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# The Improvement of Students' Ability to Read Mathematical Proof in the Subject of Probability Theory

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**Abstract.** Probability Theory is one of the courses for mathematics undergraduate that might be considered difficult. One of the skills required to learn Probability is skill to read mathematical proof. The aim of this research was to analyze the skill to read mathematical proof in the subject of Probability Theory. The method used was qualitative. The research subjects were students of the Statistics study program at Hasanuddin University Makassar who joined Probability Theory subject. The results of the data analysis show that the students still encounter difficulties in checking the truth and write the concepts used in each step of proof. They have not understood the prerequisite materials as well. This is one of the factors that make their ability to read mathematical proof is not good. Based on this research, it seems that the ability to read mathematical proof of the students on Probability Theory subject is still not good.

## INTRODUCTION

Mathematics material for college level is difficult to learn because the material presented is more abstract. In studying mathematics, students need mathematical abilities. One of them is the ability to read mathematical proof. Reading mathematical proof is an activity of finding the truth or error and giving the reasons for each step of the proving process. One of the courses in the Statistics Study Program which requires the ability to read mathematical proof is the probability theory subject. Probability Theory is one of the courses that emphasize the aspect of deductive reasoning towards mathematical proof. Probability theory is categorized as one of the subjects considered difficult by students [1]. The students' weakness is found when they are dealing with mathematical proof [2]. This research aims to see the students' ability to read mathematical proof in the subject of probability theory, while the problem raised was "how is the ability to read mathematical evidence of students in the Subject of Probability Theory?" The ability to read mathematical proof is currently not seen in the student when studying Probability Theory. They have not been able to optimize all their mathematical abilities in learning so they tend to give up on doing tasks when they have trouble. Through this research, is expected to be a reference and discourse for the practitioners of mathematics education in an effort to improve the ability to read mathematical proof through appropriate learning.

## The Ability to Read Mathematical Evidence

According Sumarmo [3], the ability to read mathematical evidence is the ability to find the truth or error of a proof and the ability to give reasons for every step of proof. In reading mathematical proofs, students must be able to express the mathematical ideas of a text, both in oral and written form in their own language. Thus, a reader not only pronounces the text, but expresses the meaning contained in the text. In other words, reading mathematical proofs is a set of skills to construct the essence of information from a text [4].

## Probability Theory

Probability Theory is a science that discusses how nature, postulate and statistical formulas are derived, as well as how to create mathematical theoretical models [5]. Probability Theory is a theoretical statistics course

given to students of Mathematics and Mathematics Education. This course is given to 3rd graders. In order to study this course, basic calculus and statistics skills are required as a prerequisite course [6].

## METHOD

The method used in this research is qualitative method. The subjects in this study are students of the Statistics Study Program Hasanuddin University Makassar who enrolled in the subject of Probabily Theory. The subjects were selected according to the purpose of this research while the data analysis was conducted by using triangulation method [7].

## RESULTS AND DISCUSSION

### Analysis of Documentation Results

The following is given the problem and the completion of the test, as well as some test results of students related to the ability to read mathematical proofs in the Course Probability theory

**Problem Test**
Check the correctness of each evidentiary step below and write down the concepts that it uses.

**Statement:**
If $X$ and $Y$ is two normal distribution which has a combined moment generating function in the form of :

$$M\left(t_1,t_2\right)=\exp\left[\mu_1 t_1+\mu_2 t_2+\tfrac{1}{2}\left(\sigma_1{}^2 t_1^2+2\rho\sigma_1\sigma_2 t_1 t_2+\sigma_2{}^2 t_2^2\right)\right]$$

Then, $\operatorname{cov}\left(X,Y\right)=\rho\sigma_1\sigma_2$.

