Dynamical Analysis of Plant Disease Model with Roguing, Replanting and Preventive Treatment

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Abstract. In this paper we discuss a mathematical model of plant disease considering roguing and replanting with preventive treatment. We show the value of the Basic Reproduction Number (BRN) ($\mathcal{R}_0$) of the plant disease transmission. The BRN is computed from the largest eigen value of the next generation matrix of the model. The result shows that in the region where $\mathcal{R}_0$ greater than one there is one single stable endemic equilibrium. However, in region where $\mathcal{R}_0$ less than one this endemic equilibrium becomes unstable. The dynamics of the model are highly sensitive to changes in contact rate and infectious period. We show that the combination of roguing and preventive treatment is able to change the stability of endemic equilibrium into an unstable equilibrium. The preventive treatment for susceptiple plant give the endemic disease will be decrease, this show the effectiveness of the preventive treatment on infected plants. Some numerical simulations are also given to illustrate our analytical results.

INTRODUCTION

Plants are the source of life that is all around us. Plants are biotic elements that have important functions in the environment, and have ecological functions as producer of oxygen. Plants evolve defense mechanisms to protect themselves from insects, pests and pathogens. But when there is an outbreak of epidemics, loss of crops can have a significant impact on the economy and human life. To reduce the losses caused by the disease, it is necessary to control the disease [1]. Many alternatives are available to reduce the application of pesticides such as cultural control, the use of resistance host plants and chemical control [2]. Another alternative are roguing and replanting the plant. Many researchers have done researches on the spread of plant diseases. In 2014, N. Latif develops a mathematical model where the susceptible plant population is branched into two compartments when it is exposed to compatible pathogen: the first one is the subpopulation that become infected by the disease and the second one is the subpopulation that is able to resist from the infection via basal host defence mechanism [3]. Some authors study plant epidemic models which involve fungicide with curative treatment [4] and protectant factor [5]. Other study done by developing a mathematical model of the spread of diseases by considering physical mechanism, i.e. without roguing and with roguing mechanism [6,8]. They show that the best result is to suppress the outbreak of viral diseases in plants by implementing roguing mechanism. Further research is done by developing the model of [6] with roguing and replanting [7]. In [7] the model is transformed into a discrete form and solved using backward Euler method. In this paper we discuss the model of plant disease transmission by considering roguing and replanting and an additional treatment, i.e. preventive treatment to the plant.

MODEL FORMULATION

Before we construct our model, to understand the effect of preventive treatment, we will give the following definition and assumptions to make the reader easier to understand the model. In this model, we devide the plant population into five subpopulations: Susceptible defined as $S(t)$, Exposed defined as $E(t)$, Infected defined as $I(t)$, Post Infectious/Removed defined as $R(t)$ and Protected defined as $P(t)$. The assumptions to this model are :

- The population of the plant is not closedmdue to the replanting and natural death.
- The compartment of infected plant consists of two compartments, namely latent $E(t)$ and infected $I(t)$.
- If an infected plant shows a comprehensive symptoms then that plant will be lifted.
- The environment factor and insect vector are ignored.
- The preventive treatment (insecticide) be given to susceptible compartment
- Susceptible plants that received preventive treatment enters Protected compartment $P(t)$.
- Protected plants $P(t)$ have protection or preventive effect, but they do not immune to the disease, thus it is allowing re-entry into the Susceptible compartment.

