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Ring Structure in Set of Codons

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Abstract. *Algebraic structure is widely known in mathematics include: Groups, Ring, fields, modules and so on. Overview investigated structures are usually on the sets that exist in the study of mathematics, but it may also associatwith other fields such as the field of biology that is the set of codons. Codon is thethree nucleotide base pairs of {U, G, C, A} as a compossser of the RNA chain and each codon will encode a particular amino acid to be spliced proteins.In the study of mathematics set of Codon will have certain Algebraic Structures. In this paper ,the set of codons form a ring structure. This review will begin by defining a cyclic group of order 4 which will be matched by the members of the set of nucleotide bases that can be reviewed as a group structure. Furthermore, the set of bases will be converted into a set of standard genetic code and are matched with $\mathbb{Z}_{64}$members so that it can be reviewed as a ring structure with a binary operation defined.*

INTRODUCTION

DNA (deoxyribonucleic acid) is the genetic material that holds a-long-term genetic information of living beings. This information will be inherited to the next generation. In addition to the genetic material DNA, there are also RNAs (Ribosanucleic Acid). Both of these materials are known as nucleic acids.

DNA is a macromolecule that consists of two polynucleotide chains linked to each other. Each nucleotide consists of three components: a nitrogenous base, a pentose sugar (deoxyribose) and phosphate groups. Nitrogenous bases include purine and pyrimidine bases. The purine bases guanine (G) and adenine (A), pyrimidine bases include cytosine (C) and thymine (T) are also included [4].

Just like DNA, RNA is also a polynucleotide. If sugar component in DNA form as deoxyribose sugar, then the component form of RNA is ribose. Purine and pyrimidine bases are also found in RNA, but one of pirimidine bases is uracil (U).RNA can be formed by DNA because DNA contained in the codes that play a role in the formation of RNA. The codes contained in double helix (double-stranded) DNA and then be printed to form RNA. RNA is formed in the process of transcription, the process of the formation of RNA using the codes contained in DNA. This process is one stage of protein synthesis in the cell. RNA formed in the transcription process will form a single-chain RNA. Sequences of nitrogen bases arranged in the RNA that is complementary (pair) of genetic code in the DNA sequence of the nitrogenous bases.[6]

Every three pairs (triplet) nitrogenous bases in RNA called codon. In other words, RNA is a chain line of standard genetic code that is composed of a set of nucleotide bases {U, G, C, A}.

RNA that composed of nucleotide bases, here in after seen as the set of codons may be viewed from the mathematical sciences by considering the sets of these elements as an algebraic structure based on binary operations and certain axioms

THEORETICAL MODEL

Codon

Codon is nucleotide sequences in nucleic acids (DNA and RNA) that encodes the amino acids in the protein chain. DNA and RNA bases' built by the nucleotides that will specify the 20 amino acids. Thus, if each nucleotide is translated into amino acids, then there would be only 4 of the 20 amino acids that will be specified. If the nucleotide sorted by 2 pieces (example: AG, GT), then there would be 16 amino acids to be specified. This amount is not enough to specify the 20 amino acids. Therefore, there must be combination of at least three nucleotides (a triplet of nucleotides) to determine each particular amino acid. This nucleotide triplet code shall be referred to the standard genetic code or codon. Codons will provide 4 ^ 3 = 64 amino acids to be specified, this amount is more than enough so that it will no amino acids are specified by more than one codon [1]. Nucleotida at codon basic component is written in the set of nucleotide bases found in RNA is {U, G, C, A} List Codon arranged at the RNA can be seen in Table 2.1 as follows:

TABLE 2.1 List of codons contained in the RNA
(source : Campbell, 2008: 330)