**Proof :**
Will be searched first value $E\left(X\right)$ dan $E\left(Y\right)$

- $$M\left(t_1,0\right)=\exp\left[\mu_1 t_1+\tfrac{1}{2}\sigma_1{}^2 t_1^2\right] \tag{1}$$

$$\frac{\partial M\left(t_1,0\right)}{\partial t_1}=\left(\mu_1+\sigma_1{}^2 t_1\right)\exp\left[\mu_1 t_1+\tfrac{1}{2}\sigma_1{}^2 t_1^2\right] \tag{2}$$

So,

$$E\left(X\right)=\left.\frac{\partial M\left(t_1,0\right)}{\partial t_1}\right]_{t_1=0}=\mu_1 \tag{3}$$

- $$M\left(0,t_2\right)=\exp\left[\mu_2 t_2+\tfrac{1}{2}\sigma_2{}^2 t_2^2\right] \tag{4}$$

$$\frac{\partial M\left(0,t_2\right)}{\partial t_2}=\left(\mu_2+\sigma_2{}^2 t_2\right)\exp\left[\mu_2 t_2+\tfrac{1}{2}\sigma_2{}^2 t_2^2\right] \tag{5}$$

So,

$$E\left(Y\right)=\left.\frac{\partial M\left(0,t_2\right)}{\partial t_2}\right]_{t_2=0}=\mu_2 \tag{6}$$

Next, the value will be searched $E(XY)$

$$\frac{\partial M(t_1,t_2)}{\partial t_1} = \left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1{}^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2{}^2 t_2^2\right)\right]\right\}\left\{\mu_1 + \tfrac{1}{2}\left(2\sigma_1{}^2 t_1 + 2\rho\sigma_1\sigma_2 t_2\right)\right\} \quad (7)$$

$$\frac{\partial^2 M(t_1,t_2)}{\partial t_1 \partial t_2} = \left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1{}^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2{}^2 t_2^2\right)\right]\right\}(\rho\sigma_1\sigma_2) + \left\{\mu_1 + \tfrac{1}{2}\left(2\sigma_1{}^2 + 2\rho\sigma_1\sigma_2 t_2\right)\right\}$$
$$\left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1{}^2 t_1^2 + 2\rho\sigma_1 t_1 t_2 + \sigma_2{}^2 t_2^2\right)\right]\right\}\left\{\mu_2 + \tfrac{1}{2}\left(2\rho\sigma_1\sigma_2 + \sigma_2{}^2 t_2\right)\right\} \quad (8)$$

So,

$$E(XY) = \left.\frac{\partial^2 M(t_1,t_2)}{\partial t_1 \partial t_2}\right]_{t_1=t_2=0} = \rho\sigma_1\sigma_2 + \mu_1\mu_2 \quad (9)$$

As a result,

$$\operatorname{cov}(X,Y) = E(XY) - E(X)E(Y) = \rho\sigma_1\sigma_2 \quad (10)$$

Proven.

**Solution :**

**Statement:**

If $X$ dan $Y$ are two normal distribution which has a combined moment generating function in the form of :

$$M(t_1,t_2) = \exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1{}^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2{}^2 t_2^2\right)\right]$$

Then, $\operatorname{cov}(X,Y) = \rho\sigma_1\sigma_2$.

**Proof :**

Will be searched first value $E(X)$ dan $E(Y)$

$$M(t_1,0) = \exp\left[\mu_1 t_1 + \tfrac{1}{2}\sigma_1{}^2 t_1^2\right] \quad \ldots (1)$$

**Explanation :**
The concept used is substitution $t_2 = 0$. The substitution process proved to be true.

$$\frac{\partial M(t_1,0)}{\partial t_1} = \left(\mu_1 + \sigma_1{}^2 t_1\right)\exp\left[\mu_1 t_1 + \tfrac{1}{2}\sigma_1{}^2 t_1^2\right] \quad \ldots (2)$$

**Explanation**
The concept used is a partial derivative of $t_1$. Partial derivative process proves true.

so, $$E(X) = \left.\frac{\partial M(t_1,0)}{\partial t_1}\right]_{t_1=0} = \mu_1 \quad \ldots (3)$$

**Explanation :**
The concept used is the proposed degradation of moments of the moment generating function by substitution $t_1 = 0$. The decomposition process proved to be true.

$$M\left(0,t_2\right)=\exp\left[\mu_2 t_2+\tfrac{1}{2}\sigma_2{}^2 t_2^2\right] \qquad \ldots\ (4)$$