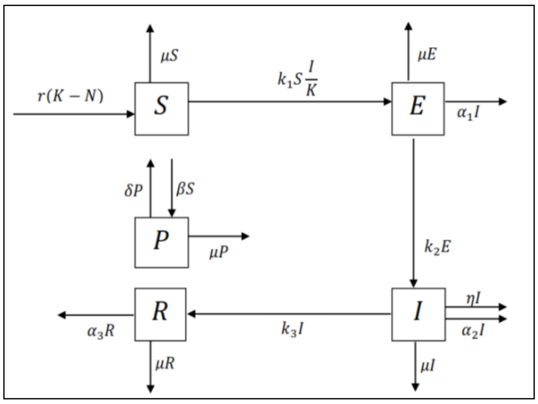


FIGURE 1. The schematic diagram of the model in equation (1)

According to above assumptions and the schematic diagram in Figure 1, the model read as :

$$
\begin{aligned}
\frac{dP}{dt} &= r\left(K-N\right)-\mu S-k_1 S\frac{I}{K}-\beta S+\delta P\\
\frac{dP}{dt} &= \beta S+\delta P-\mu P\\
\frac{dE}{dt} &= k_1 S\frac{I}{K}-\left(\mu+k_2+\alpha_1\right)E\\
\frac{dI}{dt} &= k_2 E-\left(\mu+k_3+\alpha_2+\eta\right)I\\
\frac{dR}{dt} &= k_3 I-\left(\mu+\alpha_3\right)R
\end{aligned} \tag{1}
$$

with the parameters and variable are explained in Table 1. Table 1 also give the values of parameters used in the simulation in the subsequent section. There are two data sets in Table 1, one for the case $\boldsymbol{\mathcal{R}_0} < 1$ and the other one for the case $\boldsymbol{\mathcal{R}_0} > 1$.

TABLE 1.Definition of Variabels and Parameters

Parameter/Variabel	Definition	Value ($\mathcal{R}_0 < 1$)	Value($\mathcal{R}_0 > 1$)
N	The total population of the actual plant		
K	The total maximum population of plants (agronomy)		
α_1	Effect comulative death reate for Laten Compartment	0	0
α_2	Effect comulative death reate for Infected Compartment	0	0
α_3	Effect comulative death reate for Laten Removed Compartment	0.01	0.01
k_2	conversion rate of disease progression for Infected Compartment	0.17	0.17
β	The effectiveness preventive treatment	0.0052	0.0052
δ	Preventive treatment rate	0.048	0.048
r	Replanting rate	0.011	0.013
η	Roguing rate	0.089	0.087
k_1	conversion rate of disease progression for Latent Compartment	0.06	0.3
k_3	conversion rate of disease progression for Removed Compartment	0.04	0.02
μ	Natural death rate	0.004	0.0008

MODEL ANALYSIS

Our first analysis result concerning to find the equilibrium state and to determine their local stability criteria. For this purpose, we solve the model equation in (1) for equilibrium points. We have two equilibriums, there are non endemic and endemic equilibrium point, as follows.

Equilibrium Point

a. Non Endemic Equilibrium Point

$$T = (S,P,E,I,R) = \left(\frac{rK(\mu+\delta)}{(\mu+r)(\mu+\delta+\beta)}, \frac{r\beta K}{(\mu+r)(\mu+\delta+\beta)}, 0,0,0\right)$$

with : $N_* = \frac{rK-\alpha_1 E_* - (\alpha_2+\eta)I_* - \alpha_3 R_*}{\mu+r}$, $N = \frac{rK}{\mu+r}$.

b. Endemic Equilibrium Point

$T_* = (S_*, P_*, E_*, I_*, R_*)$ with

$$S_* = \frac{K(\mu+a)(\mu+b)}{k_1 k_2}$$

$$P_* = \frac{K\beta(\mu+a)(\mu+b)}{k_1 k_2(\mu+\delta)}$$

$$E_* = -\frac{(-rk_1k_2(K-N)(\mu+\delta) + K\mu(\mu+a)(\mu+b)(\mu+d))}{k_1k_2(\mu+a)(\mu+\delta)}$$

$$I_* = -\frac{(-rk_1k_2(K-N)(\mu+\delta) + K\mu(\mu+a)(\mu+b)(\mu+d))}{k_1(\mu+a)(\mu+b)(\mu+\delta)}$$

$$R_* = -\frac{k_3(-rk_1k_2(K-N)(\mu+\delta) + K\mu(\mu+a)(\mu+b)(\mu+d))}{k_1(\mu+\delta)((\mu+a)(\mu+b)(\mu+\alpha_3))}$$

where : $a = k_2 + \alpha_1$, $b = k_3 + \alpha_2 + \eta$, and $d = \delta + \beta$.

