incoming exposed class infectious, or because of natural death.

Someone will go to infected class because of infectious viruses have been infecting the individual from a exposed class. In this class, the individual can be cured or die either naturally or death from the disease. If someone dies naturally or due to diseases it will automatically come out of the system. Furthermore if a person is cured of the disease then it will go into the recovered class.

After a certain time, a person can recover and enter class recovered. One can entergrades recovered having been vaccinated and subsequently out of class because ofnatural death is recovered.

Mathematical model of flow diagram of measles vaccination yet, as follows :

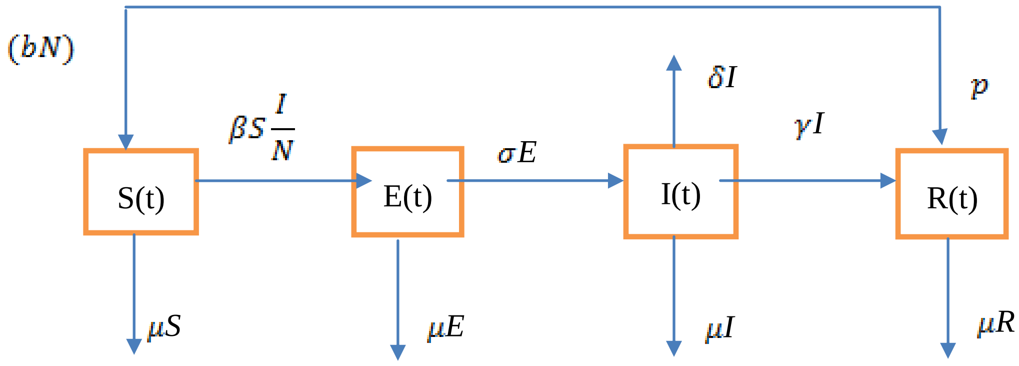

Figure 1. Transfer Diagram of Disease Spread Measles with Vaccination

b is defined for a birth rate. The number of individuals born in the population per unit time always constant. Birth population number proportionate to the total population of N. Then, the birth population is bN. Birth population will enter the class S.

μ is the natural mortality rate, based on the assumption of birth rate equal to the death rate, then the number of the population who died in each group proportional to the number of population in each group. Therefore, the number of deaths on the Group S, E, R each of μS, μE, μR, whereas in Group I of (μ + δ) δ I with is death due to measles disease.

The magnitude of the population figure is βSI infected where β is the transmission coefficient is a constant that indicates the level of contact so the transmission of disease, the exposed individuals get βI infection incidence and the number of disease in populations of infected βS I/N.

σ is the number of infected individuals who have been exposed. The notation γ is the healing of the individual figures have been infected. The notation p is the percentage of the susceptiblein vaccinations per unit of time. Notation bpN is the percentage of susceptible class to successfully vaccinated and entered the class of exposed. population

The following differential equation models to be determined for each class.

i. The magnitude of the number of individuals who are susceptible is affected by the number of individuals born in population will then decrease with diluted by the persence of the proportion of successfull vaccination at birth of bpN, numbers of individuals exposed βS (t) (I (t))/N, and natural mortality μS (t). Then, the equation obtained (dS (t))/dt = b (1-p) N-βS (t) I (t)/N-μS (t).

ii. The magnitude of the number of individuals who become infected or exposed class change with respect to time is affected by the number of individuals exposed βS (t) (I (t))/N then will decrease with the infected population and natural mortality μE (t). So the equation obtained (dE (t))/dt = βS (t) I (t)/N-(μ + σ) E (t).

The magnitude of the number of infected individuals or classes of infections over time is affected by the number of infected σE (t) then will decrease with the number of individuals cured γI (t) and natural mortality and mortality because μI measles δI(t). So the equation obtained (I(t))/dt = σE (t)-(γ + μ + δ) I (t).

iii. The magnitude of the number of individuals cured or recovered from a class change with respect to time is influenced by the proportion of successful vaccination at birth of bpN and healing of the infected individuals γI (t) and natural mortality μR (t). So the equation obtained (dR(t))/dt = bpN + γI-μR (t) (t).