**Explanation :**
The concept used is substitution $t_1=0.$ The substitution process proved to be true.

$$\frac{\partial M\left(0,t_2\right)}{\partial t_2}=\left(\mu_2+\sigma_2{}^2 t_2\right)\exp\left[\mu_2 t_2+\tfrac{1}{2}\sigma_2{}^2 t_2^2\right] \qquad \ldots\ (5)$$

**Explanation :**
The concept used is a partial derivative of $t_2$. Partial derivative process proves true.

So, $$E\left(Y\right)=\left.\frac{\partial M\left(0,t_2\right)}{\partial t_2}\right]_{t_2=0}=\mu_2 \qquad \ldots\ (6)$$

**Explanation :**
The concept used is the proposed degradation of moments of the moment generating function by substitution $t_2=0.$ The decomposition process proved to be true.

Next, the value will be searched $E\left(XY\right)$

$$\frac{\partial M\left(t_1,t_2\right)}{\partial t_1}=\left\{\exp\left[\mu_1 t_1+\mu_2 t_2+\tfrac{1}{2}\left(\sigma_1{}^2 t_1^2+2\rho\sigma_1\sigma_2 t_1 t_2+\sigma_2{}^2 t_2^2\right)\right]\right\}\left\{\mu_1+\tfrac{1}{2}\left(2\sigma_1{}^2 t_1+2\rho\sigma_1\sigma_2 t_2\right)\right\}\ldots(7)$$

**Explanation :**
The concept used is a partial derivative of $t_1$.
Because, $M\left(t_1,t_2\right)=\exp\left[\mu_1 t_1+\mu_2 t_2+\tfrac{1}{2}\left(\sigma_1{}^2 t_1^2+2\rho\sigma_1\sigma_2 t_1 t_2+\sigma_2{}^2 t_2^2\right)\right]$,
Using the concept :

$$f\left(g\left(x\right)\right)=e^{g(x)}\Rightarrow f'\left(g\left(x\right)\right)=e^{g(x)}g'\left(x\right) \text{ then :}$$

$$\frac{\partial M\left(t_1,t_2\right)}{\partial t_1}=\left\{\exp\left[\mu_1 t_1+\mu_2 t_2+\tfrac{1}{2}\left(\sigma_1{}^2 t_1^2+2\rho\sigma_1\sigma_2 t_1 t_2+\sigma_2{}^2 t_2^2\right)\right]\right\}\left\{\mu_1+\tfrac{1}{2}\left(2\sigma_1{}^2 t_1+2\rho\sigma_1\sigma_2 t_2\right)\right\}$$

The Partial Derivative Process proves true.

$$\frac{\partial^2 M\left(t_1,t_2\right)}{\partial t_1\partial t_2}=\left\{\exp\left[\mu_1 t_1+\mu_2 t_2+\tfrac{1}{2}\left(\sigma_1{}^2 t_1^2+2\rho\sigma_1\sigma_2 t_1 t_2+\sigma_2{}^2 t_2^2\right)\right]\right\}\left(\rho\sigma_1\sigma_2\right)+\left\{\mu_1+\tfrac{1}{2}\left(2\sigma_1{}^2+2\rho\sigma_1\sigma_2 t_2\right)\right\}$$
$$\left\{\exp\left[\mu_1 t_1+\mu_2 t_2+\tfrac{1}{2}\left(\sigma_1{}^2 t_1^2+2\rho\sigma_1 t_1 t_2+\sigma_2{}^2 t_2^2\right)\right]\right\}\left\{\mu_2+\tfrac{1}{2}\left(2\rho\sigma_1\sigma_2+\sigma_2{}^2 t_2\right)\right\} \qquad \ldots\ (8)$$

**Explanation :**

The concept used is a second partial derivative against $t_2$.