Basic Reproduction Number

In this section, Basic Reproduction Number of system (1) will be shown analytically. The Basic Reproduction Number (BRN) is an important threshold number in epidemiology. It is defined as the number of secondary infections caused by one primary infection in an entirely susceptible population [9,12] and usually written as according to system (1). Using the definition in [9], the BRN for system (1) is given by

$$\mathcal{R}_0 = \frac{rk_1k_2(\mu+\delta)}{(\mu+r)(\mu+a)(\mu+b)(\mu+d)}.$$

The value of the basic reproduction number depends on the value of the replanting rate. This is the reason that if we want to control plant disease, we can replanting the plant.

Local Stability

Matrix Jacobian for system (1)

$$J = \begin{bmatrix} -\mu - k_1\frac{I}{K} & 0 & -k_1\frac{S}{K} & 0 \\ k_1\frac{I}{K} & -(\mu+k_2+\alpha_1) & k_1\frac{S}{K} & 0 \\ 0 & k_2 & -(\mu+k_3+\alpha_2+\eta) & 0 \\ 0 & 0 & k_3+p & -(\mu+\alpha_3) \end{bmatrix} \quad (2)$$

The local stability of non endemic and endemic equilibrium point are given by the following theorem.

Theorem 1

The disease – free equilibrium point of the system (1) is locally asymptotically stable if $\mathcal{R}_0 < 1$.

Proof :

By following [9,12] and from (2), the Jacobian matrix for non endemic equilibrium point is

$$JT = \begin{bmatrix} -\mu-\beta & \delta & 0 & -\dfrac{rk_1(\mu+\delta)}{(\mu+r)(\mu+d)} & 0 \\ \beta & -(\mu+\delta) & 0 & 0 & 0 \\ 0 & 0 & -(\mu+a) & \dfrac{rk_1(\mu+\delta)}{(\mu+r)(\mu+d)} & 0 \\ 0 & 0 & k_2 & -(\mu+b) & 0 \\ 0 & 0 & 0 & k_3 & -(\mu+\alpha_3) \end{bmatrix}$$

The polynomial characteristic of JT are :

$$P_T\ (\lambda) = \frac{1}{(\mu+r)(\mu+d)}(\mu+\alpha_1+\lambda)p_1(\lambda)p_2(\lambda)$$

with

$$p_1(\lambda) = m_1\lambda^2 + n_1\lambda + q_1, p_2(\lambda) = m_2\lambda^2 + n_2\lambda + q_2$$

and

$$m_1 = (\mu+r)(\mu+d), n_1 = (\mu+r)(\mu+d)((\mu+b)+(\mu+a)),$$

$$q_1 = 1 - \mathcal{R}_0, m_2 = 1, n_2 = 2\mu+d, q_4 = \mu(\mu+d).$$

The eigenvalues of P_T are $-\alpha_1$ and the roots of the polynomial $p_1(\lambda)$ and $p_2(\lambda)$. The coefficient $q_1 > 0$ if $\mathcal{R}_0 <$ 1, hence all the roots of polynomial P_T have negatice real part when $\mathcal{R}_0 < 1$. This means that non endemic equilibrium point $T = (S, P, E, I, R)$ is locally asymptotically stable when $\mathcal{R}_0 < 1$ [9,10,11]. This proves the theorem.

Theorem 2

The endemic equilibrium point of the system (1) is locally asymptotically stable if $\mathcal{R}_0 > 1$.