First mRNA base (5' end of codon)	Second mRNA base: U	C	A	G	Third mRNA base (3' end of codon)
U	UUU Phe	UCU Ser	UAU Tyr	UGU Cys	U
	UUC Phe	UCC Ser	UAC Tyr	UGC Cys	C
	UUA Leu	UCA Ser	UAA Stop	UGA Stop	A
	UUG Leu	UCG Ser	UAG Stop	UGG Trp	G
C	CUU Leu	CCU Pro	CAU His	CGU Arg	U
	CUC Leu	CCC Pro	CAC His	CGC Arg	C
	CUA Leu	CCA Pro	CAA Gln	CGA Arg	A
	CUG Leu	CCG Pro	CAG Gln	CGG Arg	G
A	AUU Ile	ACU Thr	AAU Asn	AGU Ser	U
	AUC Ile	ACC Thr	AAC Asn	AGC Ser	C
	AUA Ile	ACA Thr	AAA Lys	AGA Arg	A
	AUG Met or start	ACG Thr	AAG Lys	AGG Arg	G
G	GUU Val	GCU Ala	GAU Asp	GGU Gly	U
	GUC Val	GCC Ala	GAC Asp	GGC Gly	C
	GUA Val	GCA Ala	GAA Glu	GGA Gly	A
	GUG Val	GCG Ala	GAG Glu	GGG Gly	G

Group Theory

The Group is a model example of the simplest structure, namely the classification model of a set with a binary operation and specific requirements.

Definition 2.1 (Binary Operation) (Gallian, 2010:40) Let *G* be a set. A binary operation on *G* is a function that assigns each ordered pair of elements of *G* an element of *G*.

Definition 2.2 (Group) (Gallian, 2010:41)
Let *G* a nonempty set together with a binary operation that assigns to each ordered pair (*a, b*) of elements of *G* an element in *G* denoted by ab. We say *G* is a group under this operation if the following three properties are satisfied.

1. Associativity. The operation is associative;that is $((ab)c = a(bc)$ for all *a,b,c*in *G*.
2. Identity. There is an element e (called the identity) in G such that $ae = ea = a$for all *a* in *G*.
3. Inverse. For each element a in *G*,there is an element b in *G* (called an inverse of *a*) such that $ab = ba = e$

If a group has $ab = ba$ for every *a* and *b* in *G*, then the group is called abelian group
.
Definition 2.3 (order of a group) (Galian, 2010:60)
The number of elements of a group (finite or infinite) is called is *orde*r. We will use |*G*| to denote the order of *G*.

Definition 2.4 (order of an element) (Galian, 2010 :60)
The order of an element g in a group *G* is the smallest positive integer *n* such that $g^n = e$.If no such integer exist, we say that *g* has infinite order.

Definition 2.5 (Cyclic Group) (Galian,2010: 73)
A group G is called cyclic if there is an element a in G such that G= $\{a^n | n \in Z\}$. Such an element a is called a generator of G.
We may indicate that $G$is a cyclic group generated by a by writing $G = < a >$.

Theorem 2.6 (Fraleigh, 2003:63)
Let G is a cyclic group generate by a and order of G is n, then G isomorphic with Z_n

Ring Theory

Definition (Ring) (Galian , 2010 ; 235)
A ring R is a nonempty set with two binary operation, addition (denoted by $a + b$) and multiplication(denoted by ab), such that for all a,b, c in R :

1. $a + b = b + a$
2. $(a + b) + c = a + (b + c)$
3. There is an additive identity 0. That is , there is an element 0 in R such that $a + 0 = a$ for all $a \in R$.
4. There is an element $-a \in R$ such that $a + (-a) = 0$
5. $a(bc) = (ab)c$
6. $(b + c) = a \cdot b + ac$and $(b + c)a = ba + ca$.

RESULT

In the study of [5], the collection of nitrogen bases in the RNA chain is presented in a set N = {C, U, A, G} are matched with $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{(1,1), (1,0), (0,0), (0,1)\}$. Furthermore [6], the set N = {C, U, A, G} are matched $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{(0,0), (1,0), (0,1), (1,1)\}$. With such matching, then N has the Algebraic Structure as a commutative group .

For a further discussion we will use the set of sequences of nucleotide bases N = {U, G, C, A}. Before investigating the ring structure on the set of codons, N will be investigated as a group , the review carried out by the nucleotide sequences {U, G, C , A} and it will be matched by members of a cyclic group of order 4. The set of integers modulo 4 (Z_4) is chosen as the cyclic group of order 4 since by theorem 2.6, the cyclic group of order n will be isomorphic with Z_n. Matching is done to operate the set according to the operation of the nucleotide bases used in Z_4.
Each member of the set of bases N = {U, G, C, A} are matched with members of Z_4, ie $\{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$, with matching $U = \bar{0}$, $G = \bar{1}$, $C = \bar{2}$, and $A = \bar{3}$. By matching the set of bases N can be operated with the addition operation as used on the set Z_4 resulting Cayley table as follows:

TABLE 3.1 Addition operation in $(N, +)$

$+$	U	G	C	A
U	U	G	C	A
G	G	C	A	U
C	C	A	U	G
A	A	U	G	C

The set of N with the addition operation (notation: (N, +)) can be viewed as a abelian group.
Group (N, +) also have a generator that is <G> and <A> denoted by (N +) = <G> = <A> so that (N, +) is a cyclic group. By Theorem 2.6 shows that (N +) is isomorphic to the group (Z_4, +) denoted by (N +) ≅ (Z_4+).