From Figure 1 and the explanation above obtained mathematical model with measles vaccination called system (1) as follows :

$$\frac{dS(t)}{dt} = b(1-p)N - \beta S(t)\frac{I(t)}{N} - \mu S(t)$$

$$\frac{dE(t)}{dt} = \beta S(t)\frac{I(t)}{N} - (\sigma + \mu)E(t)$$

$$\frac{dI(t)}{dt} = \sigma E(t) - (\gamma + \mu + \delta)I(t)$$

$$\frac{dR(t)}{dt} = bpN + \gamma I(t) - \mu R(t)$$

The transformation can be formed from the system (1) to the system (2) the following:

$$\frac{ds}{dt} = b - bp - \beta si - sb + s\delta i$$
$$\frac{de}{dt} = \beta si - \sigma e - eb + e\delta i$$
$$\frac{di}{dt} = \sigma e - \gamma i - \delta i - bi + \delta i^2$$
$$\frac{dr}{dt} = bp + \gamma i - br + \delta ir$$

RESULTS AND DISCUSSION

Disease Free Equilibrium Point

Disease free equilibrium point is obtained when no individuals infected $(i = 0)$. Disease free equilibrium point of a system (2) $E_0(\hat{s}, \hat{e}, \hat{\imath}, \hat{r}) = ((1-p), 0, 0, p)$.

Basic Reproduction Number

Basic reproduction number is a parameter that is used to find out the rate of spread of a disease. The basic Reproduction number is the average of the number of susceptible individuals are infected directly by another individual who was already infected when the infected individuals enter into a population who are all still vulnerable. The basic reproduction number can be retrived using the method Next Generation Matrix and obtained from the maximum eigenvalue. This matrix is formed by the sub-sub class population exposed and infection. Then, the retrieved for basic reproduction number is

$$R_0 = \frac{\left(\frac{\sigma + b}{\sigma + \mu} + \frac{\gamma + \delta + b}{\delta + \gamma + \mu}\right)}{2} + \frac{\sqrt{\left(\frac{\sigma + b}{\sigma + \mu} + \frac{\gamma + \delta + b}{\delta + \gamma + \mu}\right)^2 - 4\left(\left(\frac{\sigma + b}{\sigma + \mu}\right)\left(\frac{\gamma + \delta + b}{\delta + \gamma + \mu}\right) - \frac{\beta(1-p)}{(\delta + \gamma + \mu)}\left(\frac{\sigma}{\sigma + \mu}\right)\right)}}{2}$$

Stability the Equilibrium Point of Disease Free

After the point of equilibrium is obtained, then it will be on the analysis of the stability of the equilibrium point. The stability of the equilibrium point is used to know the behavior of the system by defining R_0.

Next up will be analyzed the stability of an equilibrium point around this is expressed in the following theorem.

Theorem 1

(i) If $R_0 < 1$ then the equilibrium point of disease free $E_0(\hat{s}, \hat{e}, \hat{\imath}, \hat{r})$ is stable local asymptotic.
(ii) If $R_0 > 1$ then the equilibrium point of disease free $E_0(\hat{s}, \hat{e}, \hat{\imath}, \hat{r})$ unstable.

Proof:

Jacobian matrix at the equilibrium point of disease free $E_0(\hat{s}, \hat{e}, \hat{\imath}, \hat{r}) = ((1-p), 0, 0, p)$ is :

$$J_0 = \begin{bmatrix} -b & 0 & -\beta(1-p) + (1-p)\delta & 0 \\ 0 & -(\sigma+b) & \beta(1-p) & 0 \\ 0 & \sigma & -(\gamma+\delta+b) & 0 \\ 0 & 0 & \gamma+\delta p & -b \end{bmatrix}$$

There are 3 eigenvalue :

1. $(\lambda + b) = 0$

   Then, $\lambda_1 = -b$
2. $(\lambda + b) = 0$

   Then, $\lambda_2 = -b$
3. Analysis of stability on the 3rd possibility can be obtained by using the Routh-Hurwitz table. In order to make the system stable, then the coefficients of Routh-Hurwitz should be marked positive.