Proof :

By following [9,12] and from (2), theJacobian matrix for endemic equilibrium point is

$$JT_* == \begin{bmatrix} -\mu-\dfrac{\mu(\mathcal{R}_0-1)(\mu+d)}{(\mu+\delta)}-\beta & \delta & 0 & -\dfrac{k_1r(\mu+\delta)}{\mathcal{R}_0(\mu+r)(\mu+d)} & 0 \\ \beta & -(\mu+\delta) & 0 & 0 & 0 \\ \dfrac{\mu(\mathcal{R}_0-1)(\mu+d)}{(\mu+\delta)} & 0 & -(\mu+a) & \dfrac{k_1r(\mu+\delta)}{\mathcal{R}_0(\mu+r)(\mu+d)} & 0 \\ 0 & 0 & k_2 & -(\mu+b) & 0 \\ 0 & 0 & 0 & k_3 & -(\mu+\alpha_3) \end{bmatrix}$$

The polynomial characteristic of JT_* are :

$$P_{T_*}(\lambda) = \frac{1}{\mathcal{R}_0(\mu+r)(\mu+d)(\mu+\delta)}(\lambda+\mu+\alpha_3).p_3(\lambda)$$

$$p_3(\lambda) = a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4.$$

The eigenvalues of P_{T_*} are $-(\mu + \alpha_3)$ and the roots of the polynomial $p_3(\lambda)$. From Routh-Hortwitz criterian we have $a_0, a_1, a_2, a_3, a_4 > 0$, $a_2a_3 - a_1a_4 > 0$, and $a_1a_2a_3 - a_1{}^2a_4 - a_0a_3{}^2 > 0$, hence all the roots of polynomial P_{T_*} have negative real part when $\mathcal{R}_0 > 1$ [10,11]. This means that non endemic equilibrium point $T_* = (S_*, P_*, E_*, I_*, R_*)$ is locally asymptotically stable when $\mathcal{R}_0 > 1$ [9,10,11]. This proves the theorem.

NUMERICAL SIMULATION

In order to illustrate the dynamics of each compartment, we give numerical example with preventive treatment and without preventive treatment. We use the values of the parameters shown in Table 1 and the initial condition for each compartment as seen in Table 2. The value in Table 1 is taken from [7, 8], while in Table 2 is given hypothetically.

TABLE 2. The initial condition for model (1)

Variabel	Initial condition
K	1000
$S(0)$	100
$P(0)$	30
$E_1(0)$	60
$E_2(0)$	30
$I(0)$	60
$R(0)$	60