In the next section will be shown the review group and ring structure on the set of codons.
In this part of the nucleotide bases set {U, G, C, A} will be converted into a set of codons (nucleotide triplets) denoted by C_g. Conversion is done by sorting the three nucleotide bases as a codon so that there will be a 4 × 4 × 4 = 64 possible sequences formed as a set codon C_g.

The set of codons C_g can be sorted from the smallest to the largest, by reference to the nucleotide sequences of {U, G, C, A}. Changes made from a base sequence of the third, then the first base and second base last. This occurs because of changes in the codon base mutations are more common in third base than second base, while changes to the first base are rare. The average change in the third base of about 100 times that of the first base and first base about 10 times a second base.

Each member of the set of codons C_g will be matched with a member of the set of the order of 64 to form a cyclic group because it will be needed at the binary operation of multiplication codons that will be discussed later. The set of selected is the set of integers modulo 64 (Z_{64}) as isomorphic to the cyclic group of order 64 (by Theorem 2.21). Member of the set$C_g$and yield matching can be seen in the following table:

TABLE 3.2The set of codons C_g
(source: Robersy Sanchez, 2007: 5)

	U			G			C			A			
	No	(1)	(2)	**No**	(1)	(2)	**No**	(1)	(2)	**No**	(1)	(2)	
U	**0**	UUU	F	**16**	UGU	C	**32**	UCU	S	**48**	UAU	Y	U
	1	UUG	L	**17**	UGG	W	**33**	UCG	S	**49**	UAG	-	G
	2	UUC	F	**18**	UGC	C	**34**	UCC	S	**50**	UAC	Y	C
	3	UUA	L	**19**	UGA	-	**35**	UCA	S	**51**	UAA	-	A
G	**4**	GUU	V	**20**	GGU	G	**36**	GCU	A	**52**	GAU	D	U
	5	GUG	V	**21**	GGG	G	**37**	GCG	A	**53**	GAG	E	G
	6	GUC	V	**22**	GGC	G	**38**	GCC	A	**54**	GAC	D	C
	7	GUA	V	**23**	GGA	G	**39**	GCA	A	**55**	GAA	E	A
C	**8**	CUU	L	**24**	CGU	R	**40**	CCU	P	**56**	CAU	H	U
	9	CUG	L	**25**	CGG	R	**41**	CCG	P	**57**	CAG	Q	G
	10	CUC	L	**26**	CGC	R	**42**	CCC	P	**58**	CAC	H	C
	11	CUA	L	**27**	CGA	R	**43**	CCA	P	**59**	CAA	Q	A
A	**12**	AUU	I	**28**	AGU	S	**44**	ACU	T	**60**	AAU	N	U
	13	AUG	M	**29**	AGG	R	**45**	ACG	T	**61**	AAG	K	G
	14	AUC	I	**30**	AGC	S	**46**	ACC	T	**62**	AAC	N	C
	15	AUA	I	**31**	AGA	R	**47**	ACA	T	**63**	AAA	K	A

Table 3.2 shows the members of the set $C_g$codon, amino acid symbols are translated by each codon and matching members $Z_{64}$with C_g.

Codon matching pair of integers modulo XYZ∈$C_g$to 64$\bar{k} \in \mathbb{Z}_{64}$or otherwise can be determined by using the division algorithm, in order to get a general form as follows **:**

$$\bar{k} = 16 \cdot Y + 4 \cdot X + 1 \cdot Z, \bar{k} \in \mathbb{Z}_{64}, \text{dan } X, Y, Z \in \{U, G, C, A\}$$

codon XYZ will be described first into the nucleotide bases, namely X, Y, and Z. Furthermore, each nucleotide will be seen as a couple matchng with $\mathbb{Z}_4$ ($U = \bar{0}$, $G = \bar{1}$, $C = \bar{2}$, dan $A = \bar{3}$). Lastly, matching results substitution into a common form.