Because $\dfrac{\partial M(t_1,t_2)}{\partial t_1} = \left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2\right)\right]\right\}\left\{\mu_1 + \tfrac{1}{2}\left(2\sigma_1^2 t_1 + 2\rho\sigma_1\sigma_2 t_2\right)\right\}$,

Using the concept :

$$f(g(x)) = e^{g(x)} \Rightarrow f'(g(x)) = e^{g(x)} g'(x) \text{ and}$$

$$f(x) = u(x)v(x) \Rightarrow f'(x) = u'(x)v(x) + u(x)v'(x) \text{ then :}$$

$$\frac{\partial^2 M(t_1,t_2)}{\partial t_1 \partial t_2} = \left\{\mu_1 + \tfrac{1}{2}\left(2\sigma_1^2 t_1 + 2\rho\sigma_1\sigma_2 t_2\right)\right\}\left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2\right)\right]\right\}$$
$$\left\{\mu_2 + \tfrac{1}{2}\left(2\rho\sigma_1\sigma_2 t_1 + 2\sigma_2^2 t_2\right)\right\} + \left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2\right)\right]\right\}(\rho\sigma_1\sigma_2)$$

$$\frac{\partial^2 M(t_1,t_2)}{\partial t_1 \partial t_2} = \left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2\right)\right]\right\}(\rho\sigma_1\sigma_2) + \left\{\mu_1 + \tfrac{1}{2}\left(2\sigma_1^2 t_1 + 2\rho\sigma_1\sigma_2 t_2\right)\right\}$$
$$\left\{\exp\left[\mu_1 t_1 + \mu_2 t_2 + \tfrac{1}{2}\left(\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2\right)\right]\right\}\left\{\mu_2 + \tfrac{1}{2}\left(2\rho\sigma_1\sigma_2 t_1 + 2\sigma_2^2 t_2\right)\right\}$$

The Partial Derivative Process proves to be wrong.

So, $E(XY) = \left.\dfrac{\partial^2 M(t_1,t_2)}{\partial t_1 \partial t_2}\right]_{t_1=t_2=0} = \rho\sigma_1\sigma_2 + \mu_1\mu_2 \quad \ldots (9)$

**Explanation :**

Using the concept of the momentary degradation of the moment generating function, ie :

$E(XY) = \dfrac{\partial^2 M(t_1,t_2)}{\partial t_1 \partial t_2}$ untuk $t_1 = t_2 = 0$, then :

$$\left.\frac{\partial^2 M(t_1,t_2)}{\partial t_1 \partial t_2}\right]_{t_1=t_2=0} = \left\{\exp\left[\mu_1(0) + \mu_2(0) + \tfrac{1}{2}\left(\sigma_1^2(0)^2 + 2\rho\sigma_1\sigma_2(0)(0) + \sigma_2^2(0)^2\right)\right]\right\}(\rho\sigma_1\sigma_2) + \left\{\mu_1 + \tfrac{1}{2}\left(2\sigma_1^2(0) + 2\rho\sigma_1\sigma_2(0)\right)\right\}$$
$$\left\{\exp\left[\mu_1(0) + \mu_2(0) + \tfrac{1}{2}\left(\sigma_1^2(0)^2 + 2\rho\sigma_1\sigma_2(0)(0) + \sigma_2^2(0)^2\right)\right]\right\}\left\{\mu_2 + \tfrac{1}{2}\left(2\rho\sigma_1\sigma_2(0) + 2\sigma_2^2(0)\right)\right\}$$
$$= \{\exp[0]\}(\rho\sigma_1\sigma_2) + \{\mu_1 + 0\}\{\exp[0]\}\{\mu_2 + 0\}$$
$$= (1)(\rho\sigma_1\sigma_2) + \mu_1(1)\mu_2$$
$$= \rho\sigma_1\sigma_2 + \mu_1\mu_2$$

The decomposition process proved true.