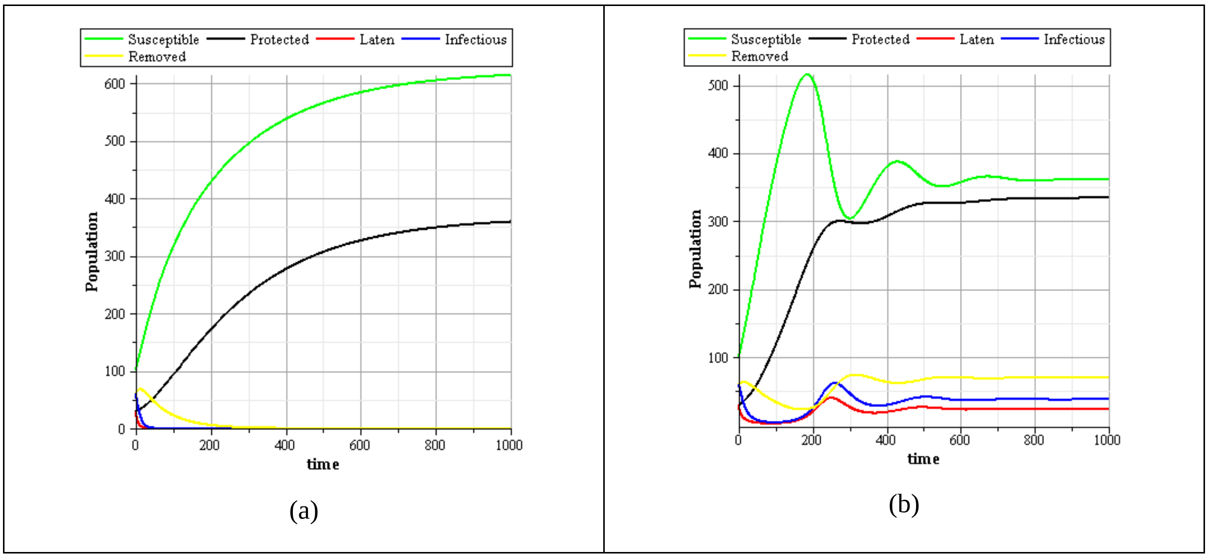


FIGURE 2. The dynamics of Susceptible-, Protected-, Laten-, Infected- and Removed- Compartments with (a) $\mathcal{R}_0 < 1$ and (b) $\mathcal{R}_0 > 1$

Figure 2 shows the dynamic of Susceptible-, Protected-, Laten-, Infected- and Removed- Compartments with (a) $\mathcal{R}_0 < 1$ and (b) $\mathcal{R}_0 > 1$. Figure 3 shows the dynamics of Susceptible-, Protected-, Laten-, Infected- and Removed- Compartments plant when $\mathcal{R}_0 < 1$ with preventive treatment and without preventive treatment. Figure 4 shows the dynamics of treatment plant when $\mathcal{R}_0 > 1$ with preventive treatment and without preventive treatment. Figure 3, 4 show that all compartment of plants reduce when preventive treatment is given. This is because the susceptible plants when they are given the preventive treatment will enter into a protected compartment (P). Figure 5 shown the dynamics of Susceptible Compartment with different value of *roguing* and *replanting* where (a) $\mathcal{R}_0 < 1$ and (b) $\mathcal{R}_0 > 1$. In figure 5(a), it shown that if the value of replanting parameter *r* increase and the value of roguing parameter η decrease, then the susceptible population will be increase when the Basic Reproduction Number is

$\mathcal{R}_0$< 1. In the contrary, in figure 5(b) when $\mathcal{R}_0$>1, if the value of replanting parameter *r* decreases and the value of roguing parameter η increase, then the susceptible population will decreases.