Example 3.1: Suppose XYZ = CAG, meaning that X = C, Y = A, and Z = G. Based on the matching nucleotide $C = \bar{2}$, $A = \bar{3}$, and $G = \bar{1}$, then $\bar{k} \in \mathbb{Z}_{64}$ is determined as follows:

$$\bar{k} = 16 \cdot Y + 4 \cdot X + 1 \cdot Z = 16 \cdot A + 4 \cdot C + 1 \cdot G = 16 \cdot 3 + 4 \cdot 2 + 1 \cdot 1 = \overline{57}$$

Binary operations (addition and multiplication codon) on the set C_g will defined first to be reviewed as ring stucture. Next we will be discussed the definition of a binary operation on the set of codons C_g .

Sum Operation at the Set of codons C_g.

Suppose $XYZ,\ X'Y'Z' \in C_g$with X and X' is the first nucleotide bases, Y and Y 'is a second base, and Z and Z' is the third in the base of codons. Summation algorithm XYZ + X'Y'Z 'this results in a X''Y''Z' 'is expressed as follows:

1. The corresponding bases in the third position are added according to the sum table (table 3.1)
2. If the resultant base of the sum operation is previous in order to the added bases(the orders in the set of bases $\{U, G, C, A\}$).,then the new value is written and the base G is added to the next position

3. The other bases are added according to the sum table, step 2, going from the forst base to the second base.

Example 3.2: $GCG + CAA$, with $XYZ = GCG$ and $X'Y'Z' = CAA$
1. $Z'' = G + A = U$. U generated precede bases G and A at the nucleotide sequences {U, G, C, A} so that the base G is added to the sum of the next base.
2. $X'' = (G + C) + G = A + G = U$. U generated precede bases A and G at nucleotide base sequence {U, G, C, A} so that the base G is added to the sum of the next base.
3. $Y'' = (C + A) + G = G + G = C$
4. $X''Y''Z'' = UCU$ or $GCG + CAA = UCU$

Example 3.3 (closure)
Let $u = CAU, v = GUA \in C_g$, then $u + v = CAU + GUA = AAA \in C_g$.

Example 3.4 (Assosiativity)
Let $u = GUC, v = AGU,$ and $w = UAU \in C_g$.

$$\begin{aligned}(u + v) + w &= (GUC + AGU) + UAU \\ &= UCC + UAU \\ &= UGC \\ &= GUC + AUU \\ &= GUC + (AGU + UAU) \\ &= u + (v + w)\end{aligned}$$

Example 3.5 (commutative)
Let l $u = GUC$ and $v = AUG \in C_g$.

$$\begin{aligned}u + v &= GUC + AUG \\ &= UGA \\ &= AUG + GUC \\ &= v + u\end{aligned}$$

The set of codons C_g have an element of identity to the sum operation is $UUU \in C_g$ such that $XYZ \in C_g$; $XYZ + UUU = XYZ = UUU + XYZ$.
Each member has an inverse to the addition operation so that for every $XYZ \in C_g$ there is $(XYZ)^{-1} \in C_g$ such that $XYZ + (XYZ)^{-1} = UUU = (XYZ)^{-1} + XYZ$.The inverse of each member C_g can be seen from the following table.

TABLE 3.3 Inverselement in $(C_g, +)$.

XYZ	$(XYZ)^{-1}$	XYZ	$(XYZ)^{-1}$	XYZ	$(XYZ)^{-1}$	XYZ	$(XYZ)^{-1}$
UUG	AAA	UGG	ACA	AAA	UUG	ACA	UGG
UUC	AAC	UGC	ACC	AAC	UUC	ACC	UGC
UUA	AAG	UGA	ACG	AAG	UUA	ACG	UGA
GUU	AAU	GGU	ACU	AAU	GUU	ACU	GGU
GUG	CAA	GGG	CCA	CAA	GUG	CCA	GGG
GUC	CAC	GGC	CCC	CAC	GUC	CCC	GGC
GUA	CAG	GGA	CCG	CAG	GUA	CCG	GGA
CUU	CAU	CGU	CCU	CAU	CUU	CCU	CGU
CUG	GAA	CGG	GCA	GAA	CUG	GCA	CGG
CUC	GAC	CGC	GCC	GAC	CUC	GCC	CGC
CUA	GAG	CGA	GCG	GAG	CUA	GCG	CGA
AUU	GAU	AGU	GCU	GAU	AUU	GCU	AGU
AUG	UAA	AGG	UCA	UAA	AUG	UCA	AGG
AUC	UAC	AGC	UCC	UAC	AUC	UCC	AGC
AUA	UAG	AGA	UCG	UAG	AUA	UCG	AGA
UGU	UAU	UCU	UCU	UAU	UGU	UUU	UUU

