A Modeling Study Of Viscoelastic Material of Wool-Lycra 36 Tex Yarn

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Abstract. This research examines the mathematical model of viscoelastic material with a classical mechanics approach. A mathematics equation of Wool-Lycra yarn having 36 (in unit tex or g/km) of yarn count number has been expressed using approximation of springs and dashpots. In this study, we found that this model can be applied to determine the viscoelastic material of yarn.

1. Introduction

The application of physics, especially classical mechanics in the field of textiles has been applied and done by several researchers [1-11]. According to several studies, the mechanical properties of textile yarn materials depend on a function of time due to the presence of viscoelastic properties that combine the presence of a natural viscous and also elastic properties, namely stress relaxation and creep behavior. Both of these properties are important properties to explain the properties of a yarn material viscoelastic [8]. The Maxwell and Voigt-Kelvin models are examples of models consisting of a single spring and a dashpot arranged in series or parallel. The weakness of the two models is the inaccuracy of predictive results (modeling) to the experimental validation to explain the properties of viscoelastic material, especially in cases of stress relaxation and creep behavior. Textile polymeric materials usually exhibit a viscoelastic property for a certain time with the same stress level. Linear viscoelastic properties are usually modeled using a physics model that is a spring that follows the Hook law formulation of a material that undergoes elastic deformation and also a dashpot which follows Newton's fluid formulation with a stress level proportional to strain. The Maxwell model has a spring and dashpot arranged in series as in Figure 1. The form of Maxwell modeling has a form of modeling where stress has the same value for the whole model, while the strain size is the sum of two elements.

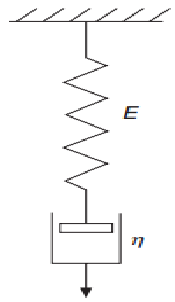

Figure 1. Form of Maxwell Modeling

In the series circuits of Maxwell model, the force is constant, so stress is constant, there is a change in strain as follows:

$$\sum \frac{d\varepsilon}{dt} = 0 \quad (1)$$

$$\frac{d\varepsilon_s}{dt} = \frac{1}{E}\frac{d\sigma_e}{dt} + \frac{\sigma_\eta}{\eta} \quad (2)$$

Because $\sigma_e = \sigma_\eta = \sigma$, then

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \quad (3)$$

For stress relaxation conditions, the strain is constant, so

$$0 = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \quad (4)$$

$$\frac{1}{E}\frac{d\sigma}{dt} = -\frac{\sigma}{\eta} \quad (5)$$

So

$$\frac{d\sigma}{\sigma} = -\frac{E}{\eta}dt \quad (6)$$

$$\int \frac{d\sigma}{\sigma} = -\int \frac{E}{\eta}dt \quad (7)$$

Which results the equation

$$\sigma = \sigma_o e^{-\frac{E}{\eta}t} \quad (8)$$

$$\sigma = \sigma_o e^{-\frac{E}{\eta}t} = \sigma_o e^{-at} \quad (9)$$

For creep behavior conditions, stress is constant

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} \quad (10)$$

$$\varepsilon = \frac{\sigma}{\eta}t \quad (11)$$

$$\varepsilon = \frac{\sigma}{\eta}t = m_{grad}t \quad (12)$$

Maxwell modeling on the equation (3) shows poor results in describing viscoelastic material properties for creep behavior in constant stress values. In the case of the Voigt-Kelvin model (Figure 2) it can be described as follows: in parallel circuits, the magnitude of the strain is constant, so there is a change in stress, namely:

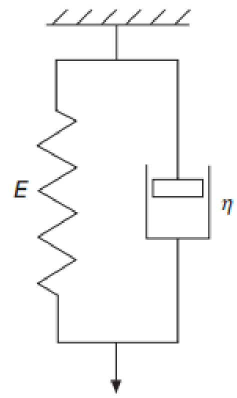

Figure 2. The Voigt-Kelvin model

$$\sum \sigma = 0 \tag{13}$$

$$\sigma = E\varepsilon_e + \eta \frac{d\varepsilon_\eta}{dt} \tag{14}$$

For the case of stress relaxation (constant strain), then

$$\sigma = E\varepsilon \tag{15}$$

The magnitude of σ with time will be constant. For the case of creeps, stress is constant, so

$$\sigma - E\varepsilon = \eta \frac{d\varepsilon}{dt} \tag{16}$$

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta}\left(1 - \frac{E}{\sigma}\varepsilon\right) \tag{17}$$