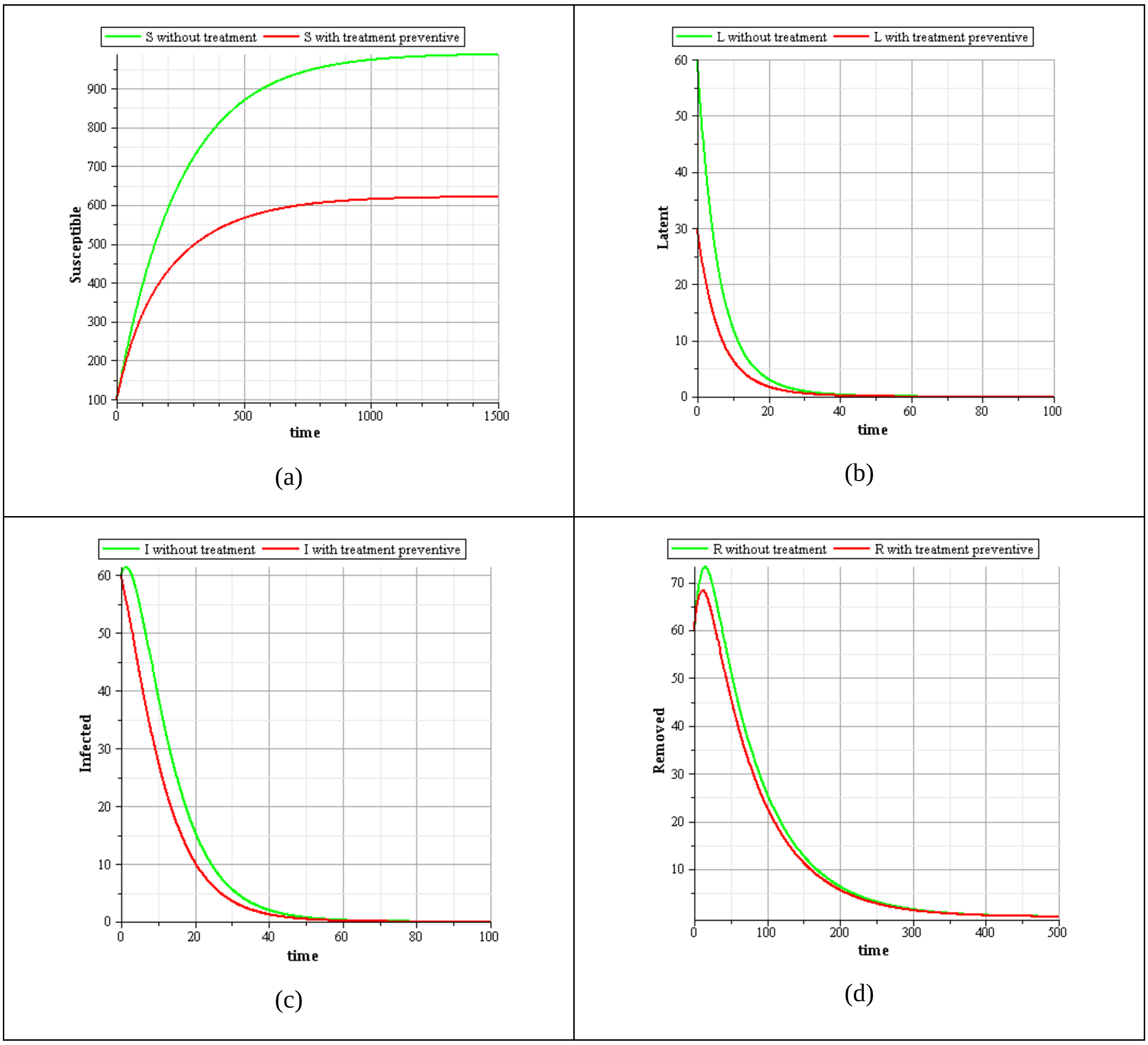


FIGURE 3. Comparison of the dynamics of the population in the model with and without pretective treatment, with $\mathcal{R}_0$< 1 (a) Susceptible, (b) Latent, (c) Infected, (d) Removed

Figure 6 shown dynamical compartment of Infected with different value of *roguing* and *replanting* (a) for $\mathcal{R}_0$< 1 and (b) for $\mathcal{R}_0$>1. This graph shown that the different value of parameter roguing and replanting does not effect the infected population if $\mathcal{R}_0$< 1, but if $\mathcal{R}_0$> 1, then the infected population plant will increase when the value of replanting parameter decrease and the value of roguing parameter increases. In the contrary, if the value of replanting parameter decrease and roguing parameter increase, then the infected population will decrease.