   $$(\sigma+b)(\gamma+\mu+\delta) - \beta(1-p)(\sigma) > 0$$

   $$\frac{1}{(\sigma+b)(\gamma+\mu+\delta)}\beta(1-p)(\sigma) < 1$$

The equilibrium point of disease free $E_0(\hat{s}, \hat{e}, \hat{\imath}, \hat{r}) = ((1-p), 0, 0, p)$ is stable asymptotic if $\frac{1}{(\sigma+b)(\gamma+\mu+\delta)}\beta(1-p)(\sigma) < 1$, indicates that the population did not occur at a spread of disease.

Results and Discussion

Based on Central Bureau of Statistics Sleman 2015, the population numbered 1.167.481 in Sleman Regency soul birth 14.134 inhabitants, the number of individual infected 104, and the number of deaths due to measles disease 0.

Parameter Estimation Model

1. Infectious rate $(\sigma) =$ 1/incubation period $= \frac{1}{12} = 0{,}0833$
2. Recovery rate $(\gamma) = \frac{1}{9} = 0{,}1111$

3. Birth date $(b) = \frac{14134}{1167481 \times 365} = 3{,}317 \times 10^{-5}$

4. Mortality rate $(\mu) =$ the average age of people of Indonesia are 70 years or 25550 days $= \frac{1}{25550} = 0{,}000039$

5. Differential mortality due to measles $(\delta) = 0$

6. Based on data and information centre Prov. DIY 2015, the percentage of successful vaccination $(p) = 95\% = 0.95$

7. Contact rate = $\beta = \frac{15}{9} = 1.6667$

Simulation

Simulation of mathematical model on the spread of measles disese SEIR in Sleman Regency.

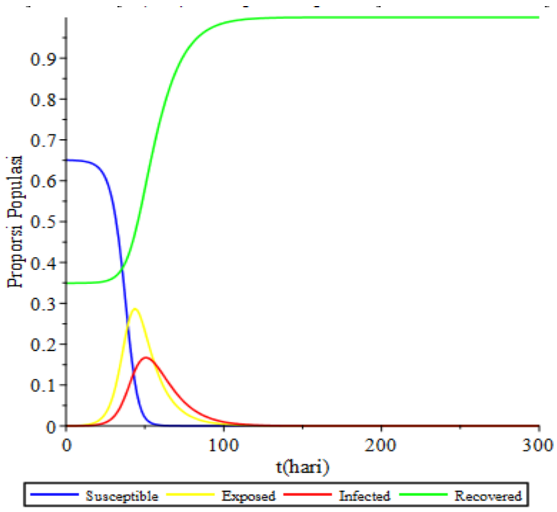

Figure 2. Simulation of the Spread of Measles Disease

If the value of the parameter in substituting to the system (2) then the retrieved value $R_0 = 2.87169$ for the point of equilibrium $((1-p), 0, 0, p)$. Furthermore, analysis of stability of the equilibrium point in disease free, retrieved the real eigenvalues first and second is negative. While the value of the third real eigenvalues, the coefficient Routh is positive. As a result, relations R_0 with coefficients Routh became $1 > R_0 > Koefisien\ Routh$ then the System unstable.

Cases with Different Vaccination Effectiveness

This section will be seen about the influence the effectiveness of vaccination against a disease. Starting with vaccinations 10% and 45% will be indicated with charts.