Table 3.3 shows each member of the set C_g has invers. Thus the set of C_g can be viewed as a abelian group by the addition operation.

Group $(C_g,+)$ has generator as follow :

$$C_g = \langle UUG\rangle = \langle UUA\rangle = \langle GUG\rangle = \langle GUA\rangle = \langle CUG\rangle = \langle CUA\rangle = \langle AUG\rangle = \langle AUA\rangle = \langle UGG\rangle$$
$$= \langle UGA\rangle = \langle GGG\rangle = \langle GGA\rangle = \langle CGG\rangle = \langle CGA\rangle = \langle AGG\rangle = \langle AGA\rangle = \langle UCG\rangle$$
$$= \langle UCA\rangle = \langle GCG\rangle = \langle GCA\rangle = \langle CCG\rangle = \langle CCA\rangle = \langle ACG\rangle = \langle ACA\rangle = \langle UAG\rangle$$
$$= \langle UAA\rangle = \langle GAG\rangle = \langle GAA\rangle = \langle CAG\rangle = \langle CAA\rangle = \langle AAG\rangle = \langle AAA\ \rangle$$

Thus $(C_g,+)$ can be viewed as cyclic group according **theorem 2.6**,$(C_g,+)$ ishomorfic with $(\mathbb{Z}_{64},+)$ or $(C_g,+) \cong (\mathbb{Z}_{64},+)$.

Multiplicative operation inC_g

Multiplication operation on the set of codons C_g defined by Robersy Sanchez in a paper entitled "Gene Algebra from a Genetic Structure Algebraic Code".
This operation on the set of codons C_g multiplication denoted by $\otimes$and defined as $(X_1Y_1Z_1 \otimes X_1Y_1Z_1) = X_1Y_1Z_1$ with $X_1Y_1Z_1 = UUG$ in witch $X_1Y_1Z_1$ one of the generator in the group (C_g, +)). Furthermore, for $u, v \in C_g$and $k, k' \in \mathbb{Z}_{64}$; $u \otimes v = k(X_1Y_1Z_1) \otimes k'(X_1Y_1Z_1) = k \cdot k'(X_1Y_1Z_1)$ where $'' \cdot ''$ is multiplicative operation in$\mathbb{Z}_{64}$.
So the multiplication on the setC_g show that is closed, associative and commutative. Examples of the closure, the nature of associative and commutative properties of the set (C_g, $\otimes$) will be awarded as follows:

Example 3.6 (closure)
Let $u = GUU$ and $v = AGC \in C_g$

$$\begin{aligned} u \otimes v &= GUU \otimes AGC \\ &= \overline{4}(UUG) \otimes \overline{30}(UUG) \\ &= \overline{4} \cdot \overline{30}(UUG \otimes UUG) \\ &= \overline{56}(UUG) \\ &= CAU \in C_g \\ u \otimes v &\in C_g \end{aligned}$$

Example 3.7 (Associative)
Let $u = CUA, v = GGA,$ and $w = UGG \in C_g$.

$$\begin{aligned} (u \otimes v) \otimes w &= (CUA \otimes GGA) \otimes UGG \\ &= (\overline{11}(UUG) \otimes \overline{23}(UUG)) \otimes \overline{17}(UGG) \\ &= (\overline{11} \cdot \overline{23}(UUG)) \otimes \overline{17}(UGG) \\ &= \overline{61}(UUG) \otimes \overline{17}(UGG) \\ &= \overline{13}(UGG) \\ &= AUG \\ &= \overline{13}(UGG) \\ &= \overline{11}(UUG) \otimes \overline{7}(UGG) \\ &= \overline{11}(UUG) \otimes \left(\overline{23} \cdot \overline{17}(UUG)\right) \\ &= \overline{11}(UUG) \otimes (\overline{23}(UUG) \otimes \overline{17}(UUG)) \\ &= CUA \otimes (GGA \otimes UGG) \\ (u \otimes v) \otimes w &= u \otimes (v \otimes w) \end{aligned}$$