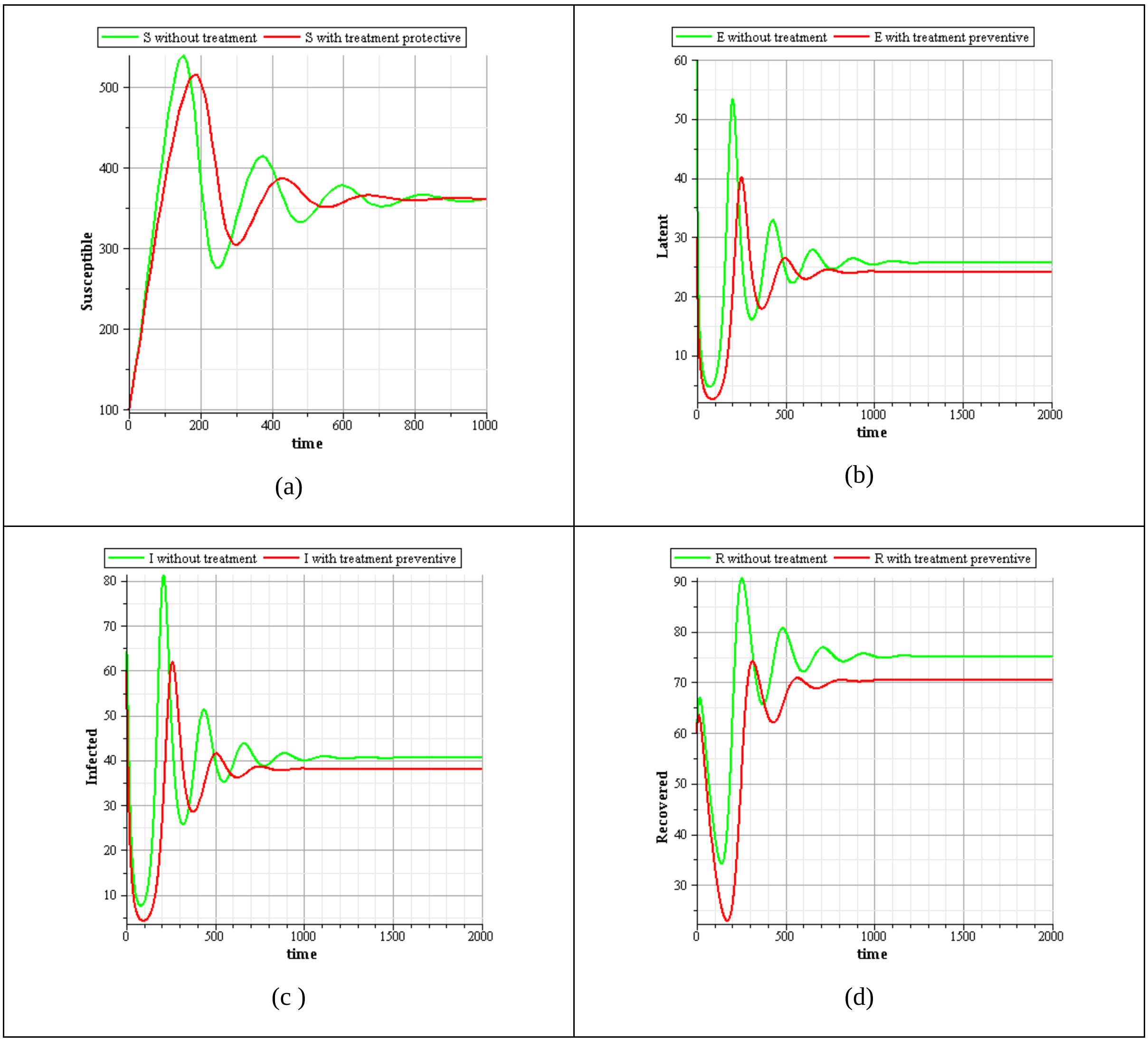


FIGURE 4. Comparison of the dynamics of the population in the model with and without pretective treatment with $\mathcal{R}_0$>1 (a) Susceptible, (b) Latent, (c) Infected, (d) Removed