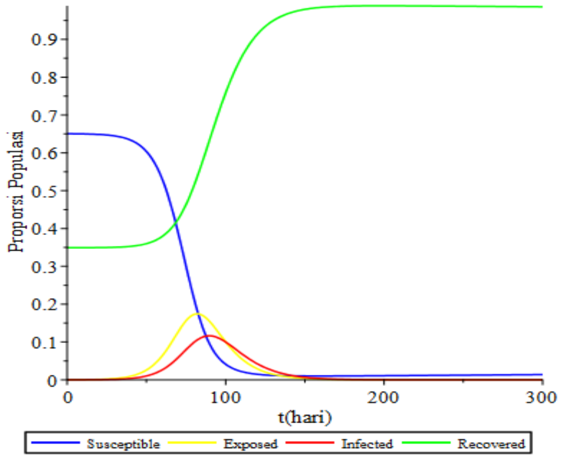

Figure 3. Cases with 10% Vaccine Effectiveness

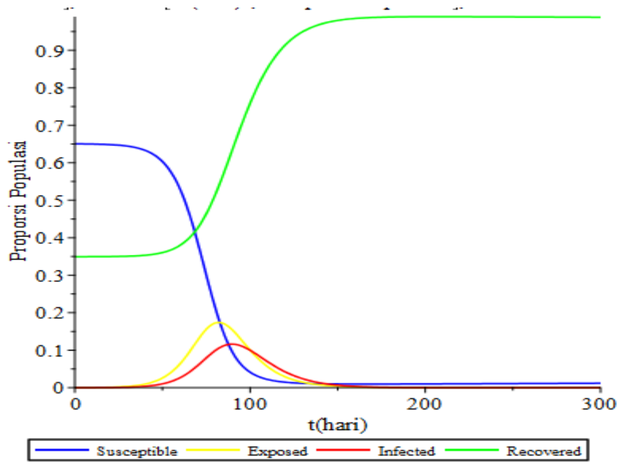

Figure 4. Cases with 45% Vaccine Effectiveness

At the vaccination 10% and 45% resulted in a period of the spread of disease and the infected individuals become longer that is more than 100 days. From figure 2 shows that the percentage of the value of vaccines to 95% the proportion of infected population increase then decrease in the span of less than 100 days. It can be concluded that the magnitude of the percentage of successful vaccination can accelerate the spread of the period and the rate of individuals infected. However, because the system is unstable, the percentage of vaccines amounting to 95% have not been able to prevent the occurrence of endemic measles in Sleman Regency, Yogyakarta.

CONCLUSION

Based on the characteristics of the spread of the disease measles vaccination can be formed with a mathematical model of SEIR with 4 classes are susceptible, exposed, infected, and recovered. Disease free equilibrium point $((1-p),0,0,p)$

$$R_0 = \frac{\left(\frac{(\sigma+b)}{(\sigma+\mu)}+\frac{(\gamma+\delta+b)}{(\delta+\gamma+\mu)}\right)+\sqrt{\left(\frac{(\sigma+b)}{(\sigma+\mu)}+\frac{(\gamma+\delta+b)}{(\delta+\gamma+\mu)}\right)^2+4\left(\left(\frac{\sigma+b}{\sigma+\mu}\right)\left(\frac{\gamma+\delta+b}{\delta+\gamma+\mu}\right)-\frac{\beta(1-p)}{(\delta+\gamma+\mu)}\left(\frac{\sigma}{\sigma+\mu}\right)\right)}}{2}$$

and furthermore can be obtained for the value of the percentage of vaccines of 95% the proportion of infected individuals experience increased then decrease in the span of less than 100 days.

The magnitude of the percentage of successful vaccination can accelerate the spread of the period of individuals infected, but haven't been able to prevent the occurrence of endemic measles in Sleman Regency, Yogyakarta.

REFERENCES

1. Braueer, Fred, dkk. 2008. *Mathematical Epidemiology: Mathematical Biosciences Subseries*. Berlin: Springer-Verlag Berlin Heidelberg.
2. Departemen Kesehatan Republik Indonesia. 2016. *Profil Kesehatan Indonesia* 2015. Jakarta: Kementrian Kesehatan Republik Indonesia.
3. Olsder, G. J & Woude, J. W. van der. 2004. *Mathematical System Theory*. Netherland: VVSD.
4. Ross, Shepley L.1984. *Differential Equations: Third Edition*. New York: John Wiley and Sins.
5. Wiggins, Stephen. 2003. *Introduction to Applied Nonlinear Dynamical System and Chaos: Second Edition*. New York: Springer-Verlag.