The result of the completion of the above equation is

$$\frac{d\varepsilon}{\left(1 - \frac{E}{\sigma}\varepsilon\right)} = \frac{\sigma}{\eta} dt \tag{18}$$

If $1 - \frac{E}{\sigma}\varepsilon = u$, then $du = -\frac{E}{\sigma} d\varepsilon$, so

$$-\frac{\sigma}{E}\frac{du}{u} = \frac{\sigma}{\eta} dt \tag{19}$$

$$\int \frac{du}{u} = -\int \frac{E}{\eta} dt \tag{20}$$

$$u = u_o e^{-\frac{E}{\eta}t} \tag{21}$$

$$1 - \frac{E}{\sigma}\varepsilon = e^{-\frac{E}{\eta}t} \tag{22}$$

$$\sigma - E\varepsilon = \sigma e^{-\frac{E}{\eta}t} \tag{23}$$

$$\varepsilon = \frac{\sigma}{E}\left(1 - e^{-\frac{E}{\eta}t}\right) = a(1 - e^{-bt}) \tag{24}$$

The form of Voigt-Kelvin modeling on the equation (14) cannot explain the stress relaxation conditions in constant strain conditions.

2. Modeling Viscoelastic Theory Using Classical Mechanics

In this modeling viscoelastic material is modeled using three springs system and one dashpot arranged as shown in Figure 3.

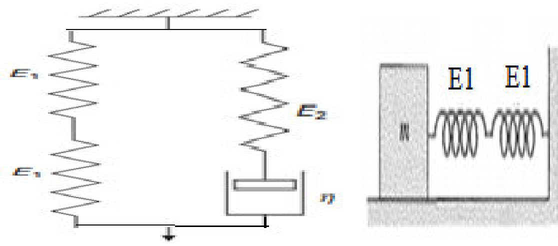

Figure 3. Form of Classical Mechanics Modeling of Viscoelastic Materials

In this modeling the values of spring constants are E_1 and E_2 and the viscosity coefficient is η. The description of this system can be solved using Newton's 3rd law, so if it is reviewed in mass m, it is found that

$$\sum F = 0 \tag{25}$$

$$F_{ext} - F_{left\ spring} = 0 \tag{26}$$

$$F - E\Delta x_l = 0 \tag{27}$$

$$F = E\Delta x_l \tag{28}$$

Reviewed at the connection points of two springs, namely

$$\sum F = 0 \tag{29}$$

$$F_{right\ spring} - F_{left\ spring} = 0 \tag{30}$$

$$F_{right\ spring} = F_{left\ spring} \tag{31}$$

$$E\Delta x_r = E\Delta x_l \tag{32}$$

$$\Delta x_r = \Delta x_l \tag{33}$$

So that the amount of total shift is

$$\Delta \text{x} = \Delta x_r + \Delta x_l = 2\Delta x_l \tag{34}$$

To find the effective combined spring coefficient, then

$$\sum F = 0 \tag{35}$$

$$F_{ext} - F_{mixed\ spring} = 0 \tag{36}$$

$$F = F_{mixed\ spring} \tag{37}$$

$$F = E_{eff}\Delta \text{x} \tag{38}$$

Substitute $F = E\Delta x_l$ to $F = E_{eff}\Delta \text{x}$, then

The application of modeling and prediction of the study of viscoelastic properties in the textile field is a study to determine the properties of viscoelastic material in fibers, yarns or fabrics, for example in a Wool-

Lycra yarn which has a yarn count of 36 tex or 36 g / km. The data results for the stress relaxation process (constant strain) are as follows (Table 1).