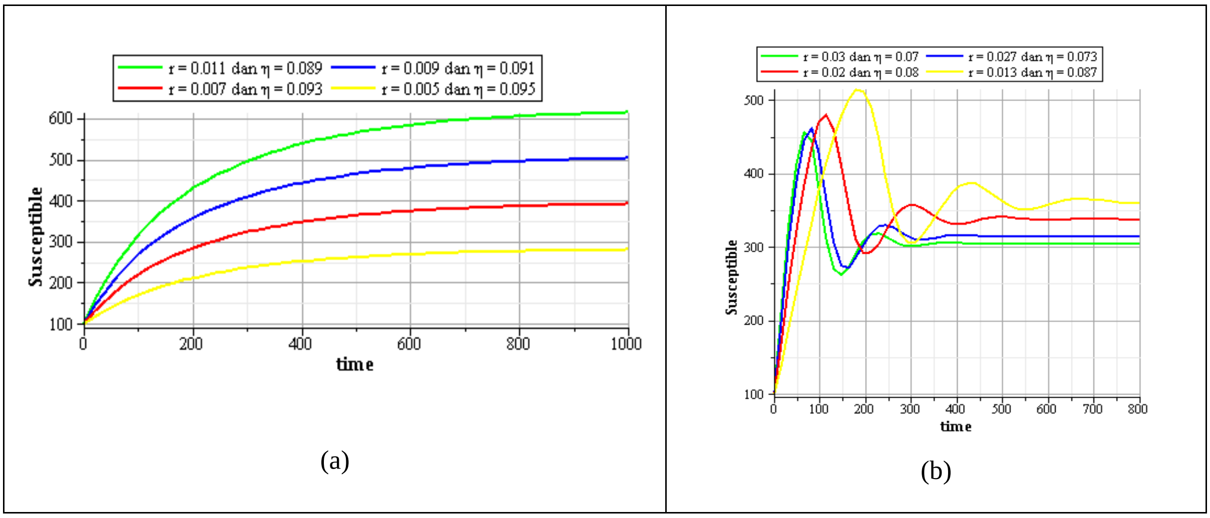
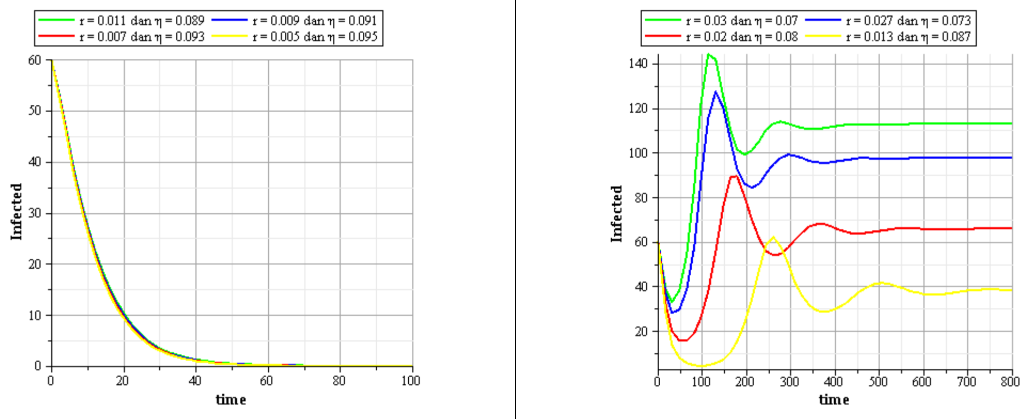


FIGURE 5. The dynamics of Susceptible Compartment with different value of *roguing* and *replanting* (a) $\mathcal{R}_0$< 1, (b) $\mathcal{R}_0$>1



(a) (b)

FIGURE 6. Dynamical Compartment of Infected with different value of *roguing* and *replanting* (a) $\mathcal{R}_0 < 1$, (b) $\mathcal{R}_0 > 1$

CONCLUSION

Mathematical model for the Plant Disease Model with Roguing, Replanting and Preventive treatment has been introduced in this article. The equilibrium points have been shown explicitly with positive and local stability. It can be concluded that the non endemic equilibrium will be stable if $\mathcal{R}_0 < 1$, and the endemic equilibrium point will be stable when $\mathcal{R}_0 > 1$. From numerical solution, it shown that if we give the preventive treatment for susceptiple plant then the disease will be decrease, this show the effectiveness of the preventive treatment on infected plants.

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