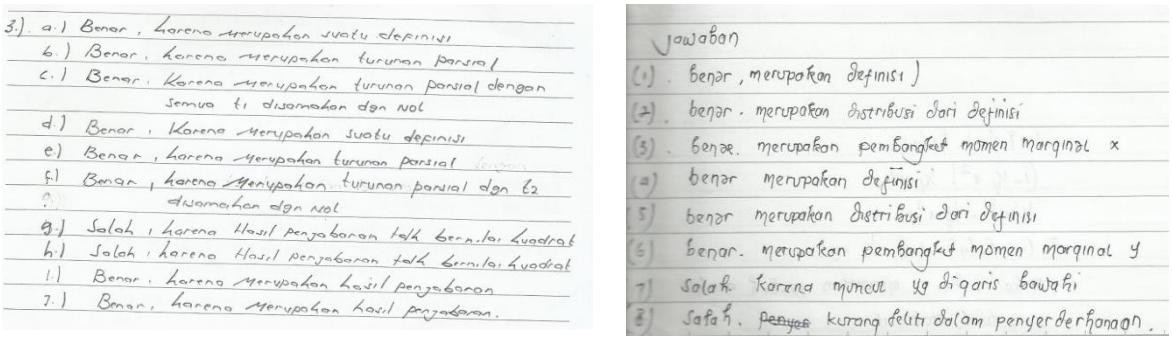
as a result, $\operatorname{cov}(X,Y) = E(XY) - E(X)E(Y) = \rho\sigma_1\sigma_2 \quad \ldots (10)$

**Explanation :**

By using the concept of proposition of the formulation of general covariance, then

$$\operatorname{cov}(X,Y) = E(XY) - E(X)E(Y)$$
$$= (\rho\sigma_1\sigma_2 + \mu_1\mu_2) - \mu_1\mu_2 \text{ The decomposition process proved true.}$$
$$= \rho\sigma_1\sigma_2$$

After elaborated on the subject matter probability tests related to the ability to read mathematical proofs and their completion, the next is to analyze the test answers from some students. The description is as follows.




(a) (b)

**FIGURE 1.** Answers Test of Reading a Proof of Student's 1 (a) and Student 2 (b)

For the test answer of Student 1, it can be seen that the verification of the verification on all steps along with the writing of the concept used is not complete, there are even some steps that are not appropriate and not completed. For the 1st, 2nd, 3rd, 4th, 5th, 6th and 8th step, the verification of correctness of proof is correct, but the concept writing used is incomplete and there are some less precise ones. The technical explanation (mathematically) in the steps is not available and the statistical explanation is only a few steps closer to the right (steps 3 and 6), while the other steps are considered to be less precise (steps 1, 2, 4, 5, And 8). For the 7th step, the verification of the verification of the verification is not accurate and the concept of writing used is not correct. For the 8th step, even though the verification results of the verification are correct, but no actual results are written. This is because the result obtained in the step is wrong. For steps 9 and 10, student 1 does not verify the verification and conceptual writing used, so the answer does not exist.

For the answer of Student 2 test is almost the same case with Student 1, it is seen that the examination of the verification of evidences on all the steps along with the writing of the concept used is incomplete, there are even some inappropriate steps. In Student 2's answer, there is an error writing the step item that should have used the number, but using the letters. For the 1st, 2nd, 3rd, 4th, 5th, 6th, 8th, 9th and 10th steps, the verification of correctness of proof is correct, but the concept writing used is incomplete and there are some less precise ones. Some steps are only elaborated technically (math) only without statistical explanation. For the 8th step, even though the verification results of the verification are correct, but no actual results are written. This is because the result obtained in the step is wrong. For the 7th step, the verification of the verification of the verification is not quite accurate as well as the answer of the Student 1 test and the use of the concept is not appropriate.