Table 1. *Stress relaxation*

No	Time (10^3detik)	*stress* (cN/ tex)
1	0	6.1
2	0.23	5.3
3	1.2	5.2
4	1.3	5.1
5	3.3	4.8

$$E\Delta x_l = E_{eff}\Delta \mathrm{x} \tag{39}$$

$$E_{eff} = \frac{E\Delta x_l}{\Delta \mathrm{x}} = \frac{E\Delta x_l}{2\Delta x_l} = \frac{E}{2} = \frac{E^2}{2E} \tag{40}$$

$$\frac{1}{E_{eff}} = \frac{2E}{E^2} = \frac{1}{E} + \frac{1}{E} \tag{41}$$

So that the general formula for springs arranged in series is obtained

$$\frac{1}{E_{eff}} = \frac{1}{E_1} + \frac{1}{E_1} = \frac{1}{E} + \frac{1}{E} = \frac{2}{E} \tag{42}$$

The description form of the equation for the series circuit in Figure 3 can be solved first. Because of series circuit, the stress value is constant, so $\sigma_e = \sigma_v = \sigma_2$ and $\varepsilon_s = \varepsilon_e + \varepsilon_v$

$$\sum\left(\frac{d\varepsilon}{dt}\right) = 0 \tag{43}$$

$$\frac{d\varepsilon_s}{dt} = \frac{d\varepsilon_e}{dt} + \frac{d\varepsilon_v}{dt} \tag{44}$$

$$\frac{d\varepsilon_s}{dt} = \frac{1}{E_2}\frac{d\sigma_e}{dt} + \frac{\sigma_v}{\eta} = \frac{1}{E_2}\frac{d\sigma_2}{dt} + \frac{\sigma_2}{\eta} \tag{45}$$

$$\frac{d\sigma_2}{dt} = E_2\frac{d\varepsilon_s}{dt} - E_2\frac{\sigma_2}{\eta} \tag{46}$$

For parallel circuits, the value of the strain is equal, so $\varepsilon_s = \varepsilon_1 = \varepsilon$

$$\sum \sigma = 0 \tag{47}$$

$$\sigma = \sigma_1 + \sigma_s \tag{48}$$

If it is reduced once to time, it is obtained

$$\sum \frac{d\sigma}{dt} = 0 \tag{49}$$

$$\frac{d\sigma}{dt} = \frac{d\sigma_1}{dt} + \frac{d\sigma_s}{dt} \tag{50}$$

$$\frac{d\sigma}{dt} = E_{eff}\frac{d\varepsilon_1}{dt} + E_2\frac{d\varepsilon_s}{dt} - E_2\frac{\sigma_2}{\eta} \quad (51)$$

$$\frac{d\sigma}{dt} = (E_{eff} + E_2)\frac{d\varepsilon}{dt} - E_2\frac{\sigma_2}{\eta} \quad (52)$$

$$\frac{d\sigma}{dt} = (E_{eff} + E_2)\frac{d\varepsilon}{dt} - E_2\frac{(\sigma - \sigma_1)}{\eta} \quad (53)$$

$$\frac{d\sigma}{dt} = \left(E_{eff} + E_2\right)\frac{d\varepsilon}{dt} - \frac{E_2}{\eta}\sigma + \frac{E_2}{\eta}\sigma_1 = (E_{eff} + E_2)\frac{d\varepsilon}{dt} - \frac{E_2}{\eta}\sigma + \frac{E_2}{\eta}E_{eff}\varepsilon \quad (54)$$

$$\frac{d\sigma}{dt} = (E_{eff} + E_2)\frac{d\varepsilon}{dt} - \frac{E_2}{\eta}\sigma + \frac{E_2}{\eta}E_{eff}\varepsilon \quad (55)$$

$$\frac{d\sigma}{dt} + \frac{E_2}{\eta}\sigma = (E_{eff} + E_2)\frac{d\varepsilon}{dt} + \frac{E_2 E_{eff}}{\eta}\varepsilon \quad (56)$$

$$\frac{\eta}{E_2}\frac{d\sigma}{dt} + \sigma = \frac{(E_{eff} + E_2)\eta}{E_2}\frac{d\varepsilon}{dt} + E_{eff}\varepsilon \quad (57)$$

$$\frac{\eta}{E}\frac{d\sigma}{dt} + \sigma = E_{eff}\varepsilon + \eta\frac{(E_{eff} + E_2)}{E_2}\frac{d\varepsilon}{dt} \quad (58)$$

For same value of E_{eff} and E_2, then

$$\frac{\eta}{E}\frac{d\sigma}{dt} + \sigma = 2\eta\frac{d\varepsilon}{dt} + E\varepsilon \quad (59)$$

The result of completion for stress relaxation or constant strain conditions is

$$\frac{\eta}{E}\frac{d\sigma}{dt} + \sigma = E\varepsilon \quad (60)$$

with stress amount as follows

$$\frac{d\sigma}{dt} + \frac{E}{\eta}\sigma = \frac{E^2}{\eta}\varepsilon \quad (61)$$

$$\sigma = \sigma_o e^{-\frac{E}{\eta}t} + E\varepsilon = 1.2e^{-2.1t} + 4.78 \quad (62)$$

Using the curve fitting method, a modeling curve of equation (62) can be presented in Figure 4 and Table 2.