Example 3.8 (commutative)
Let$u = UGA$ and $v = CUC \in C_g$.

$$\begin{aligned} u \otimes v &= UGA \otimes CUC \\ &= \overline{19}(UUG) \otimes \overline{10}(UUG) \\ &= \overline{62}(UUG) \\ &= AAC \\ &= \overline{62}(UUG) \\ &= \overline{10}(UUG) \otimes \overline{19}(UUG) \\ &= v \otimes u \end{aligned}$$

Example 3.9 (left distributive)
Let $u = GUC, v = AUG, w = UCG \in C_g$.

$$
\begin{aligned}
u \otimes (v + w) &= GUC \otimes (AUG + UCG) \\
&= GUC \otimes ACC \\
&= \overline{6}(UUG) \otimes \overline{46}(UUG) \\
&= \overline{6} \cdot \overline{46}(UUG) \\
&= \overline{20}(UUG) \\
&= GGU \\
&= AUC + GUC \\
&= \overline{14}(UUG) + \overline{6}(UUG) \\
&= \left(\overline{6} \cdot \overline{13}(UUG)\right) + (\overline{6} \cdot \overline{33}(UUG)) \\
&= \left(\overline{6}(UUG) \otimes \overline{13}(UUG)\right) + (\overline{6}(UUG) \otimes \overline{33}(UUG)) \\
&= (GUC \otimes AUG) + (GUC \otimes UCG) \\
&= (u \otimes v) + (u \otimes w)
\end{aligned}
$$

Example 3.10 (right distributive)
Misal$uGUC, v = AUG, w = UCG \in C_g$.

$$
\begin{aligned}
(u + v) \otimes w &= (GUC + AUG) \otimes UCG \\
&= UGA \otimes UCG \\
&= \overline{19}(UUG) \otimes \overline{33}(UUG) \\
&= \overline{19} \cdot \overline{33}(UUG) \\
&= 51(UUG) \\
&= UAA \\
&= GUC + AGC \\
&= \overline{6}(UUG) + \overline{45}(UUG) \\
&= \left(\overline{6} \cdot \overline{33}(UUG)\right) + (\overline{13} \cdot \overline{33}(UUG)) \\
&= \left(\overline{6}(UUG) \otimes \overline{33}(UUG)\right) + (\overline{13}(UUG) \otimes \overline{33}(UUG)) \\
&= (GUC \otimes UCG) + (AUG \otimes UCG) \\
&= (u \otimes w) + (v \otimes w)
\end{aligned}
$$

The set of codons $C_g$with binary operations addition and multiplication are denoted by $(C_g, +, \otimes)$can be viewed as an abelian group respect to the addition operation. The Group is also closed, associative and commutative to the multiplication operation and left-right distributive. Therefore, by definition 2.7, the set of codons $(C_g, +, \otimes)$has a structure as a Ring.

CONCLUSION

The set of codons wich are denoted by $(C_g, +, \otimes)$ with binary operation addition as follows :

1. The corresponding bases in the third position are added according to the sum table (table 3.1)
2. If the resultant base of the sum operation is previous in order to the added bases(the orders in the set of bases $\{U, G, C, A\}$).,then the new value is written and the base G is added to the next position
3. The other bases are added according to the sum table, step 2, going from the first base to the second base.

And multiplicative operation $\otimes$ is defined as $(X_1Y_1Z_1 \otimes X_1Y_1Z_1) = X_1Y_1Z_1$ with $X_1Y_1Z_1 = UUG$with $X_1Y_1Z_1$ one of the generator in the group (C_g, +)). Furthermore, for $u, v \in C_g$and $k, k' \in \mathbb{Z}_{64}$; $u \otimes v = k(X_1Y_1Z_1) \otimes k'(X_1Y_1Z_1) = k \cdot k'(X_1Y_1Z_1)$ where$'' \cdot ''$ is multiplicative operation in$\mathbb{Z}_{64}$, can be viewed as a Ring .

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