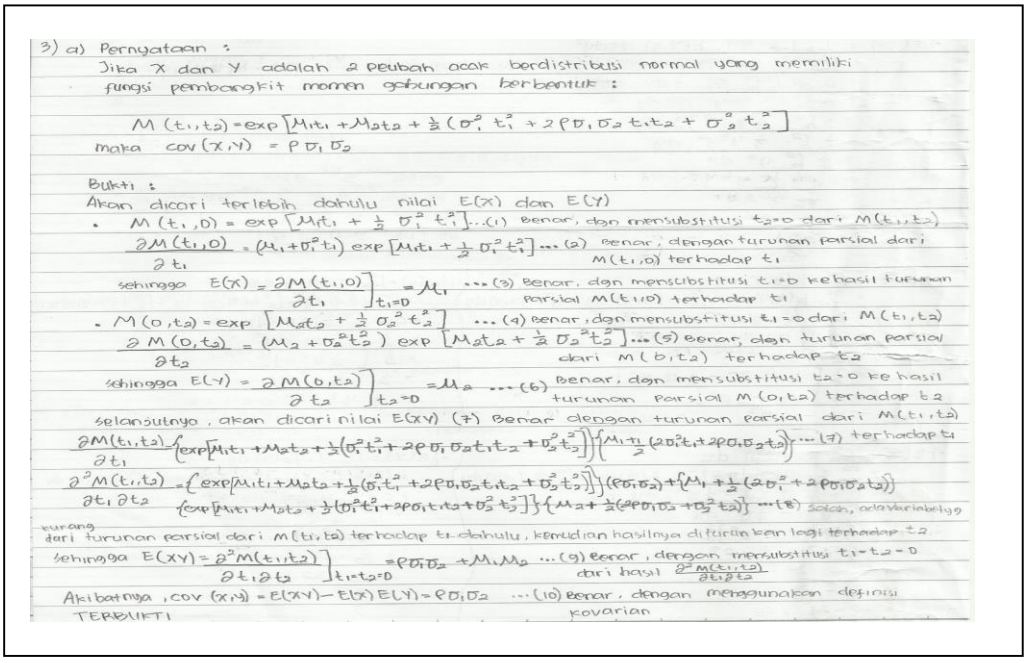



**FIGURE 1.** Answers Test of Reading a Proof of Student's 3

For the answer of Student 3 test, it can be seen that the verification of the verification in the 1st, 2nd, 4th, and 5th step along with the writing of the concept used is complete and correct; While for the 3rd, 6th, 7th, 8th, 9th and 10th steps, the verification of the verification is correct, but the concept writing used is incomplete. Student 3 only elaborates technically only (mathematically), not equipped with the statistics concept. For the 8th step, even though the verification results of the verification are correct, but no actual results are written. This is because the result obtained in the step is wrong. Overall, the answer of Student 3 test is better than the test answer of Students 1 and 2.

## Analysis of Observation and Interview Results

Based on the results of observation and interviews on students who enrolled Mathematics Statistics subject at Hasanuddin University Makassar related to the ability to read mathematical evidence, it appears that some students still have difficulty in understanding the flow of a proof, including difficulty in checking the verification of verification and Writing concept used. One of the factors that led to this is the weakness of the concept of prerequisite courses (Calculus and Basic Statistics).

For Calculus Subject, some students recognize that they are still weak in the partial derivation concept, let alone in solving problems related to more complex partial derivatives. They are sometimes confused when deriving a variable partially if the number of variables in the function is quite large. They are still weak in applying derivative techniques in solving derivative problems, such as the concept of derivatives on multiplication and division of two or more functions, and the concept of chain rules in derivatives.

For Basic Statistics Subject, some students acknowledge that they are still weak in the concept of expectations, especially regarding the moment. They are often confused between concepts by definition and proposition; As well as the lack of understanding of students about the link between the concept of expectations, average, variance, and covariance.

In order to strengthen the analysis, researchers also observed and interviewed one of the lecturers of Mathematics Statistics Course. The lecturer acknowledges that the learning model used is still conventional which emphasizes the lecturer's activity in explaining the material, question and answer, and the deepening of the material through workmanship and discussion of the exercise. Lecturers also admitted that the mastery of some students on prerequisite subjects is still weak, so often lecturers have to review the material. By reviewing the prerequisite materials, students can better understand Mathematical Statistics materials which often require the ability to read mathematical proofs.

Based on the above description shows that the ability to read mathematical proof of students in the Mathematical Statistics Course in general is still not good.

## CONCLUSION AND RECOMMENDATION

The ability to read mathematical proof of students on probability theory course is still not good enough. This is evident from some students still have difficulty in understanding the flow of a proof, including the difficulty in checking the verification of authentication and the writing of the concept used. Through this research, the lecturer of probability theory subject is recommended to try to apply a suitable learning model to improve students' ability in reading mathematical proof.

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