Table 2. Fitting curve method equation (62)

t 10^3s	σ experiment	σ_o	$\frac{E}{\eta}$	$E\varepsilon$	σ theory
0	6.1	1.1	2.1	4.78	6.08
0.23	5.3	1.1	2.1	4.78	5.51
1.2	5.2	1.1	2.1	4.78	5.18
1.3	5.1	1.1	2.1	4.78	4.95
3.3	4.8	1.1	2.1	4.78	4.78

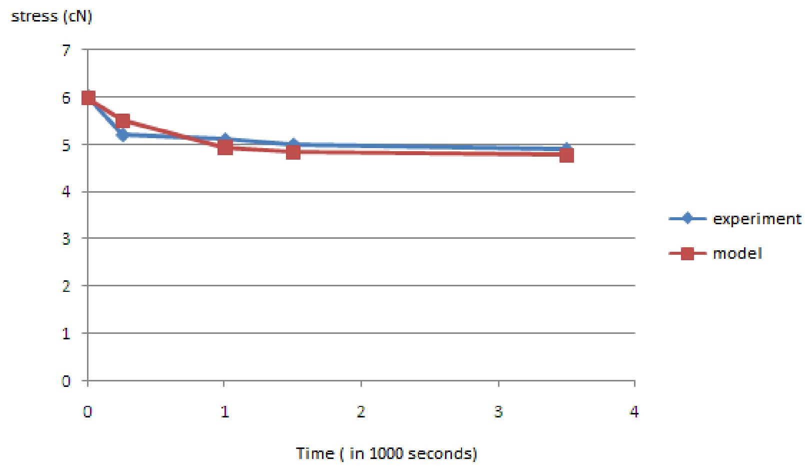

Figure 4. Stress relaxation conditions equation (62) stress curve (cN) for time (10^3 seconds)

A linear viscoelastic mechanical property is usually modeled using a physical model, namely a spring that follows the formulation of the Hook law from a material that undergoes elastic deformation and also a dashpot that follows Newton's fluid formula. Based on the results of this study, it was found that the form of modeling using Equation (62) can be used to predict stress relaxation conditions, modeling and predicting viscoelastic properties in the textile field to determine the viscoelastic material properties of a Wool-Lycra yarn which has a yarn count of 36 tex or 36 g / km.

In this study the results of R^2 values ranged from 0.81 which shows the closeness between the experimental results against the predictions of the theory. The characteristic of stress relaxation is an important characteristic for explaining the properties of a viscoelastic yarn material. Maxwell's model and Voigt-Kelvin model which consists of a single spring and a dashpot arranged in series or parallel show results that are less representative of experimental data. The weakness of the two models is the inaccuracy between the predictive results (modeling) of experimental validation. In this study, the Wool-Lycra yarn which has a yarn count of of 36 tex or 36 g / km has been studied is one example of viscoelastic material. The motion equation for modeling in Figure 3 can be written as follows:

$$\frac{\eta}{E_2}\frac{d\sigma}{dt} + \sigma = \frac{(E_{eff} + E_2)\eta}{E_2}\frac{d\varepsilon}{dt} + E_{eff}\varepsilon \qquad (63)$$

Based on the results of modeling and experimental validation, it was found that $\frac{E}{\eta} = 2.1$ Equation (62) shows that the amount of stress to time for a constant force will have an inverse relationship with the decaying exponential curve.

3. Conclusions

Textile yarn, such as Wool-Lycra yarn does not only have elastic or viscous properties, but a combination of these two properties. For example, if a constant pull or stress is applied to a thread, there will be a short strain / strain then a strain gradually occurs as in the description of viscoelastic material modeling in this study. Wool-Lycra yarn material that has viscous and elastic is called viscoelastic material. In this study, we have discussed several modeling of viscoelastic material in yarns. Based on the results of this study, both theoretically and experimental validation, it was found that there was a match between the results of theoretical predictions and experimental validation for several models.

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